15-826: Multimedia (Databases) and Data Mining

Lecture#5: Multi-key and

Spatial Access Methods – II – z-ordering

C. Faloutsos

Must-read material

- MM-Textbook, Chapter 5.1
- Ramakrinshan+Gehrke, Chapter 28.4
- J. Orenstein, *Spatial Query Processing in an Object-Oriented Database System*, Proc. ACM SIGMOD, May, 1986, pp. 326-336, Washington D.C.

Outline

Goal: ‘Find similar / interesting things’

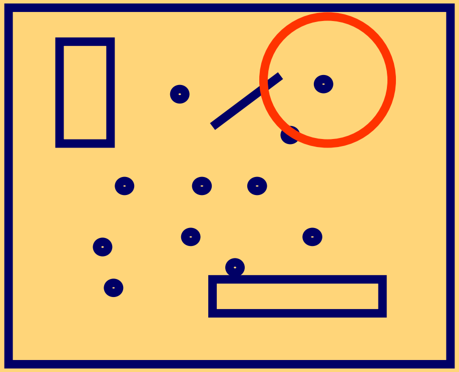
- Intro to DB
- ➡ Indexing - similarity search
- Data Mining

Indexing - Detailed outline

- primary key indexing
- secondary key / multi-key indexing
- spatial access methods
  - problem dfn
  - z-ordering
  - R-trees
  - ...
- text
- ...

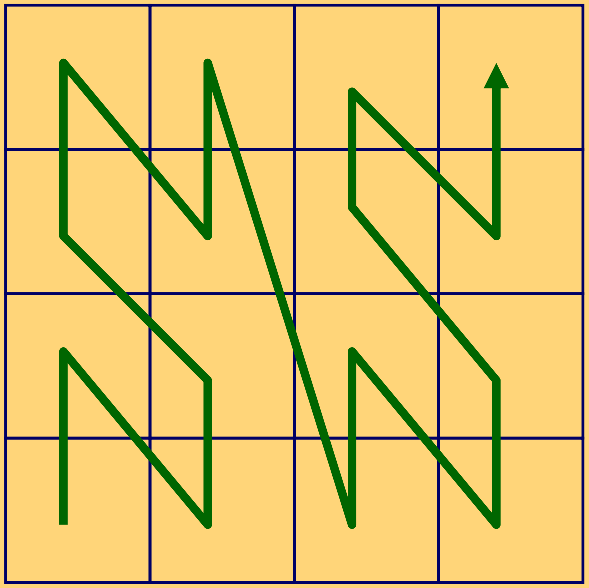
Spatial Access Methods - problem

- Given a collection of geometric objects (points, lines, polygons, ...)
- Find cities within 100mi from Pittsburgh



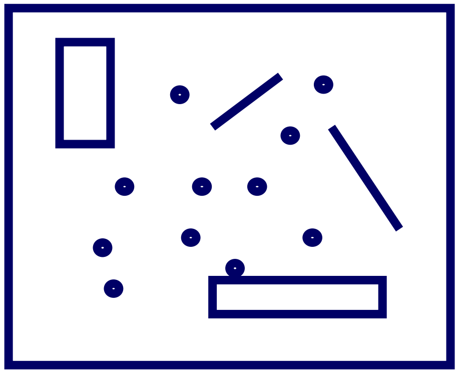
Solution#1: z-ordering

A: z-ordering/bit-shuffling/linear-quadtrees



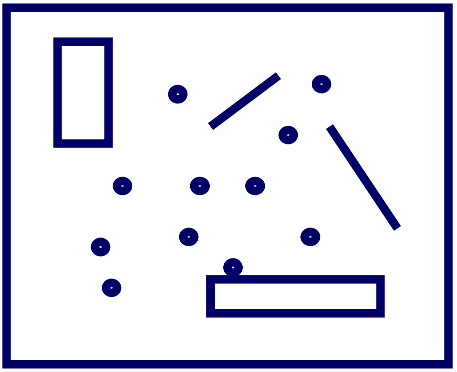
Spatial Access Methods - problem

- Given a collection of geometric objects (points, lines, polygons, ...)
- organize them on disk, to answer spatial queries (like??)



Spatial Access Methods - problem

- Given a collection of geometric objects (points, lines, polygons, ...)
- organize them on disk, to answer
  - – point queries
  - – range queries
  - – k-nn queries
  - – spatial joins (‘all pairs’ queries)



Spatial Access Methods - problem

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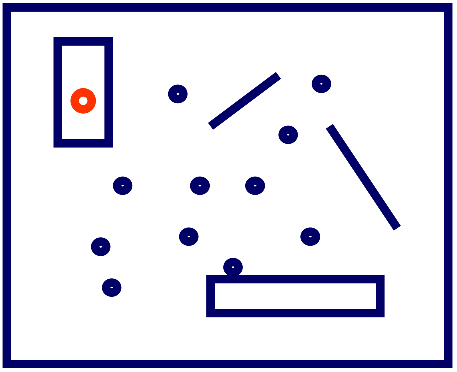

Spatial Access Methods - problem

- Given a collection of geometric objects (points, lines, polygons, ...)
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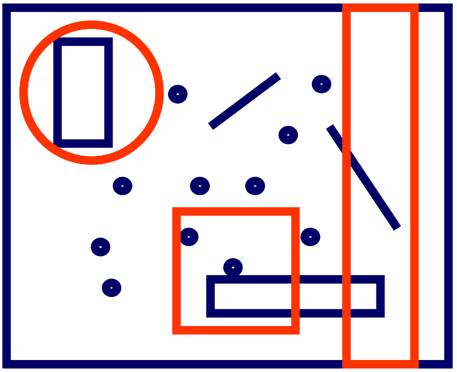

Spatial Access Methods - problem

- Given a collection of geometric objects (points, lines, polygons, ...)
- organize them on disk, to answer
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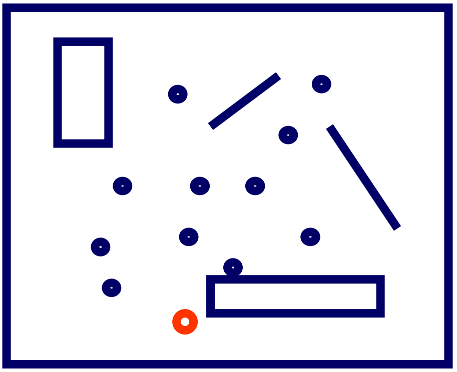
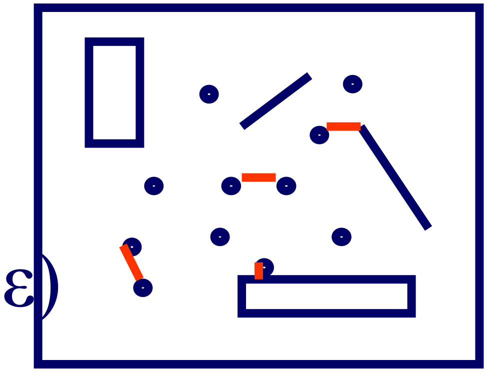


Spatial Access Methods - problem

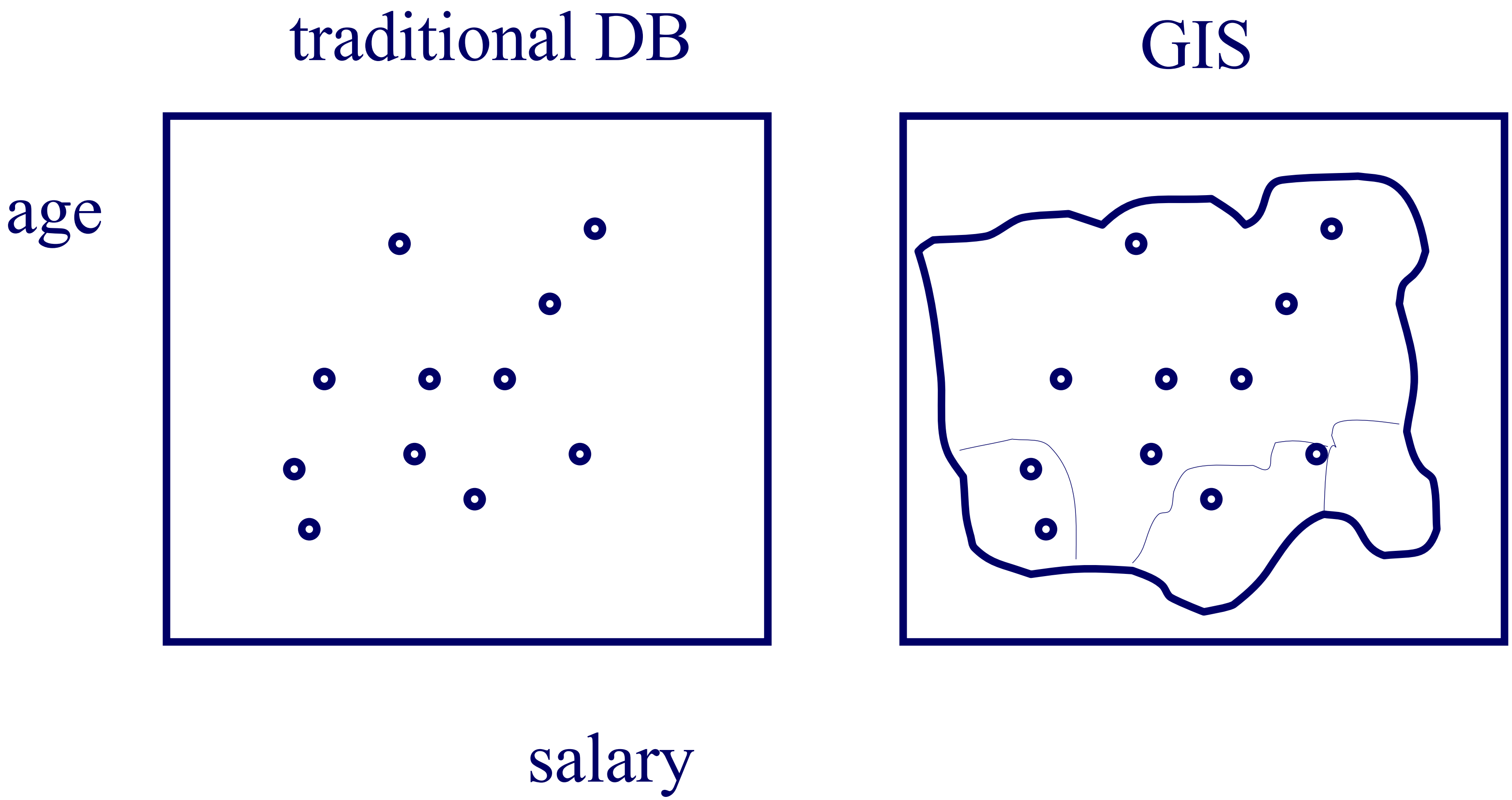
- Given a collection of geometric objects (points, lines, polygons, ...)
- organize them on disk, to answer
  - point queries
  - range queries
  - k-nn queries
  - spatial joins (‘all pairs’ within ε)



SAMs - motivation

- Q: applications?

SAMs - motivation

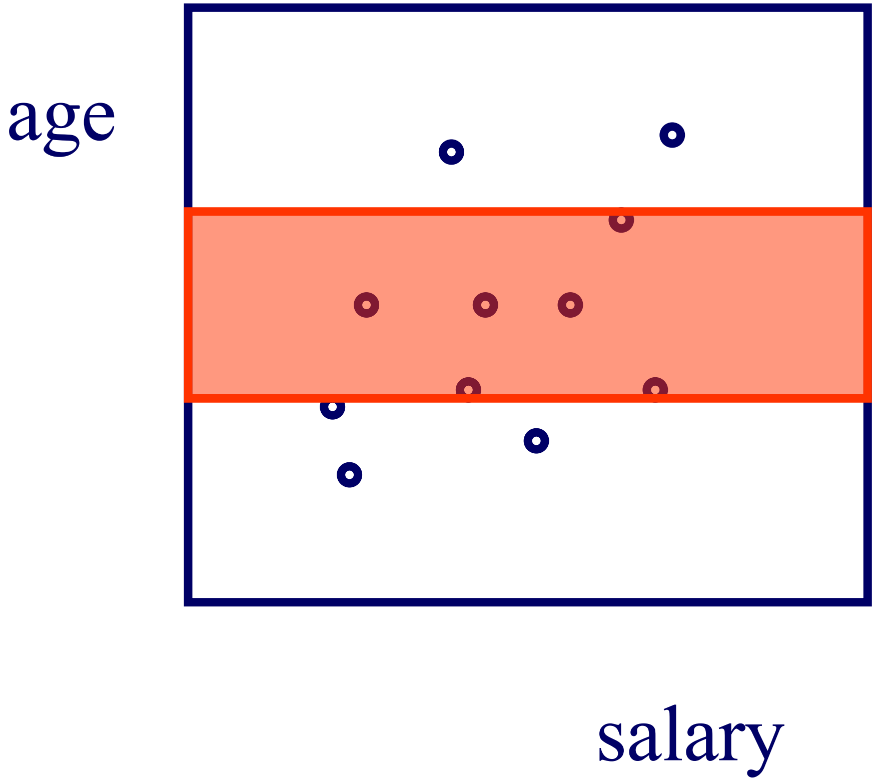


traditional DB

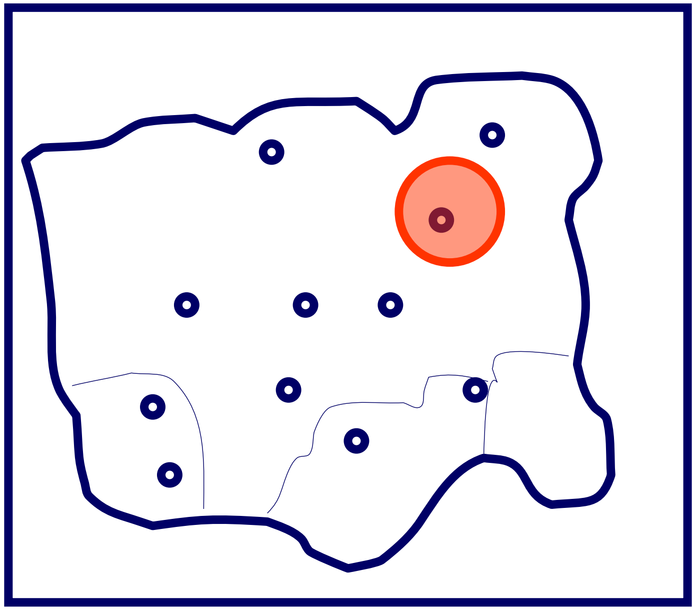
GIS

SAMs - motivation

traditional DB

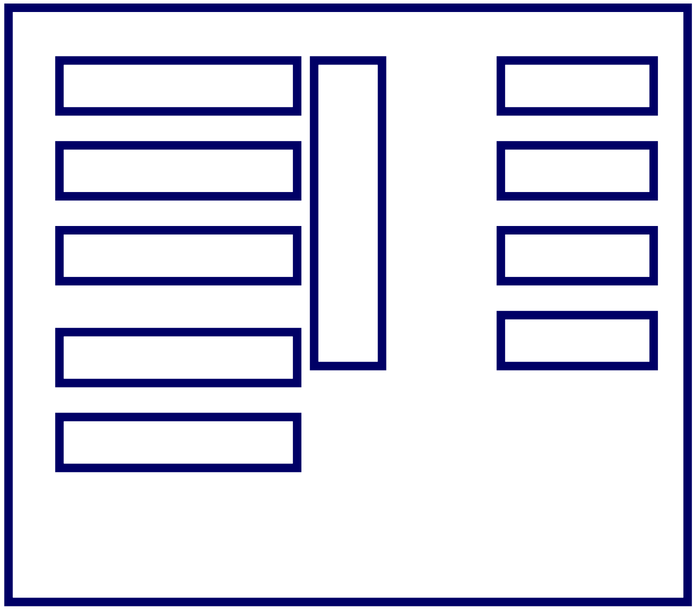


GIS



SAMs - motivation

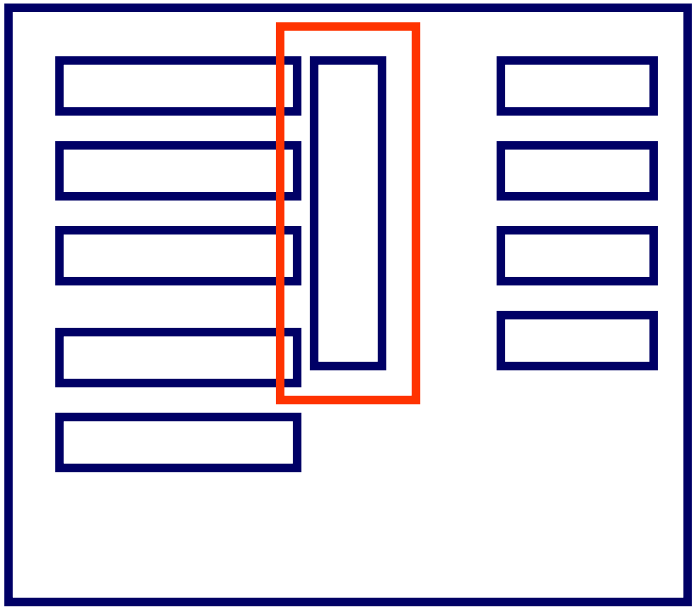
CAD/CAM



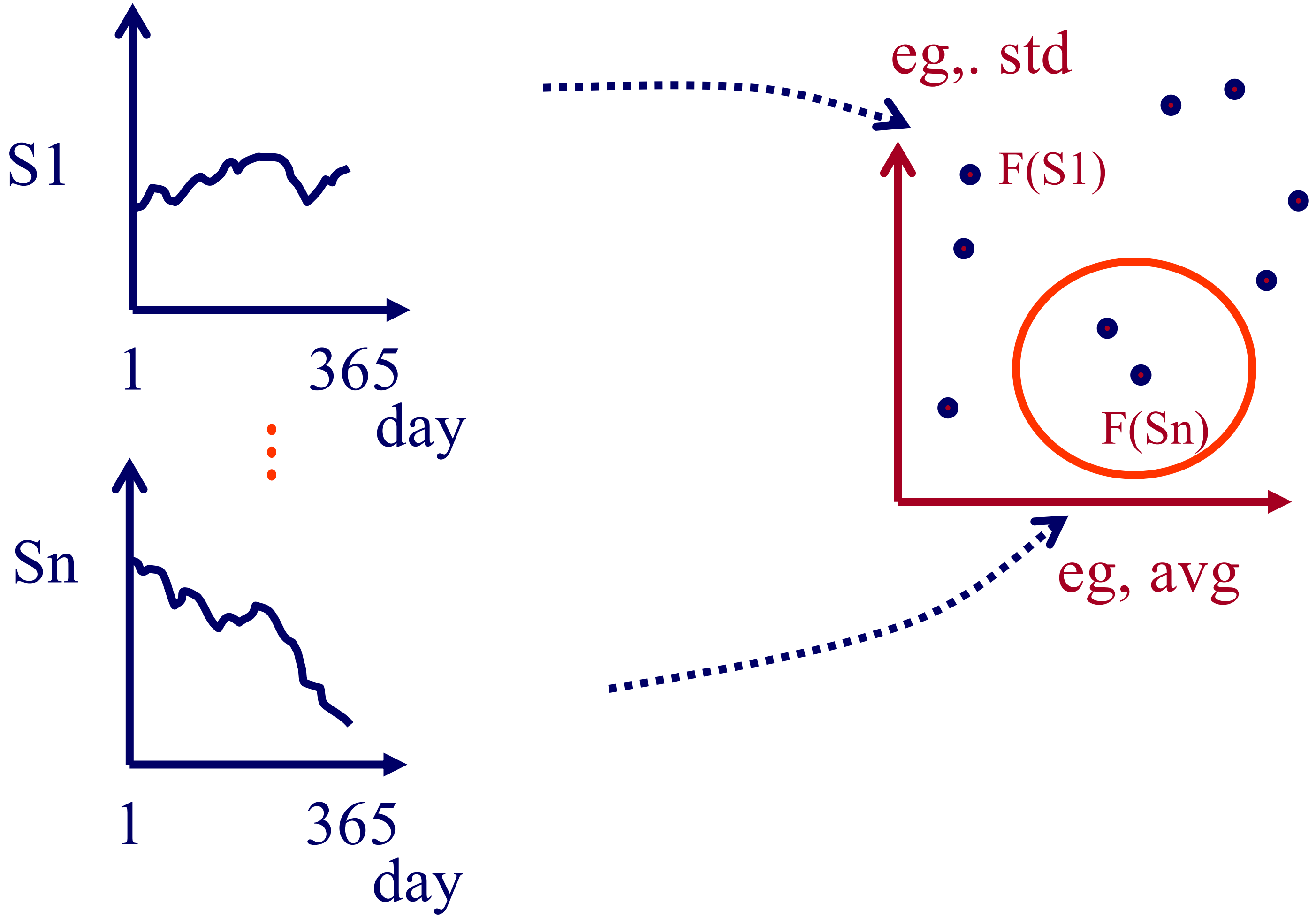
find elements
too close
to each other

SAMs - motivation

CAD/CAM



SAMs - motivation



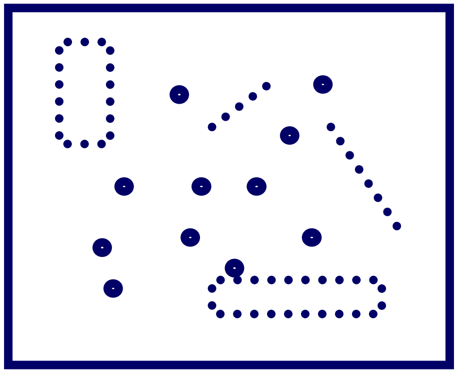
Indexing - Detailed outline

- primary key indexing
- secondary key / multi-key indexing
- spatial access methods
  - problem dfn
  - z-ordering
  - R-trees
  - ...
- text
- ...

SAMs: solutions

- z-ordering
- R-trees
- (grid files)

Q: how would you organize, e.g., *n*-dim points, on disk? (*C* points per disk page)



z-ordering

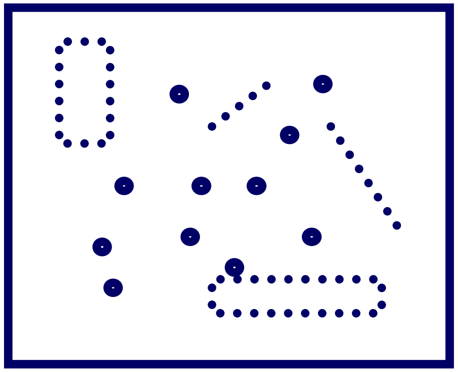
Q: how would you organize, e.g., *n*-dim points, on disk? (*C* points per disk page)

Hint: reduce the problem to 1-d points (!!)

Q1: why?

A:

Q2: how?



z-ordering

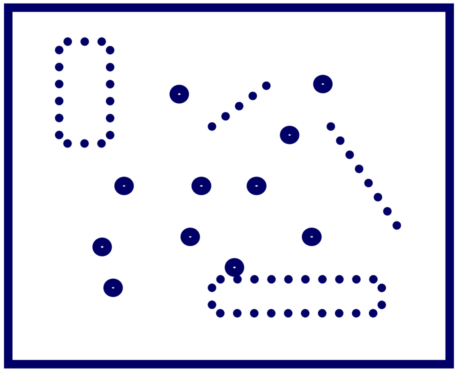
Q: how would you organize, e.g., *n*-dim points, on disk? (*C* points per disk page)

Hint: reduce the problem to 1-d points (!!)

Q1: why?

A: B-trees!

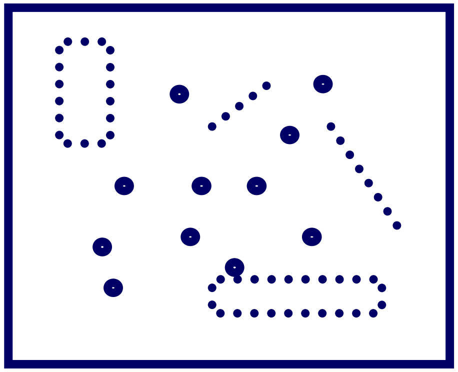
Q2: how?



z-ordering

Q2: how?

A: assume finite granularity; z-ordering = bit-shuffling = N-trees = Morton keys = geo-coding = ...



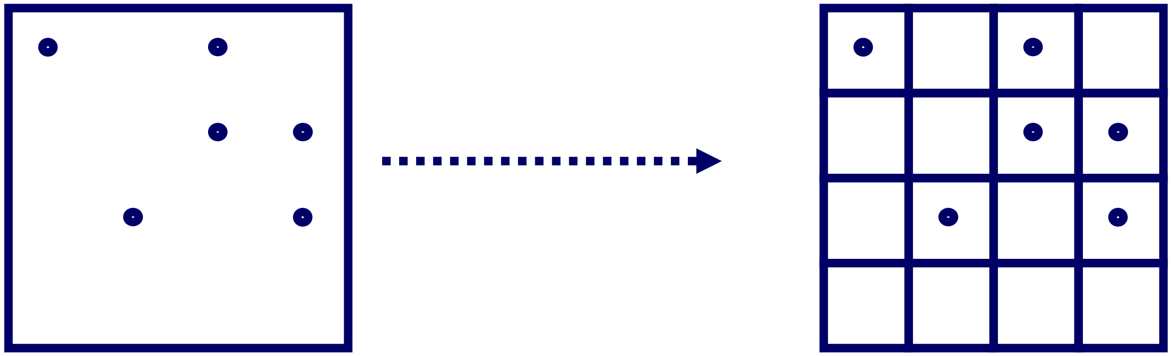
Carnegie Mellon

z-ordering

Q2: how?

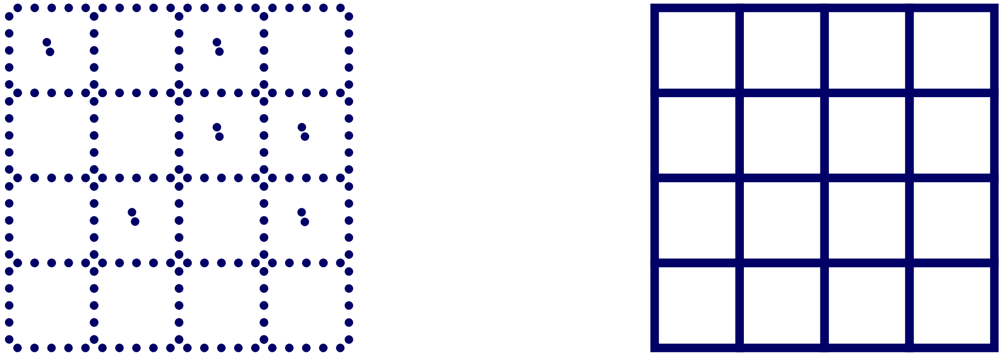
A: assume finite granularity (e.g., 2^{32}x2^{32} ; 4x4 here)

Q2.1: how to map n-d cells to 1-d cells?



z-ordering

Q2.1: how to map *n*-d cells to 1-d cells?

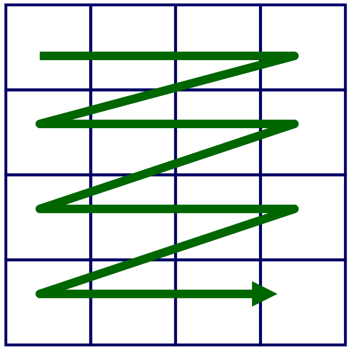


z-ordering

Q2.1: how to map *n*-d cells to 1-d cells?

A: row-wise

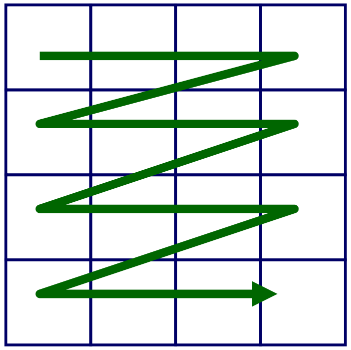
Q: is it good?



z-ordering

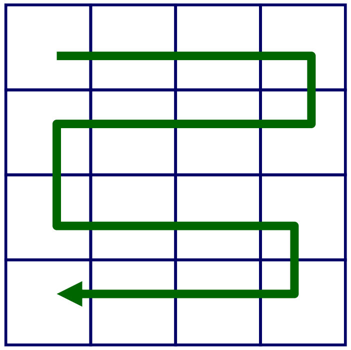
Q: is it good?

A: great for ‘x’ axis; bad for ‘y’ axis



z-ordering

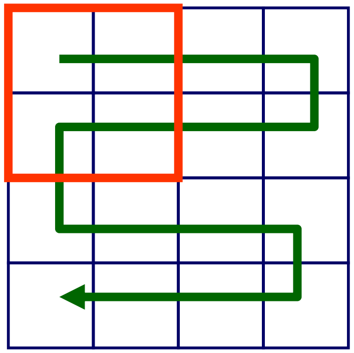
Q: How about the ‘snake’ curve?

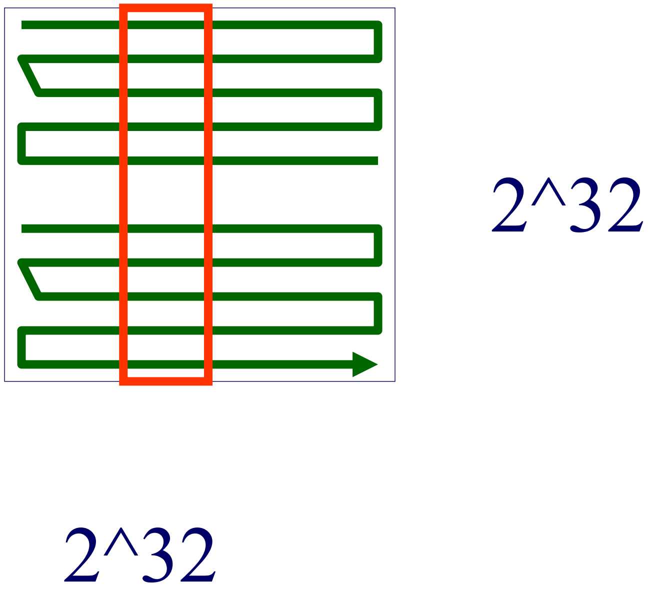


z-ordering

Q: How about the ‘snake’ curve?

A: still problems:



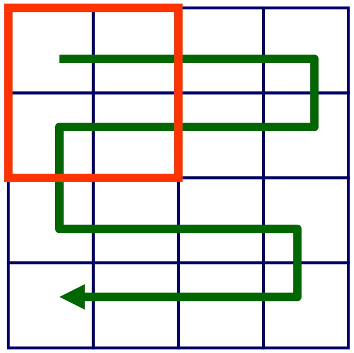


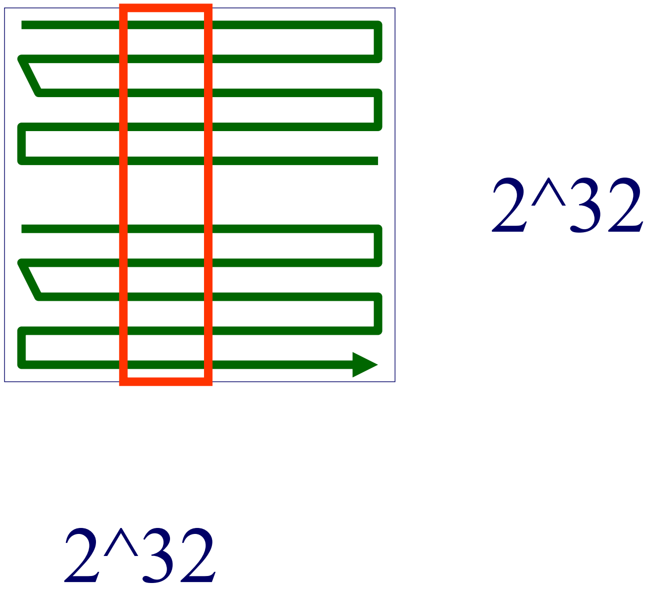
z-ordering

Q: Why are those curves ‘bad’ ?

A: no distance preservation (~ clustering)

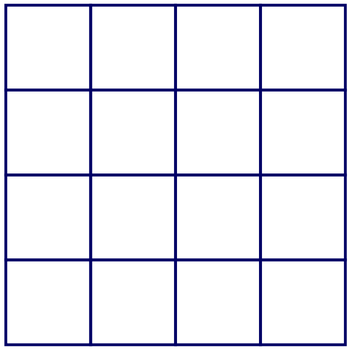
Q: solution?





z-ordering

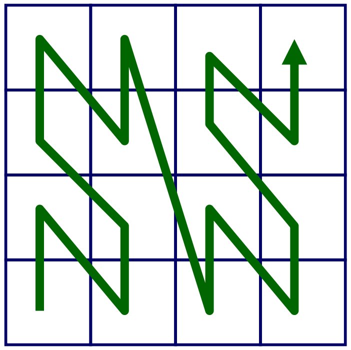
Q: solution? (w/ good clustering, and easy to compute, for 2-d and *n*-d?)



z-ordering

Q: solution? (w/ good clustering, and easy to compute, for 2-d and *n*-d?)

A: z-ordering/bit-shuffling/linear-quadtrees



'looks' better:

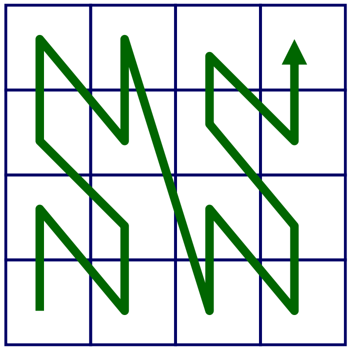
- few long jumps;
- scoops out the whole quadrant before leaving it
- a.k.a. space filling curves

z-ordering

z-ordering/bit-shuffling/linear-quadtrees

Q: How to generate this curve (*z = f(x,y)*)?

A: 3 (equivalent) answers!

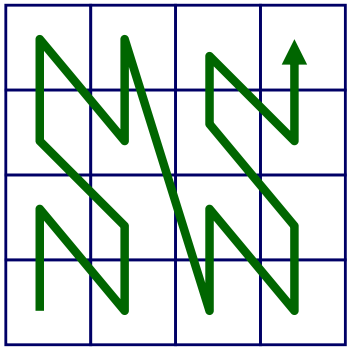


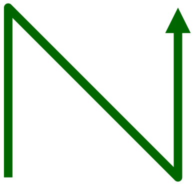
z-ordering

z-ordering/bit-shuffling/linear-quadtrees

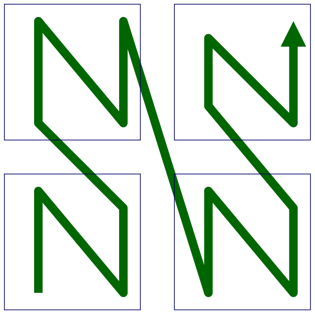
Q: How to generate this curve ($z = f(x,y)$)?

A1: ‘z’ (or ‘N’) shapes, RECURSIVELY



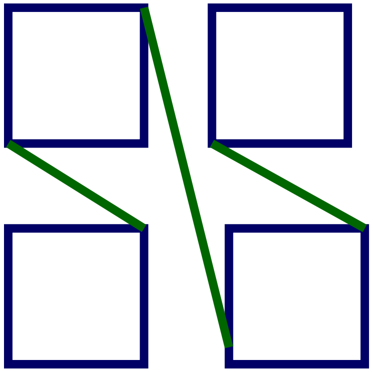


order-1



order-2

…

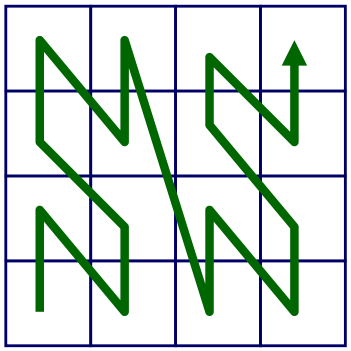


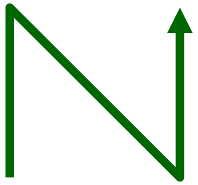
order (n+1)

z-ordering

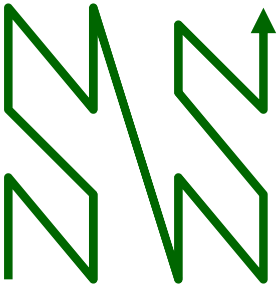
Notice:

- self similar (we' ll see about fractals, soon)
- method is hard to use: *z =? f(x,y)*



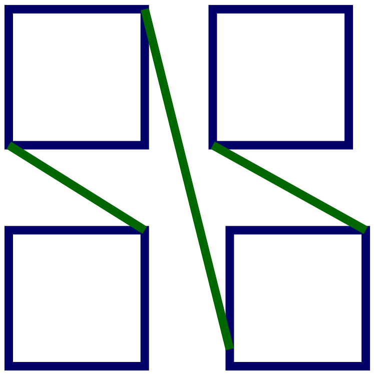


order-1



order-2

…



order (n+1)

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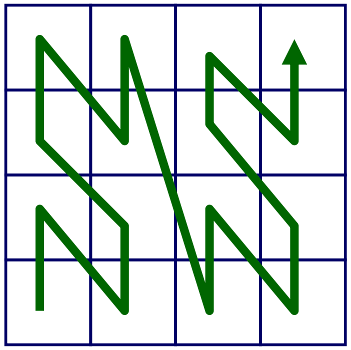
z-ordering

z-ordering/**bit-shuffling**/linear-quadtrees

Q: How to generate this curve ($z = f(x,y)$)?

A: 3 (equivalent) answers!



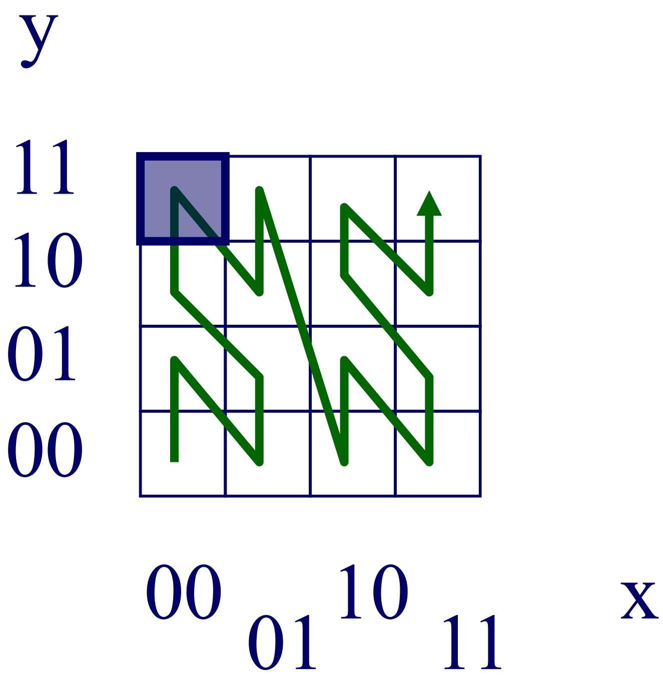
Method #2?

z-ordering

bit-shuffling

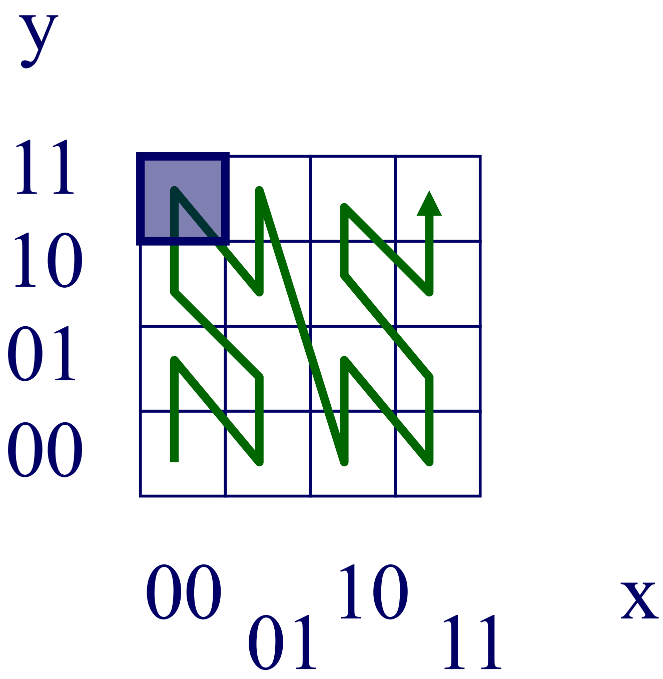
x y

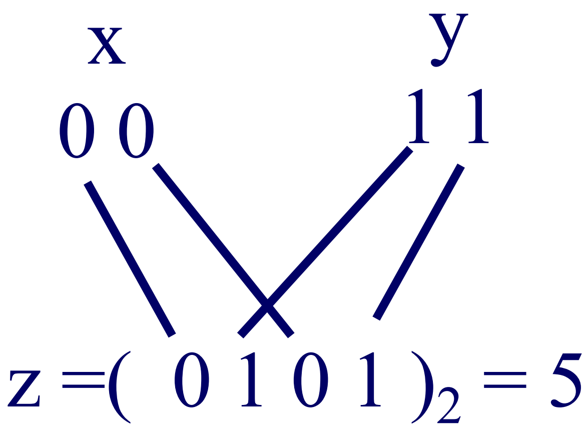
0 0 1 1



z-ordering

bit-shuffling

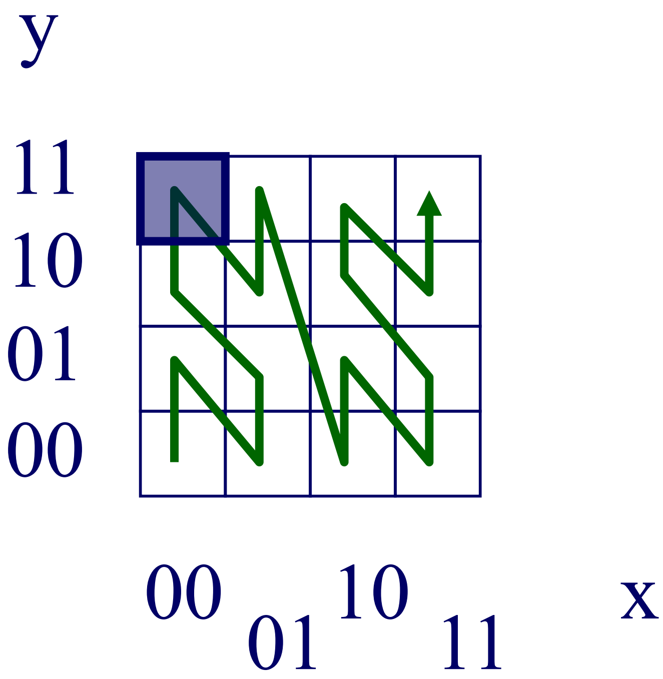


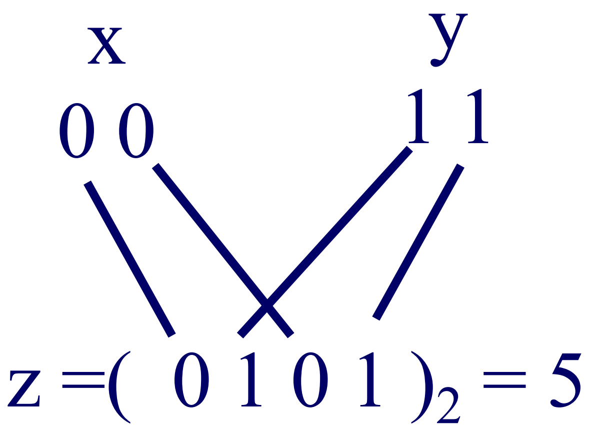


$z = (\ 0\ 1\ 0\ 1\ )_2 = 5$

z-ordering

bit-shuffling





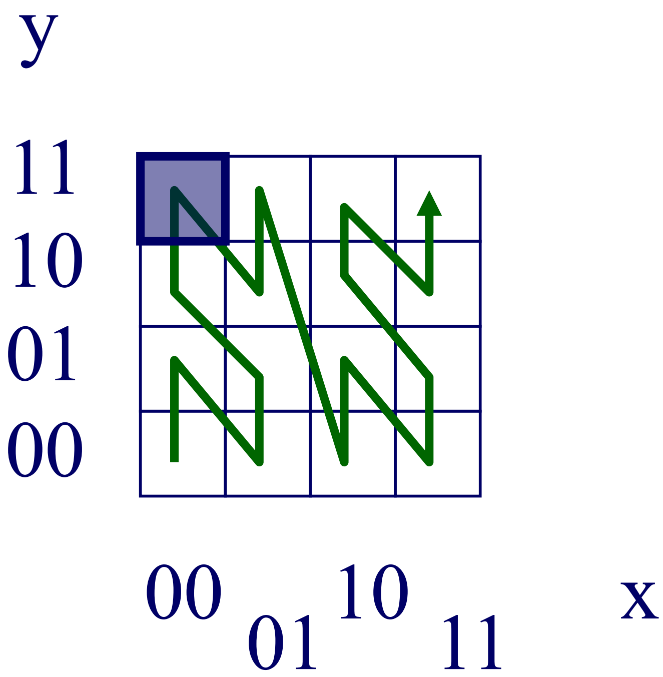
$z = (\ 0\ 1\ 0\ 1\ )_2 = 5$

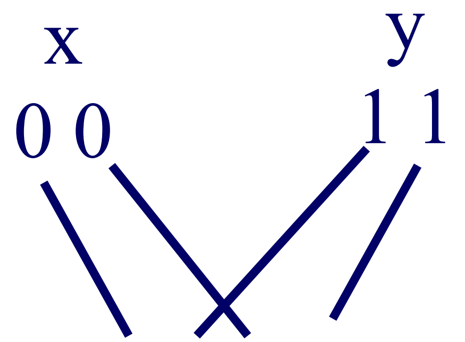
How about the reverse:

$(x,y) = g(z)$?

z-ordering

bit-shuffling





$z = (\ 0\ 1\ 0\ 1\ )_2 = 5$

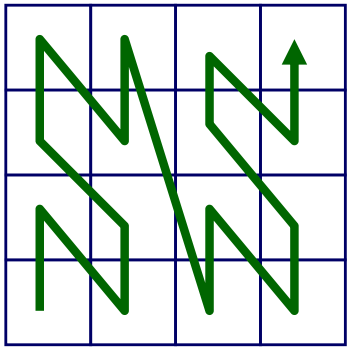
How about *n*-d spaces?

z-ordering

z-ordering/bit-shuffling/**linear-quadtrees**

Q: How to generate this curve ($z = f(x,y)$)?

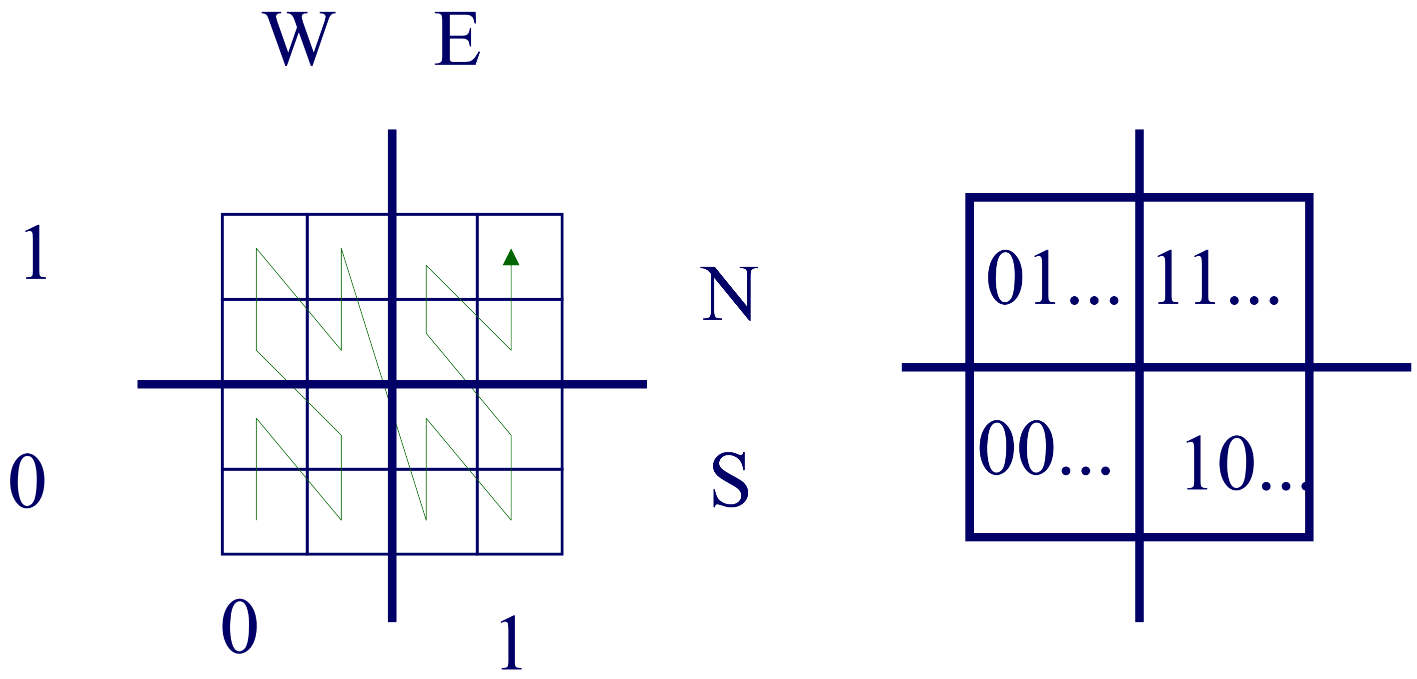
A: 3 (equivalent) answers!



Method #3?

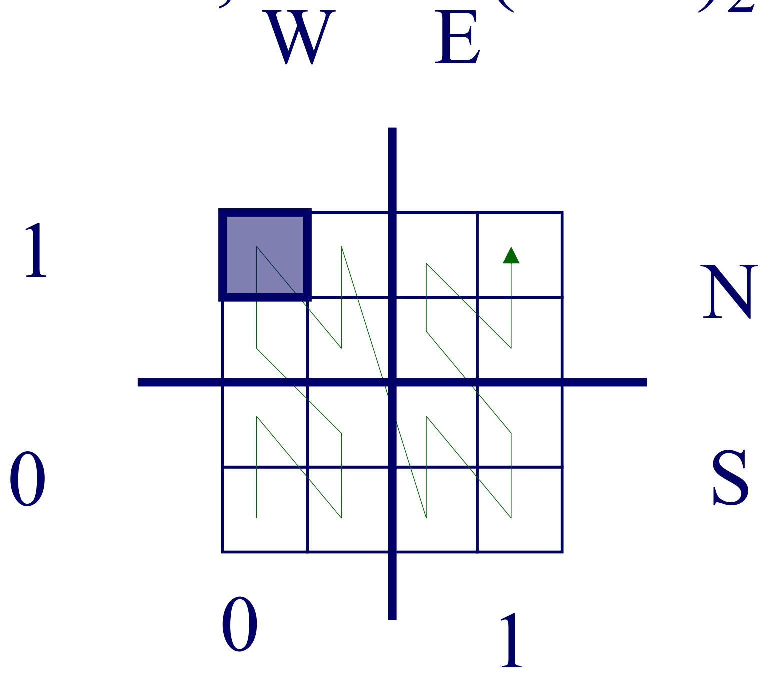
z-ordering

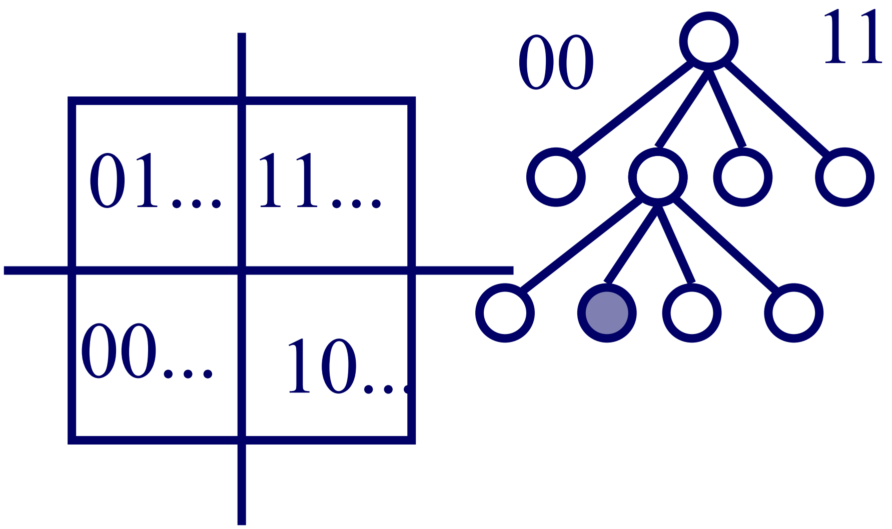
linear-quadtrees : assign N->1, S->0 e.t.c.



z-ordering

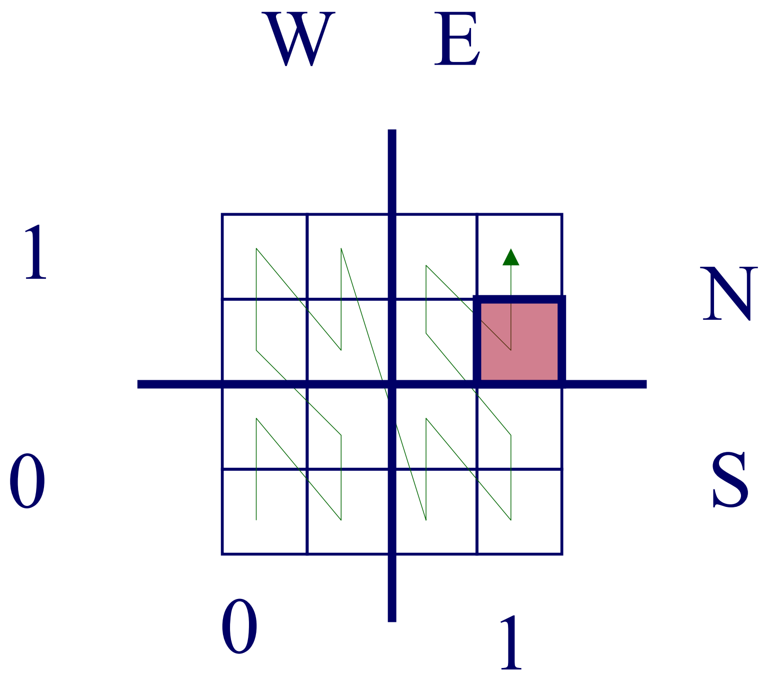
... and repeat recursively. Eg.: $z_{\text{blue-cell}}$ = WN;WN = $(0101)_2$ = 5





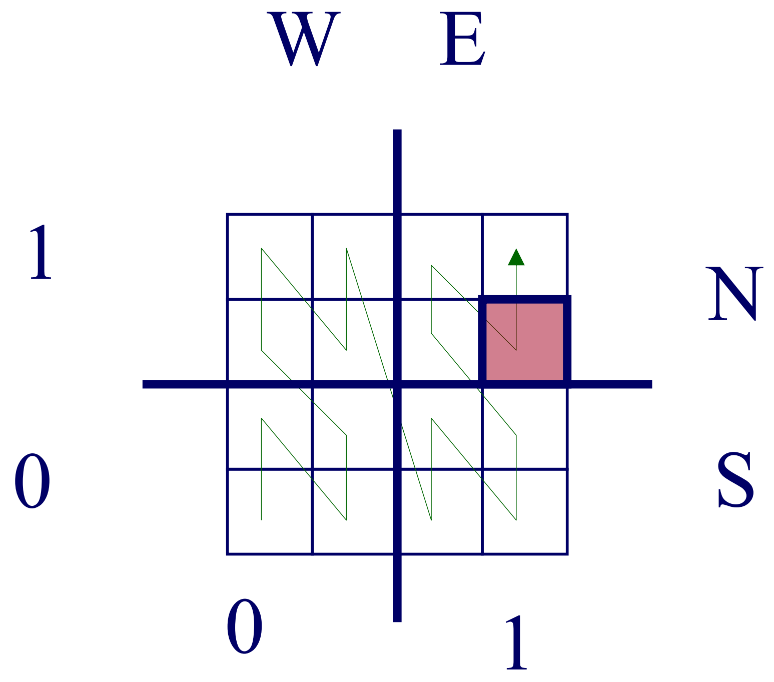
z-ordering

Drill: z-value of magenta cell, with the three methods?



z-ordering

Drill: z-value of magenta cell, with the three methods?

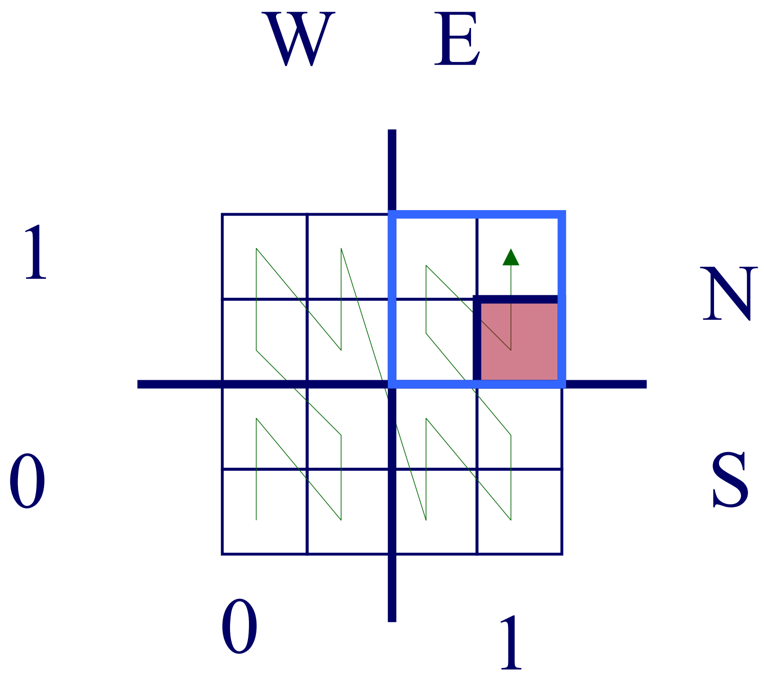


method#1: 14

method#2: shuffle(11;10)= $(1110)_2$ = 14

z-ordering

Drill: z-value of magenta cell, with the three methods?



method#1: 14

method#2: shuffle(11;10)= $(1110)_2$ = 14

method#3: EN;ES = ... = 14

z-ordering - Detailed outline

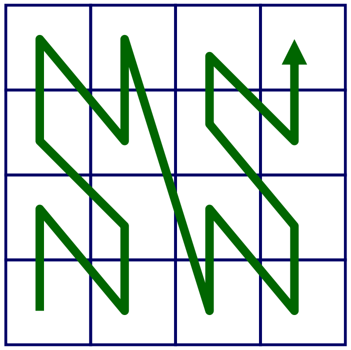
- spatial access methods
  - z-ordering
    - main idea - 3 methods
    - use w/ B-trees; algorithms (range, knn queries ...)
    - non-point (eg., region) data
    - analysis; variations
  - R-trees
  - ...

z-ordering - usage & algo’ s

Q1: How to store on disk?

A:

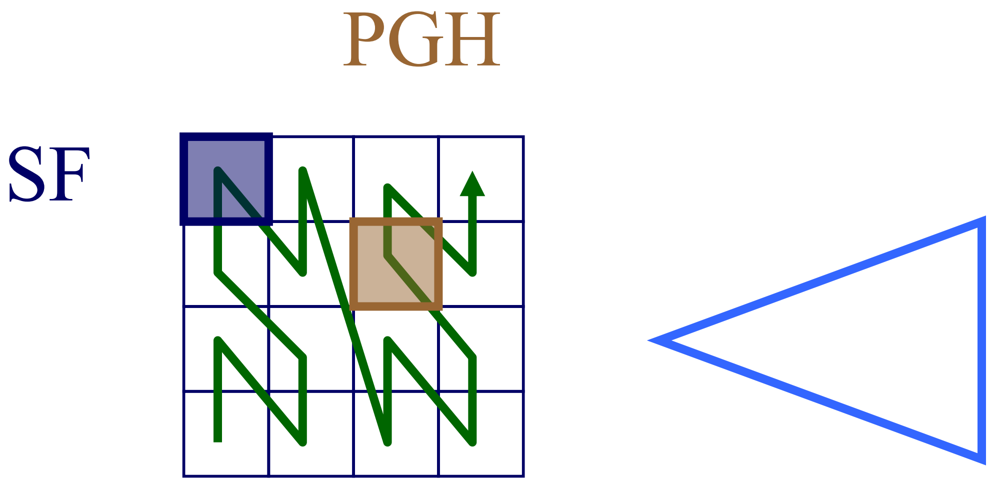
Q2: How to answer range queries etc



z-ordering - usage & algo' s

Q1: How to store on disk?

A: treat z-value as primary key; feed to B-tree

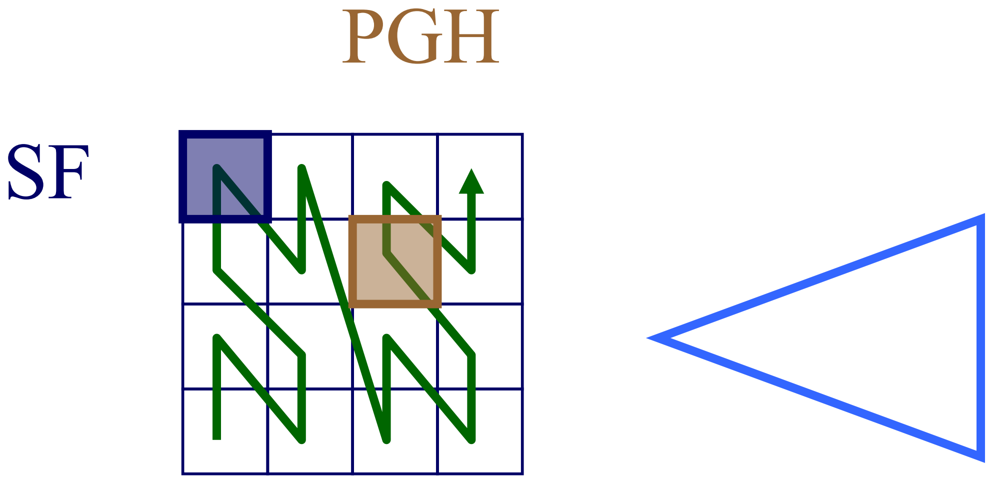


z	cname	etc
5	SF	
12	PGH	

z-ordering - usage & algo’ s

MAJOR ADVANTAGES w/ B-tree:

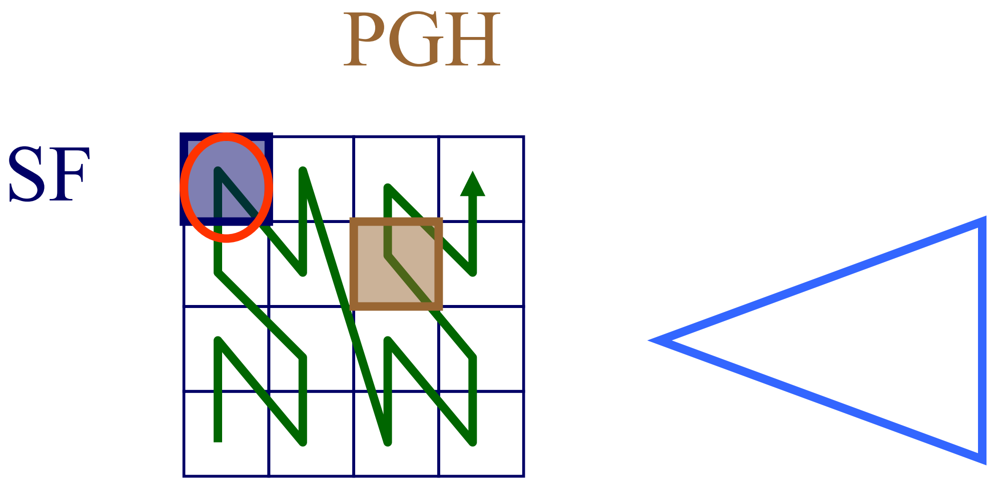
- already inside commercial systems (no coding/debugging!)
- concurrency & recovery is ready



z	cname	etc
5	SF	
12	PGH	

z-ordering - usage & algo’ s

Q2: queries? (eg.: *find city at (0,3))?*

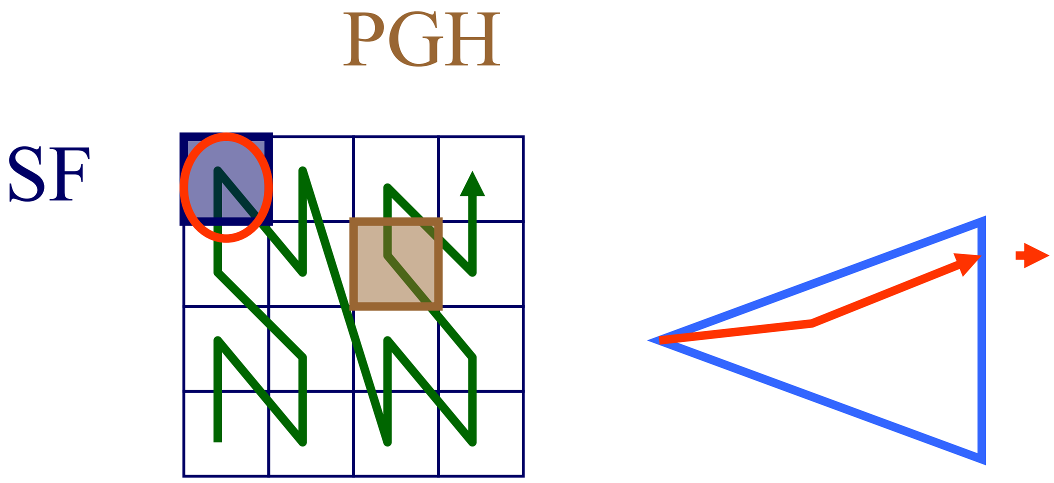


z	cname	etc
5	SF	
12	PGH	

z-ordering - usage & algo’ s

Q2: queries? (eg.: *find city at (0,3))?*

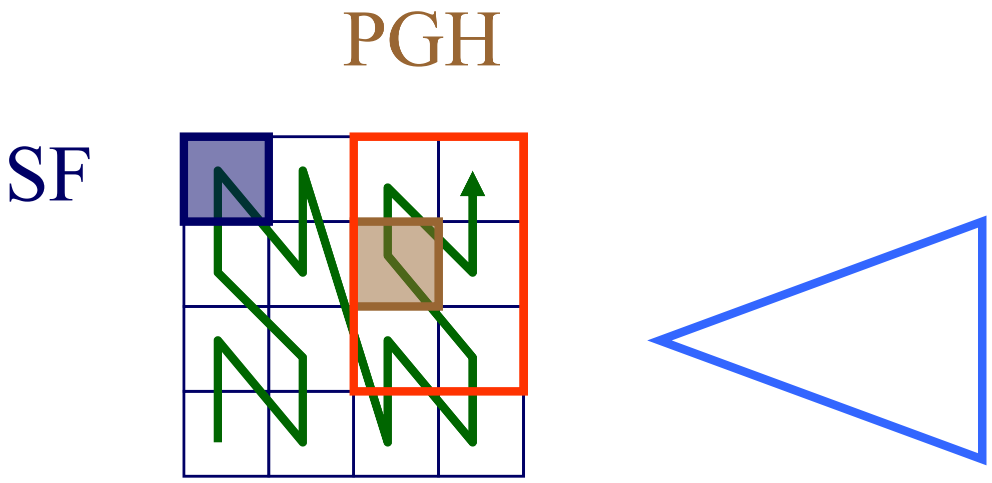
A: find z-value; search B-tree



z	cname	etc
5	SF	
12	PGH	

z-ordering - usage & algo’s

Q2: range queries?

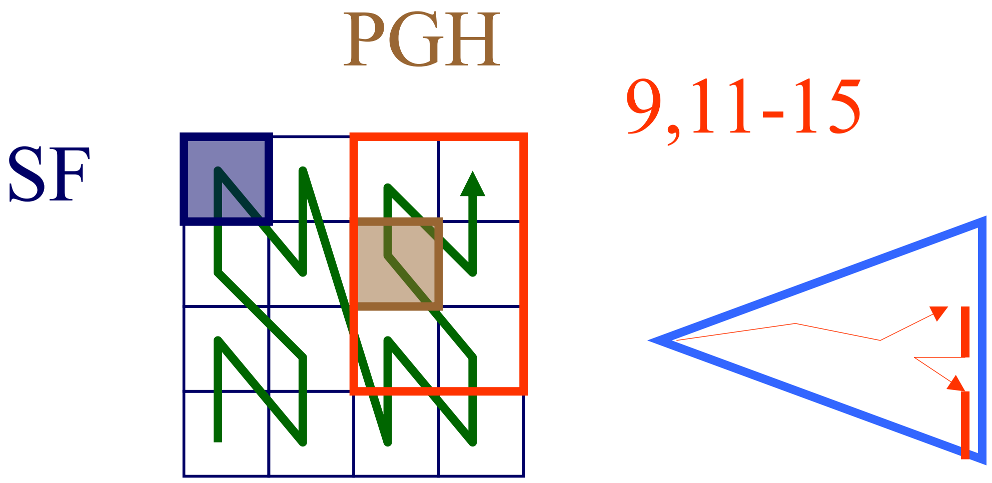


z	cname	etc
5	SF	
12	PGH	

z-ordering - usage & algo’ s

Q2: range queries?

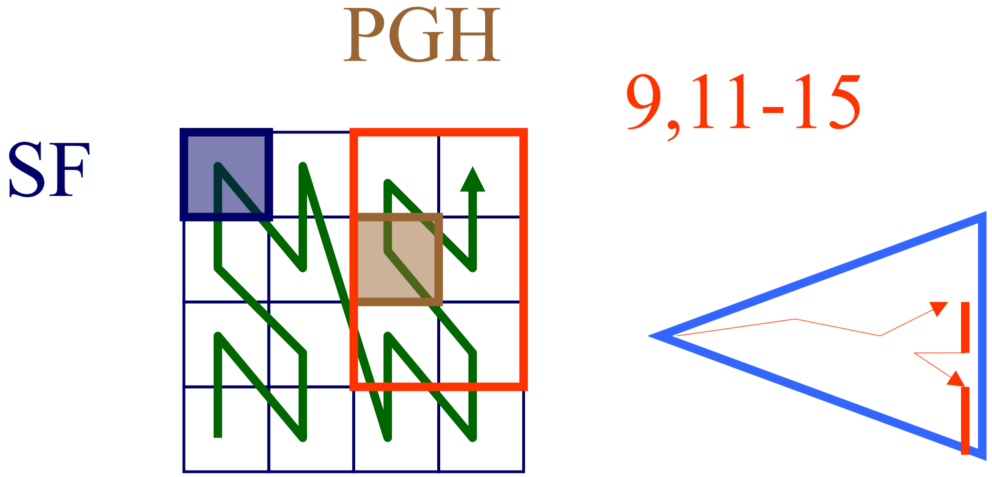
A: compute ranges of z-values; use B-tree



z	cname	etc
5	SF	
12	PGH	

z-ordering - usage & algo’ s

Q2’ : range queries - how to reduce # of qualifying of ranges?

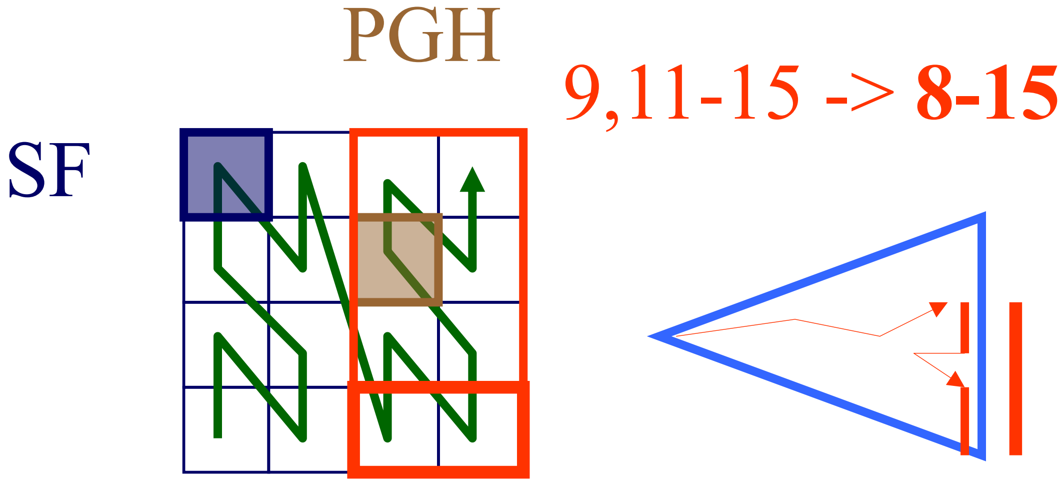


z	cname	etc
5	SF	
12	PGH	

z-ordering - usage & algo' s

Q2' : range queries - how to reduce # of qualifying of ranges?

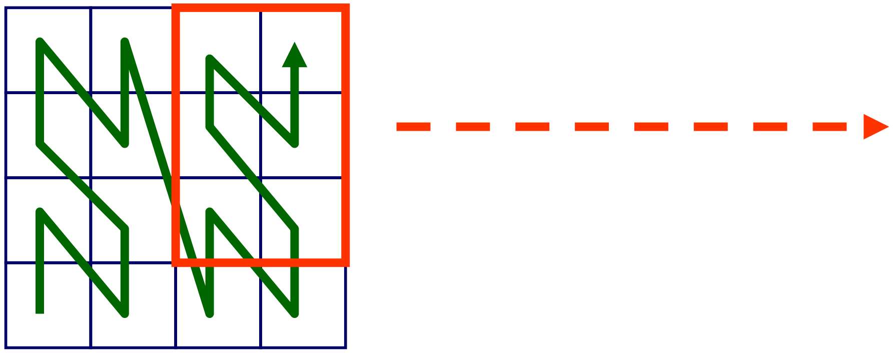
A: Augment the query!



z	cname	etc
5	SF	
12	PGH	

z-ordering - usage & algo’s

Q2’’ : range queries - how to break a query into ranges?

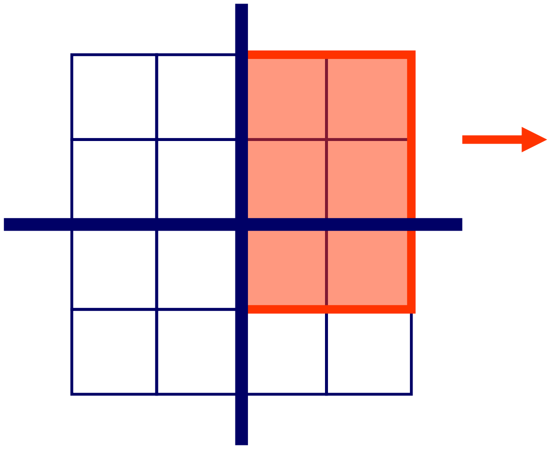


9,11-15

z-ordering - usage & algo's

Q2'': range queries - how to break a query into ranges?

A: recursively, quadtree-style; decompose only non-full quadrants

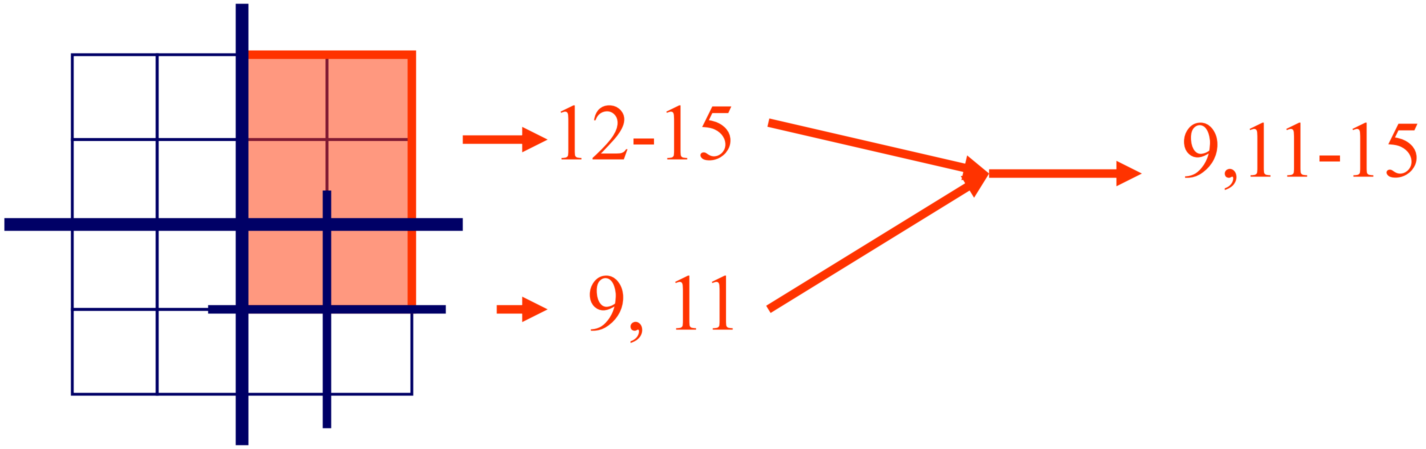


9,11-15

z-ordering - usage & algo's

Q2'' : range queries - how to break a query into ranges?

A: recursively, quadtree-style; decompose only non-full quadrants



z-ordering - Detailed outline

- spatial access methods
  - z-ordering
    - main idea - 3 methods
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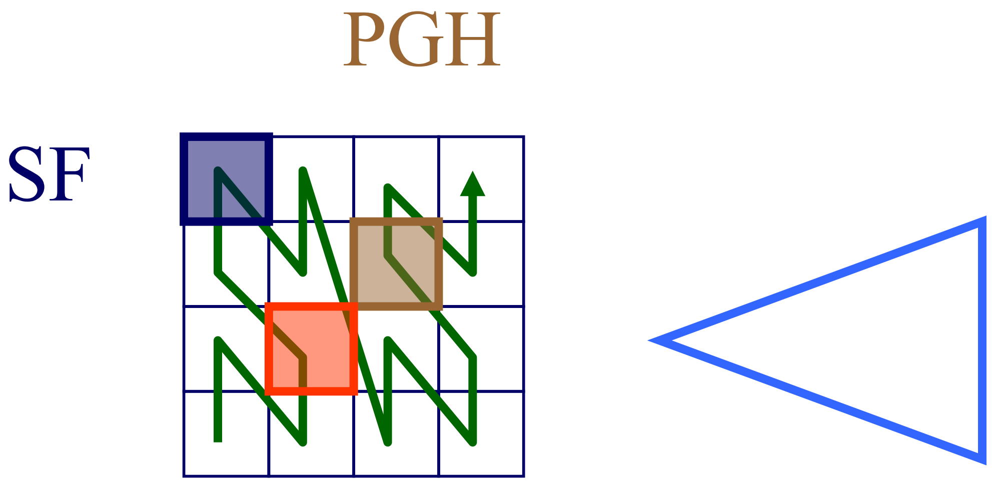
    - non-point (eg., region) data
    - analysis; variations
  - R-trees
  - ...

z-ordering - usage & algo’s

Q3: k-nn queries? (say, 1-nn)?

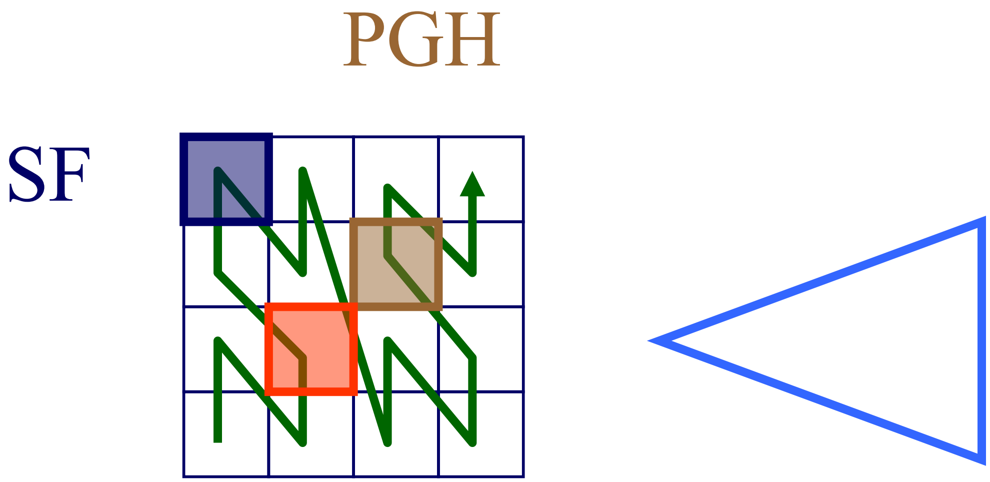


z	cname	etc
5	SF	
12	PGH	

z-ordering - usage & algo’ s

Q3: k-nn queries? (say, 1-nn)?

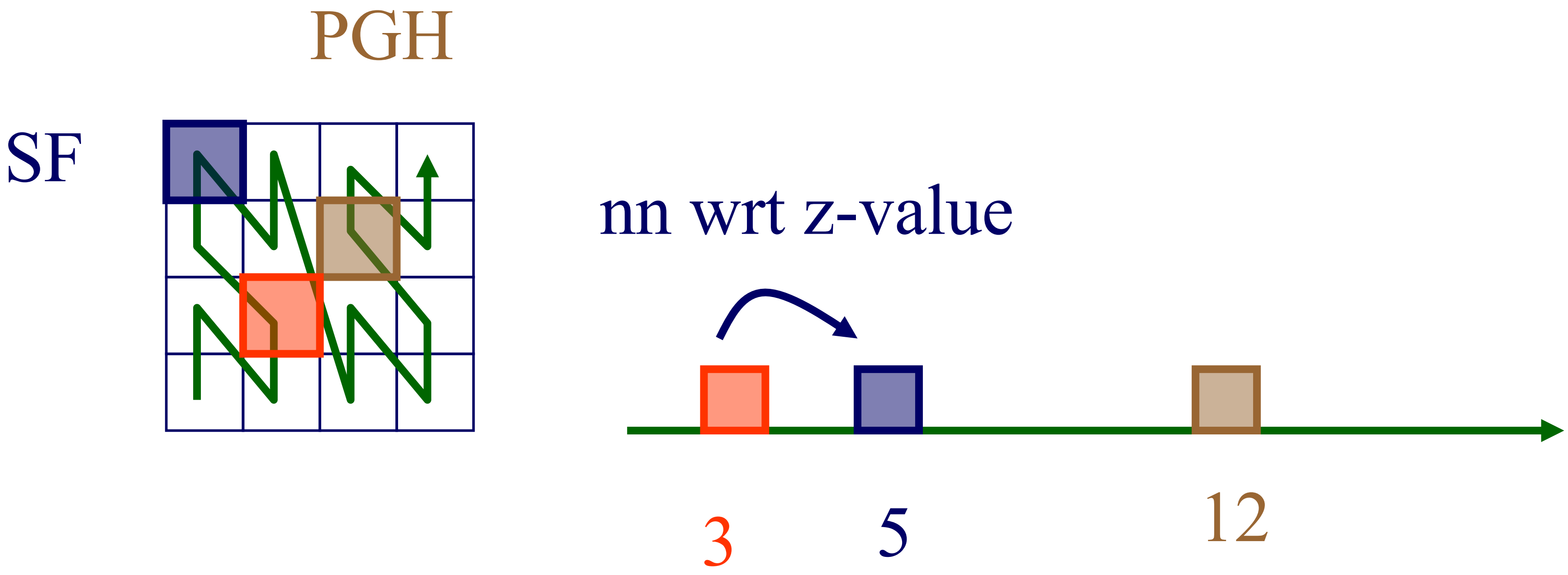
A: traverse B-tree; find nn wrt z-values and ...



z	cname	etc
5	SF	
12	PGH	

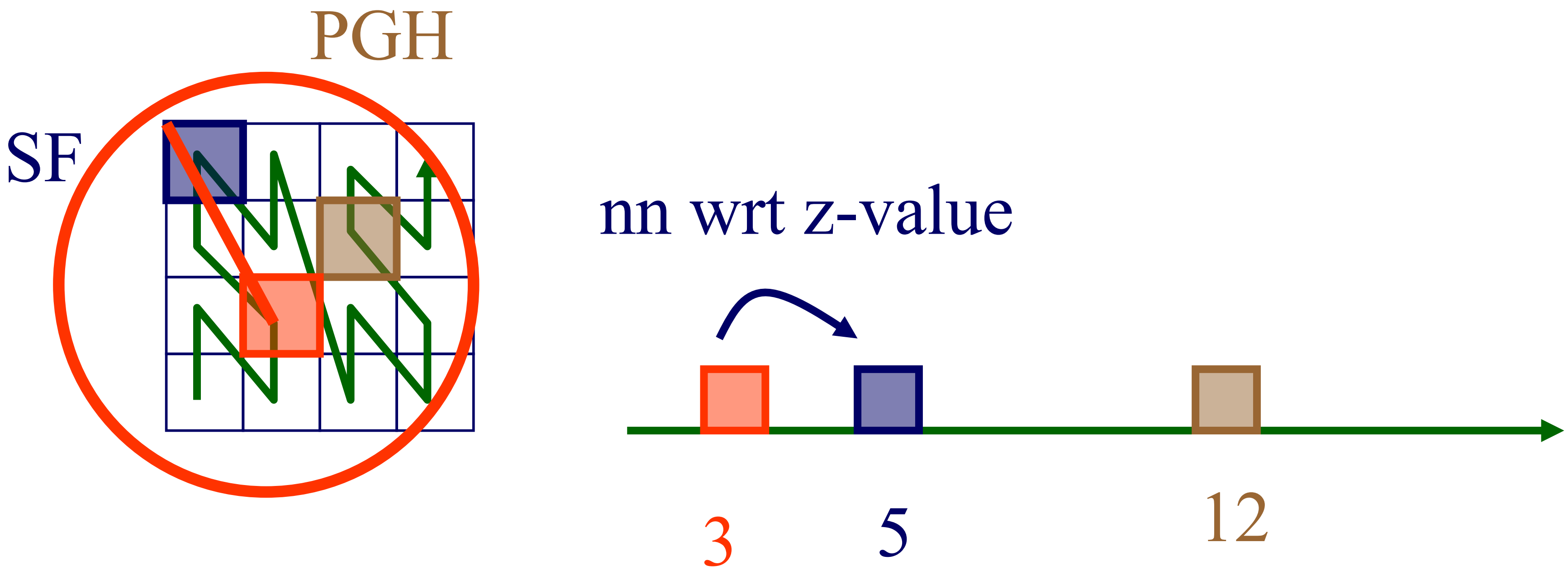
z-ordering - usage & algo’ s

... ask a range query.



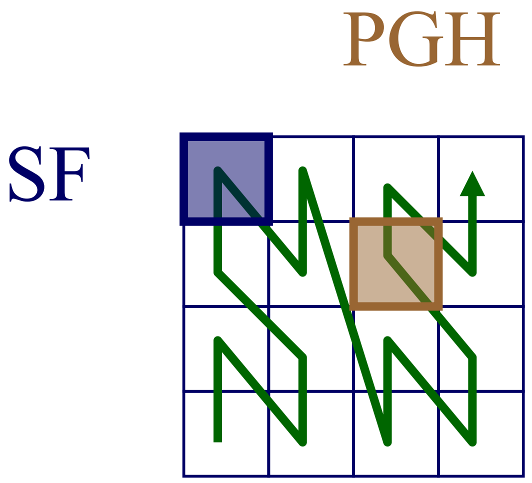
z-ordering - usage & algo’ s

... ask a range query.



z-ordering - usage & algo’s

Q4: all-pairs queries? (*all pairs of cities within 10 miles from each other?*)



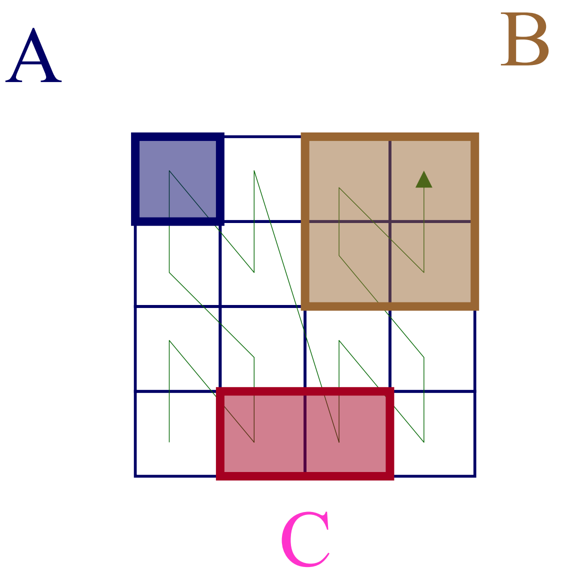
(we’ll see ‘spatial joins’ later: *find all PA counties that intersect a lake)*

z-ordering - Detailed outline

- spatial access methods
  - z-ordering
    - main idea - 3 methods
    - use w/ B-trees; algorithms (range, knn queries ...)
    - non-point (eg., region) data
    - analysis; variations
  - R-trees
  - ...

z-ordering - regions

Q: z-value for a region?



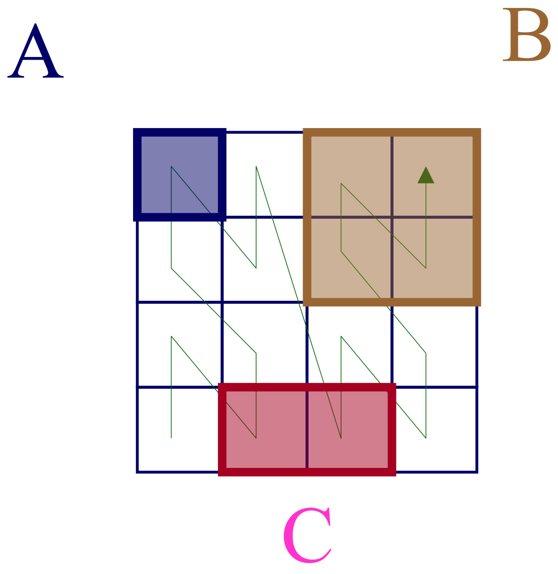
z_B = ??

z_C = ??

z-ordering - regions

Q: z-value for a region?

A: 1 or more z-values; by quadtree decomposition



$z_B = ??$

$z_C = ??$

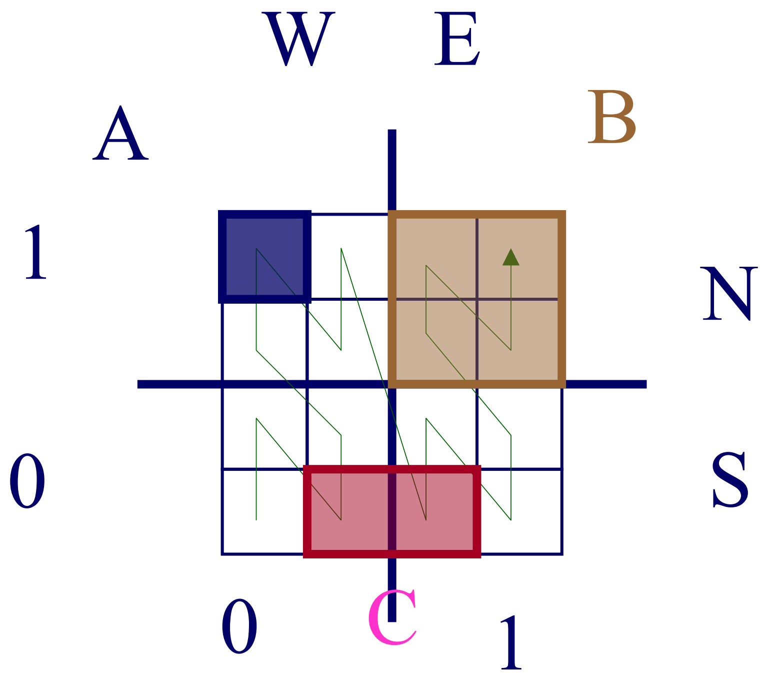
z-ordering - regions

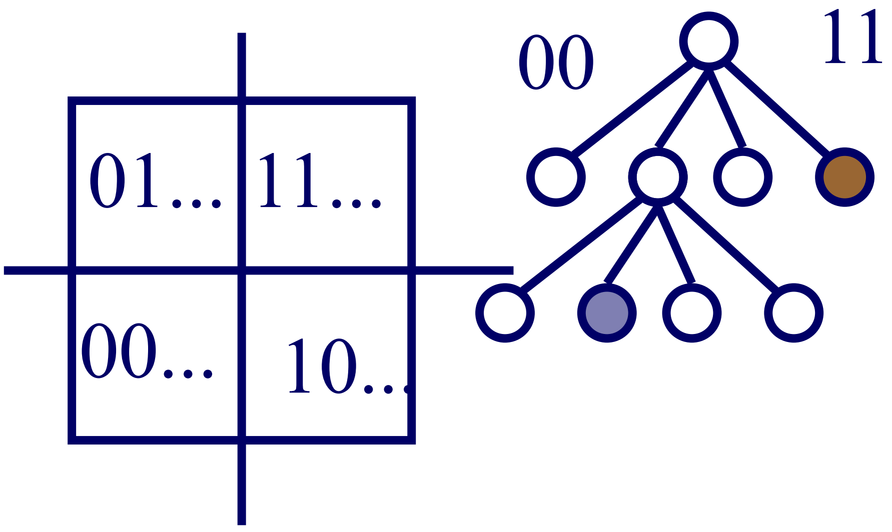
Q: z-value for a region?

“don’ t care”

$z_B = 11{*}{*}$

$z_C = ??$





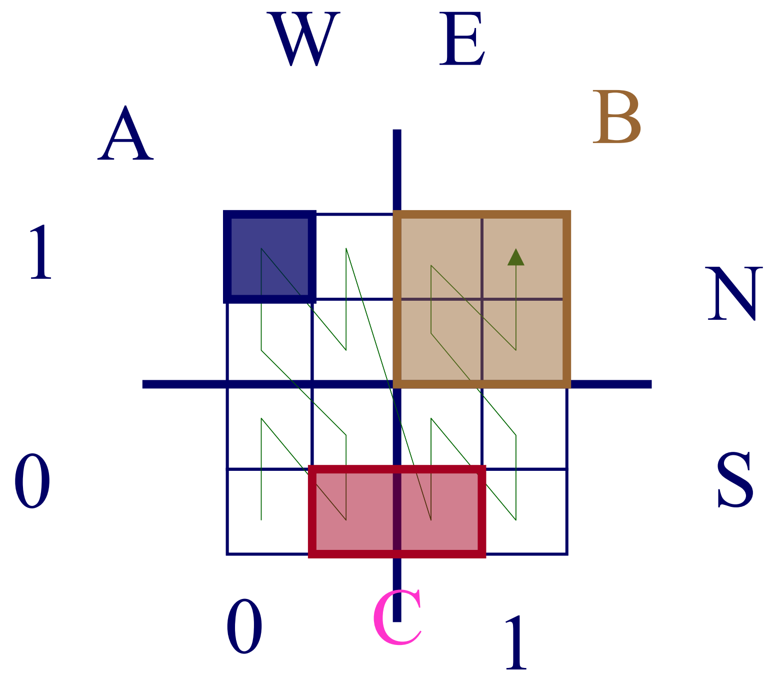
z-ordering - regions

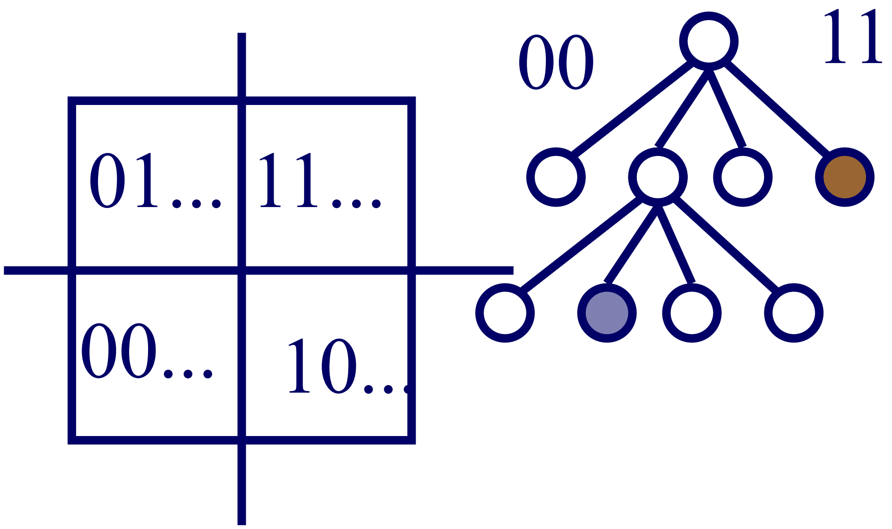
Q: z-value for a region?

“don’ t care”

$z_B = 11**$

$z_C = \{0010; 1000\}$



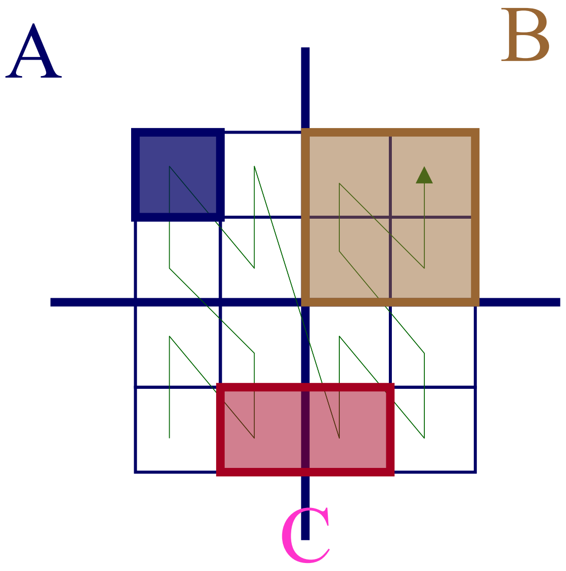


01...	11...
00...	10...

z-ordering - regions

Q: How to store in B-tree?

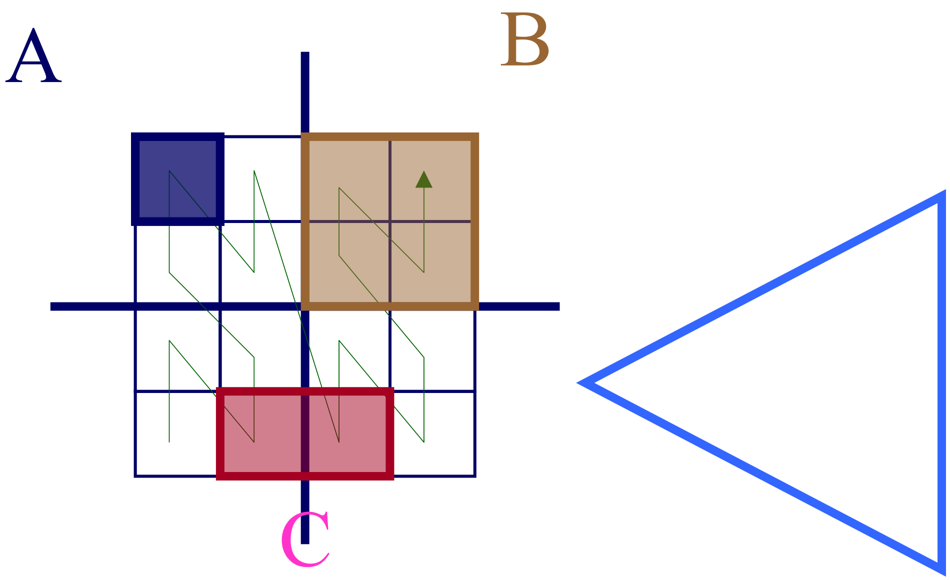
Q: How to search (range etc queries)



z-ordering - regions

Q: How to store in B-tree? A: sort (*<0<1)

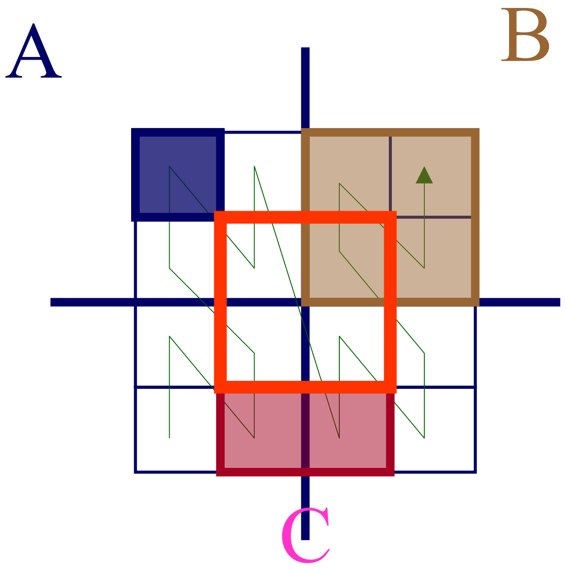
Q: How to search (range etc queries)



z	obj-id	etc
0010	C	
0101	A	
1000	C	
11**	B	

z-ordering - regions

Q: How to search (range etc queries) - eg 'red' range query

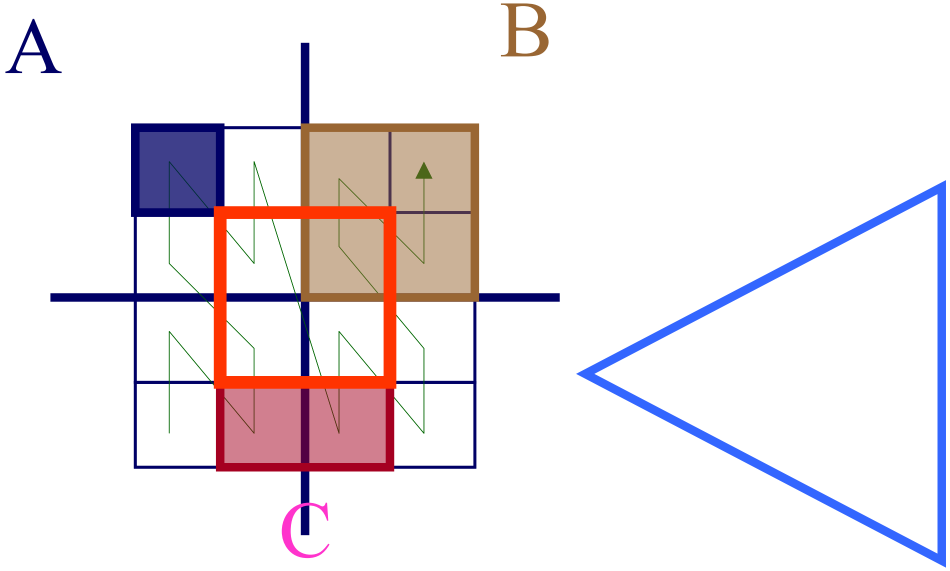


z	obj-id	etc
0010	C	
0101	A	
1000	C	
11**	B	

z-ordering - regions

Q: How to search (range etc queries) - eg ‘red’ range query

A: break query in z-values; check B-tree

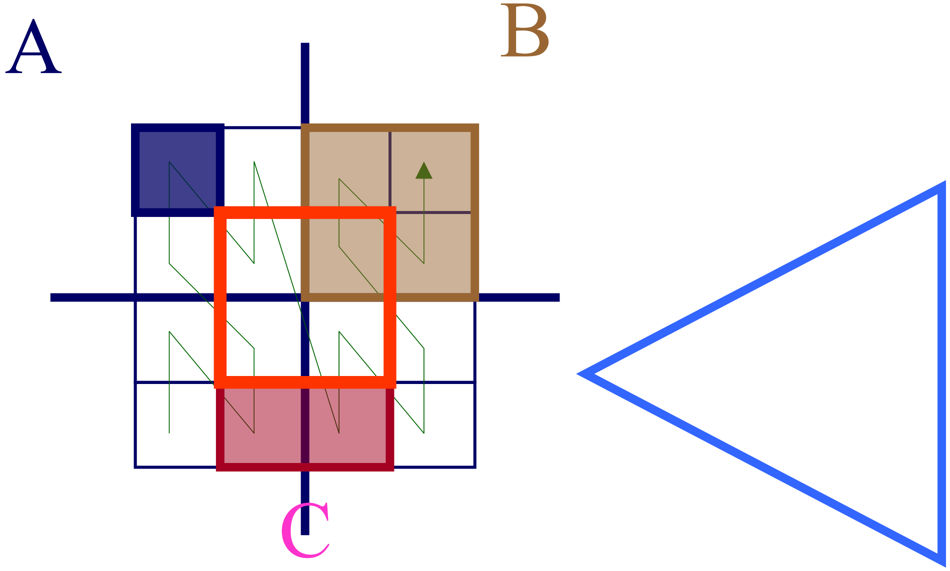


z	obj-id	etc
0010	C	
0101	A	
1000	C	
11**	B	

z-ordering - regions

Almost identical to range queries for point data, except for the “don’ t cares” - i.e.,

1100 ?? 11**



z	obj-id	etc
0010	C	
0101	A	
1000	C	
11**	B	

z-ordering - regions

Almost identical to range queries for point data, except for the "don' t cares" - i.e.,

z1= 1100 ?? 11** = z2

Specifically: does z1 contain/avoid/intersect z2?

Q: what is the criterion to decide?

z-ordering - regions

z1= 1100 ?? 11** = z2

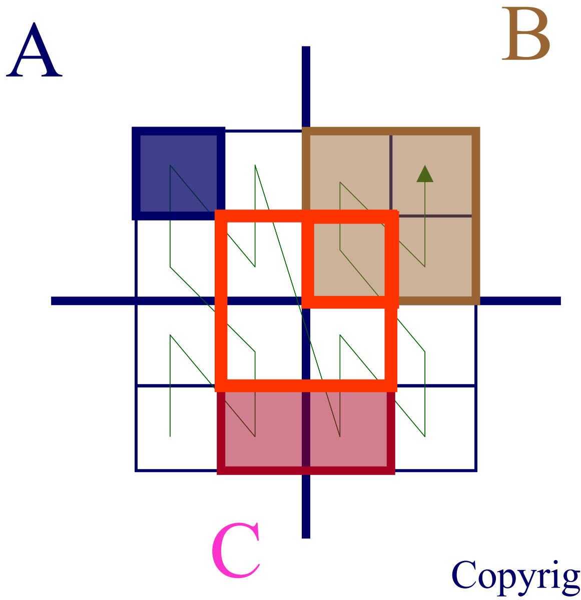
Specifically: does z1 contain/avoid/intersect z2?

Q: what is the criterion to decide?

A: **Prefix property**: let r1, r2 be the corresponding regions, and let r1 be the smallest (=> z1 has fewest '*' s). Then:

z-ordering - regions

- r2 will either contain completely, or avoid completely r1.
- it will contain r1, if z2 is the prefix of z1



1100 ?? 11**

region of z1: completely contained in region of z2

z-ordering - regions

Drill (True/False). Given:

- z1= 011001**
- z2= 01******
- z3= 0100****

T/F r2 contains r1

T/F r3 contains r1

T/F r3 contains r2

z-ordering - regions

Drill (True/False). Given:

- z1= 011001**
- z2= 01******
- z3= 0100****

T/F r2 contains r1 - TRUE (prefix property)

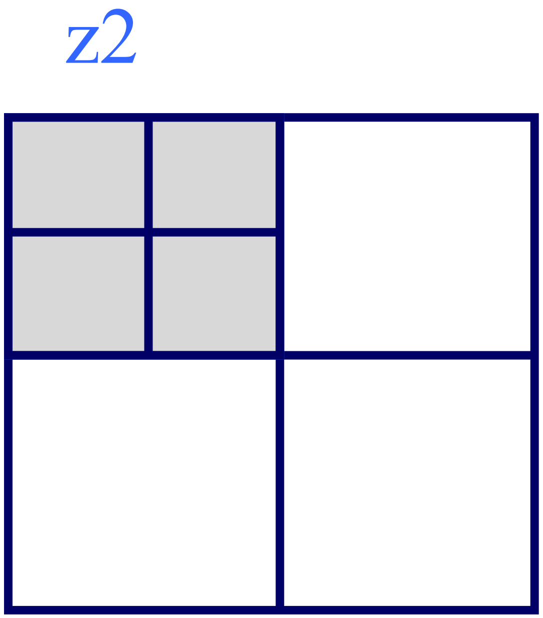
T/F r3 contains r1 - FALSE (disjoint)

T/F r3 contains r2 - FALSE (r2 contains r3)

z-ordering - regions

Drill (True/False). Given:

- z1= 011001**
- z2= 01******
- z3= 0100****

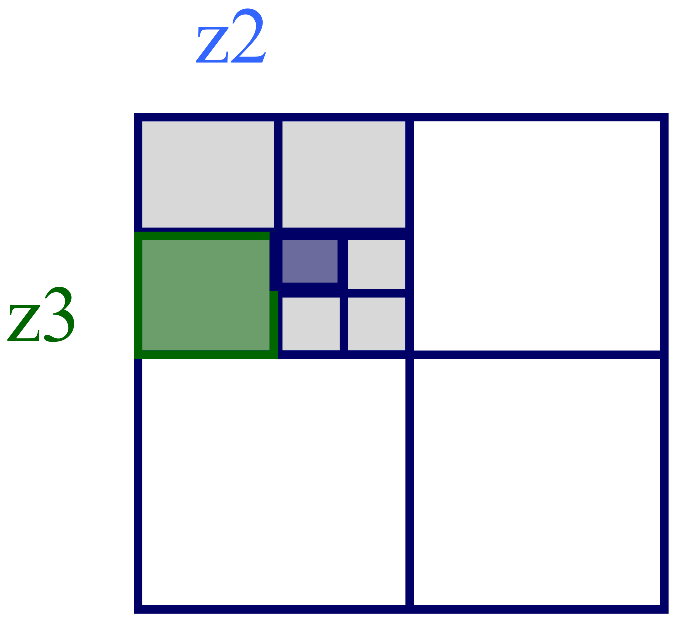


Carnegie Mellon

z-ordering - regions

Drill (True/False). Given:

- z1= 011001**
- z2= 01******
- z3= 0100****

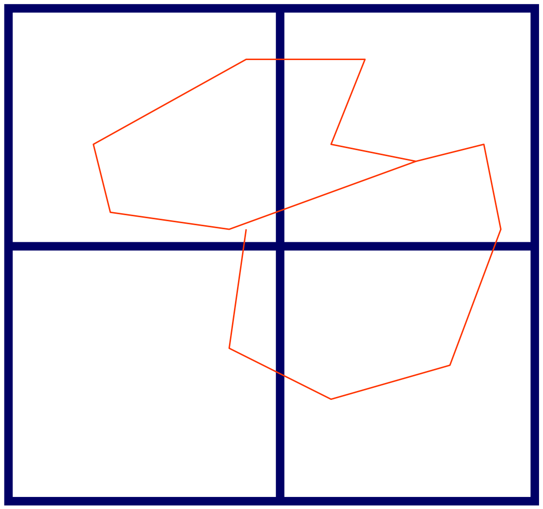


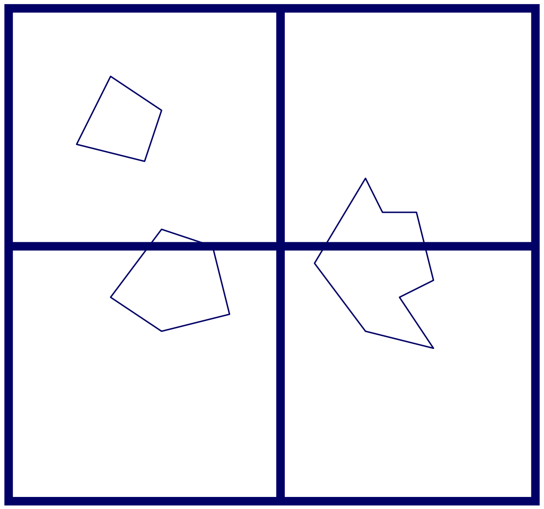
T/F r2 contains r1 - TRUE (prefix property)
T/F r3 contains r1 - FALSE (disjoint)
T/F r3 contains r2 - FALSE (r2 contains r3)

z-ordering - regions

Spatial joins: find (quickly) all

counties intersecting lakes

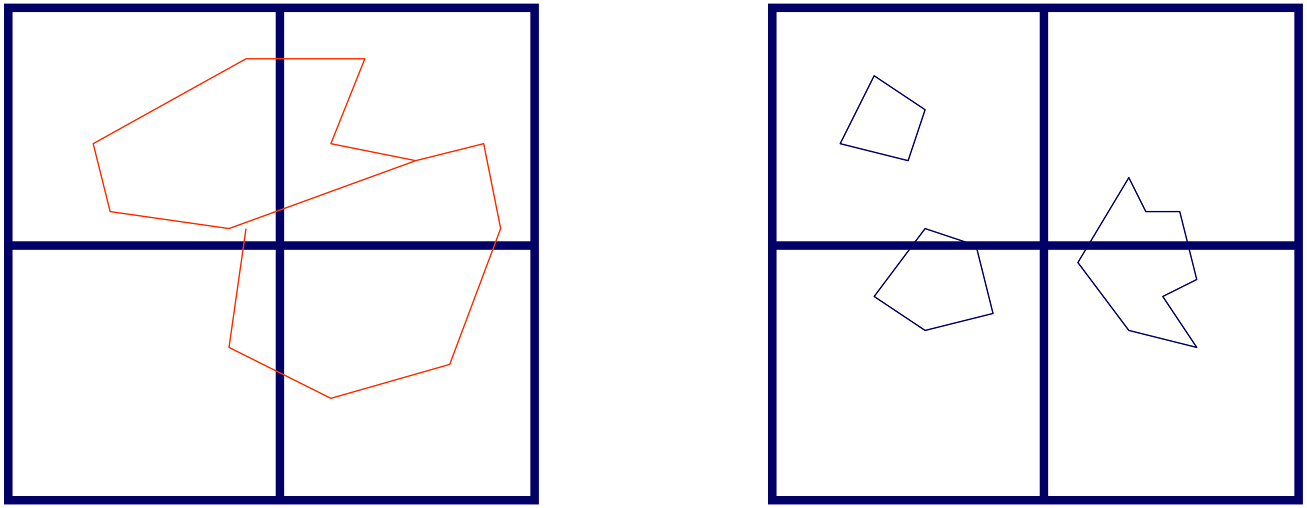




z-ordering - regions

Spatial joins: find (quickly) all

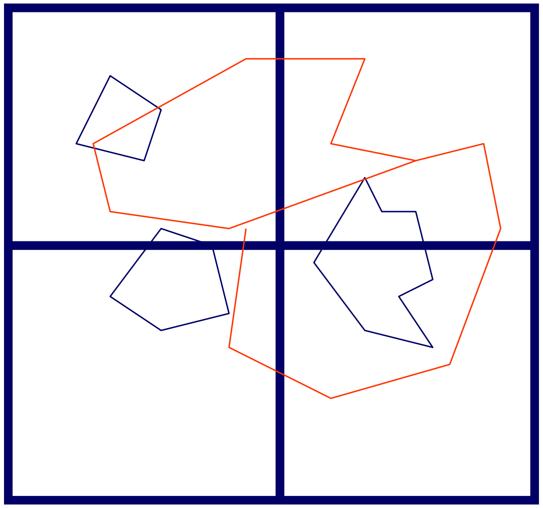
counties intersecting lakes



z-ordering - regions

Spatial joins: find (quickly) all

counties intersecting lakes



z-ordering - regions

Spatial joins: find (quickly) all

counties intersecting lakes

Naive algorithm: O(N * M)

Something faster?

z-ordering - regions

Spatial joins: find (quickly) all
counties intersecting lakes

z	obj-id	etc
0010	ALG	
…	…	
1000	WAS	
11**	ALG	

z	obj-id	etc
0011	Erie	
0101	Erie	
…		
10**	Ont.	

z-ordering - regions

Spatial joins: find (quickly) all

counties intersecting lakes

Solution: merge the lists of (sorted) z-values, looking for the prefix property

footnote#1: ‘*’ needs careful treatment

footnote#2: need dup. elimination

z-ordering - Detailed outline

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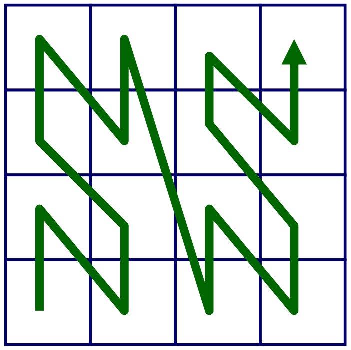
    - ➡ analysis; variations
  - R-trees
  - ...

z-ordering - variations

Q: is z-ordering the best we can do?

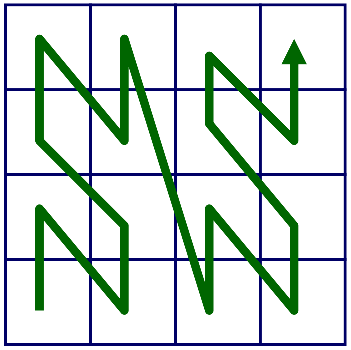


z-ordering - variations

Q: is z-ordering the best we can do?

A: probably not - occasional long 'jumps'

Q: then?

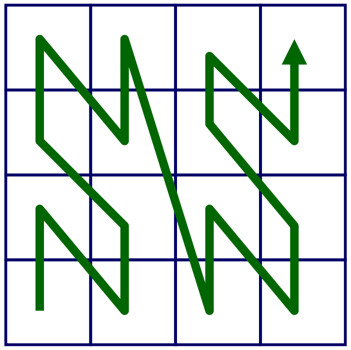


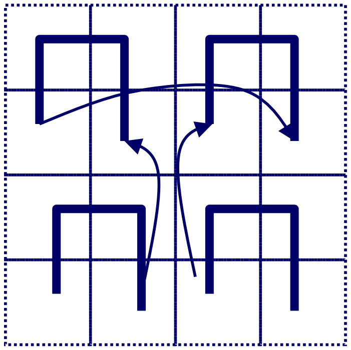
z-ordering - variations

Q: is z-ordering the best we can do?

A: probably not - occasional long ‘jumps’

Q: then? A1: Gray codes





(Gray codes)

- Ingenious way to spot flickering LED – binary:

3.5V →

000	0
001	1
010	2
011	3
100	4
101	5
110	6
111	7

F. Gray. *Pulse code communication*,
March 17, 1953
U.S. Patent 2,632,058

(Gray codes)

- Ingenious way to spot flickering LED

0
1

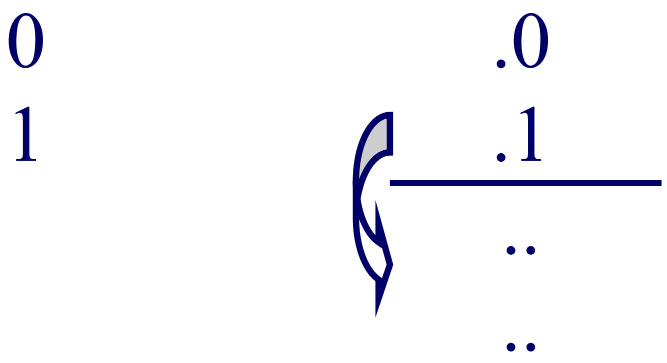
(Gray codes)

- Ingenious way to spot flickering LED

0	.0
1	.1
	..
	..

(Gray codes)

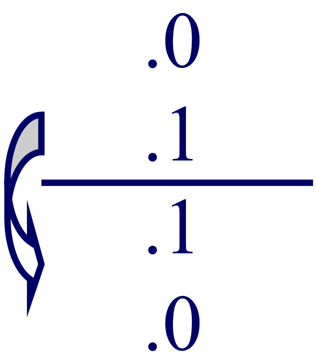
- Ingenious way to spot flickering LED



(Gray codes)

- Ingenious way to spot flickering LED

0
1



.0

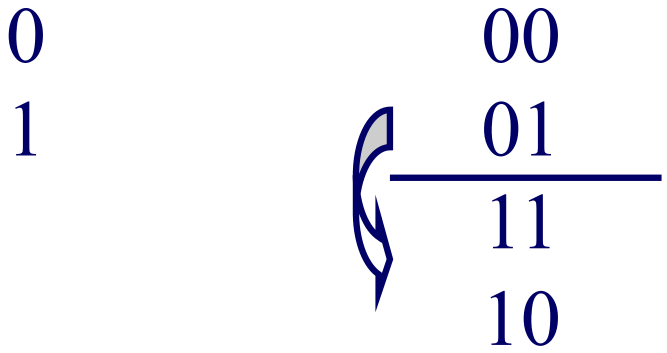
.1

.1

.0

(Gray codes)

- Ingenious way to spot flickering LED



0 00

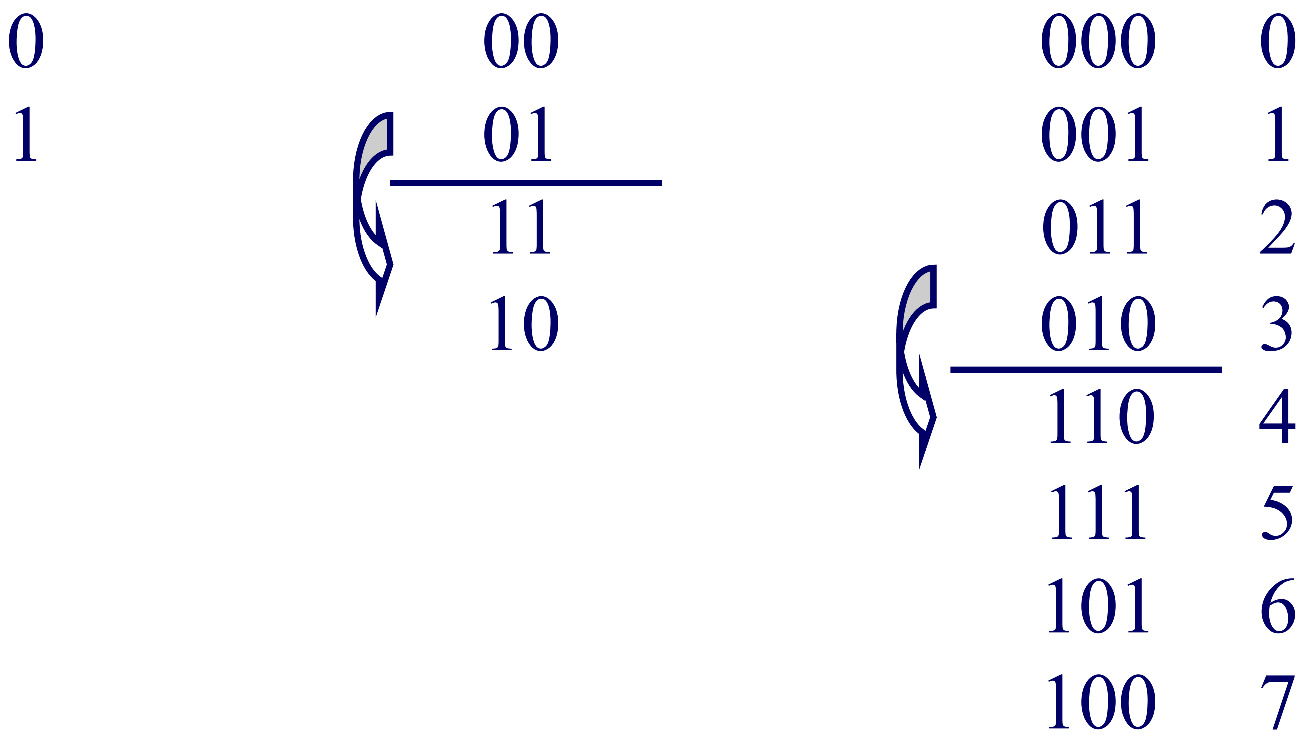
1 01

11

10

(Gray codes)

- Ingenious way to spot flickering LED



0
1

00
01
11
10

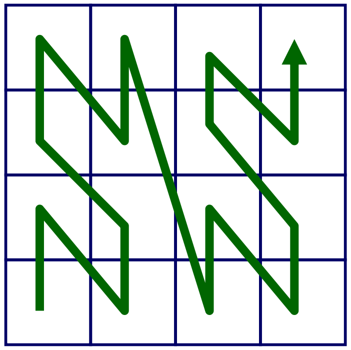
000	0
001	1
011	2
010	3
110	4
111	5
101	6
100	7

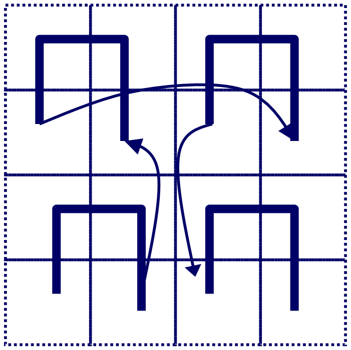
z-ordering - variations

Q: is z-ordering the best we can do?

A: probably not - occasional long ‘jumps’

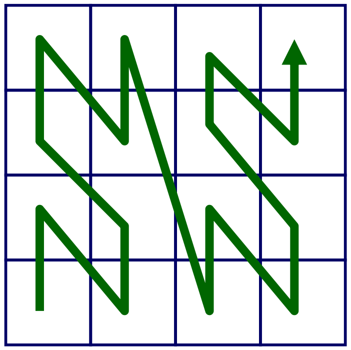
Q: then? A1: Gray codes – CAN WE DO BETTER?

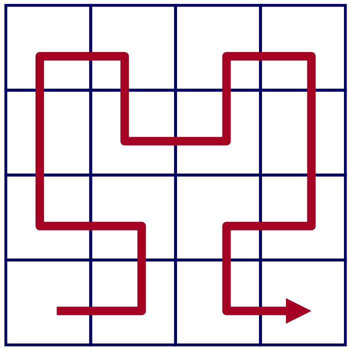




z-ordering - variations

A2: Hilbert curve! (a.k.a. Hilbert-Peano curve)





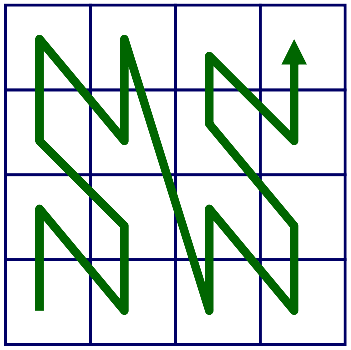
(break)

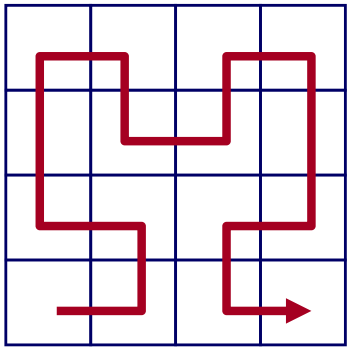
David Hilbert (1862-1943)

Giuseppe Peano (1858-1932)

z-ordering - variations

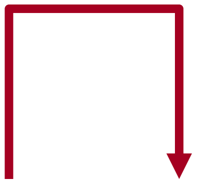
'Looks' better (never long jumps). How to derive it?



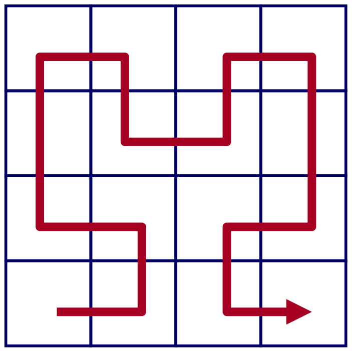


z-ordering - variations

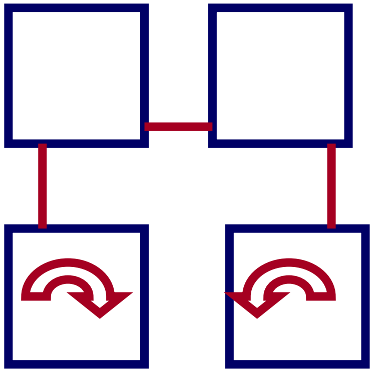
‘Looks’ better (never long jumps). How to derive it?



order-1



order-2



… order (n+1)

z-ordering - variations

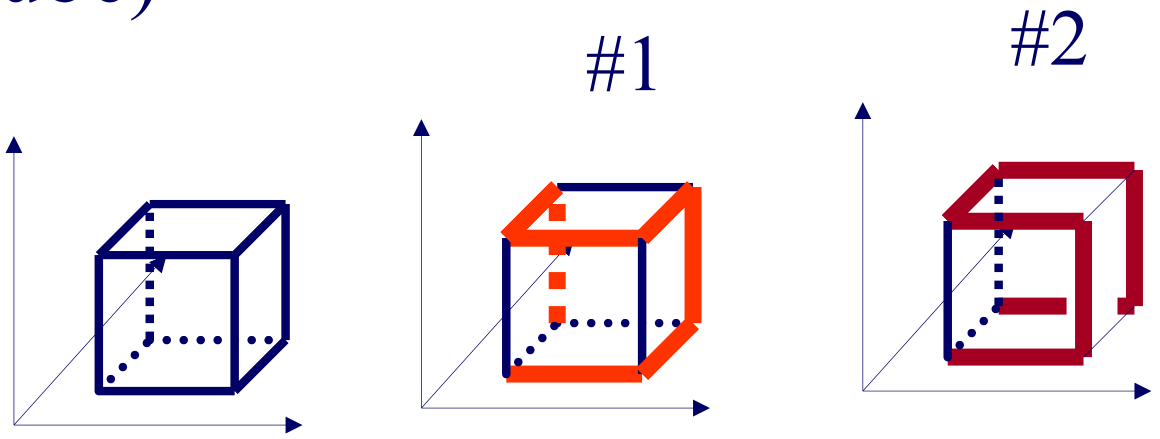
Q: function for the Hilbert curve ($h = f(x,y)$)?

A: bit-shuffling, followed by post-processing, to account for rotations. Linear on # bits. See textbook, for pointers to code/algorithms (eg., [Jagadish, 90])

z-ordering - variations

Q: how about Hilbert curve in 3-d? n-d?

A: Exists (and is not unique!). Eg., 3-d, order-1 Hilbert curves (Hamiltonian paths on cube)



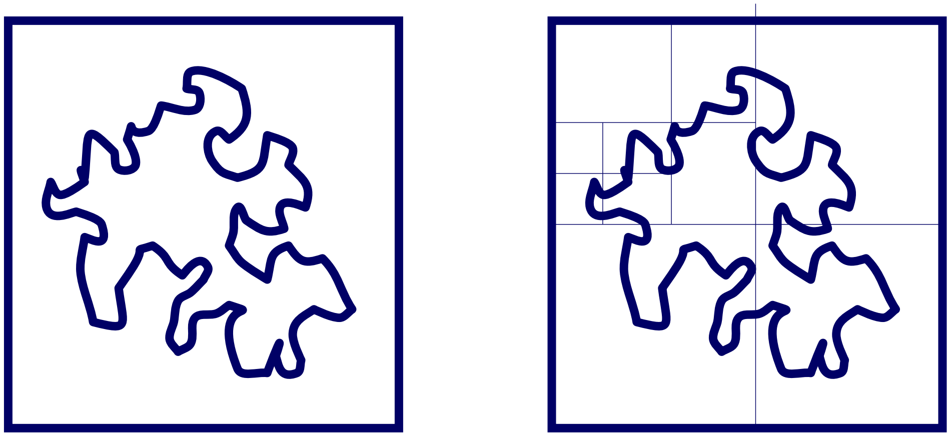
z-ordering - Detailed outline

- spatial access methods
  - z-ordering
    - main idea - 3 methods
    - use w/ B-trees; algorithms (range, knn queries ...)
    - non-point (eg., region) data
    - analysis; variations
  - R-trees
  - ...

z-ordering - analysis

Q: How many pieces (‘quad-tree blocks’) per region?

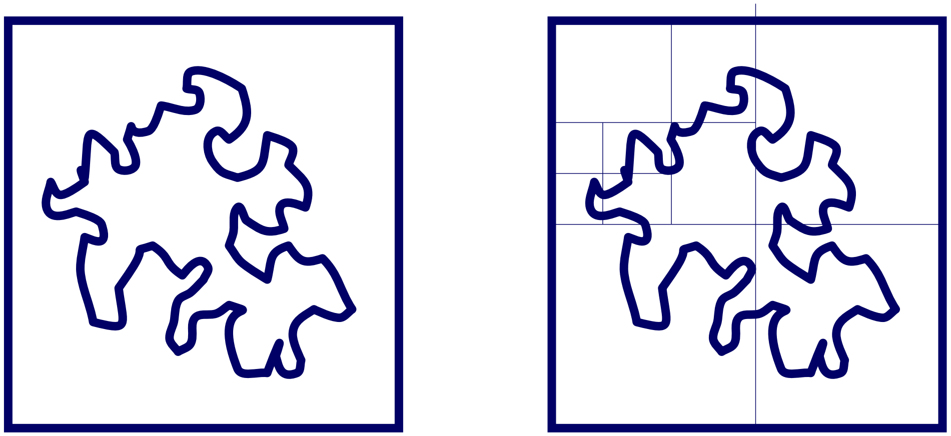
A: proportional to perimeter (surface etc)



z-ordering - analysis

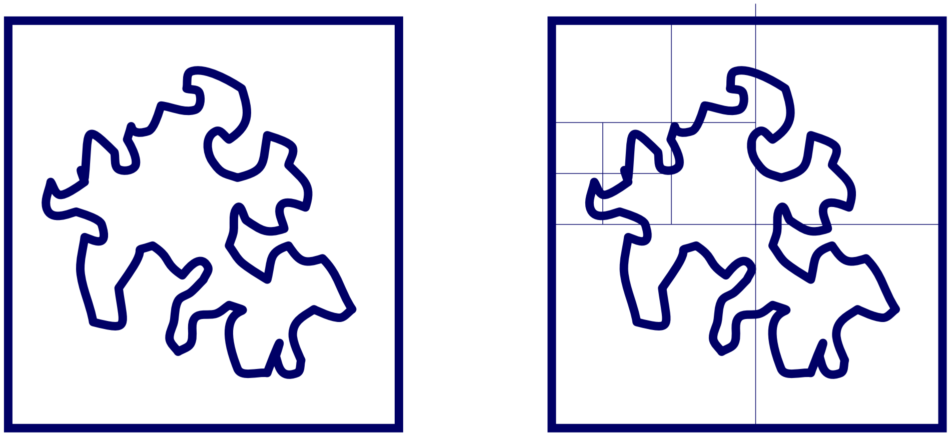
(How long is the coastline, say, of England?

Paradox: The answer changes with the yard-stick -> fractals ...)



z-ordering - analysis

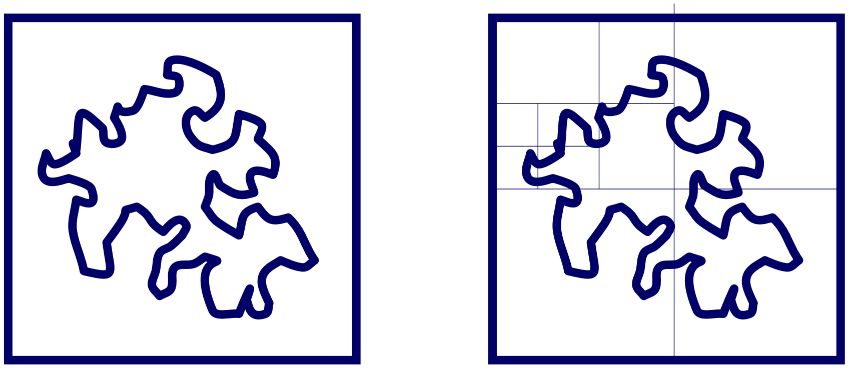
Q: Should we decompose a region to full detail (and store in B-tree)?



z-ordering - analysis

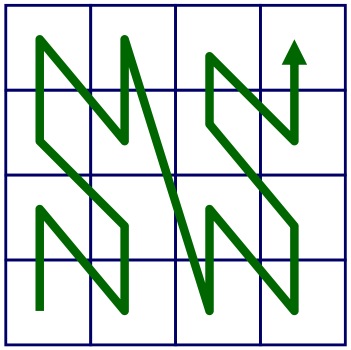
Q: Should we decompose a region to full detail (and store in B-tree)?

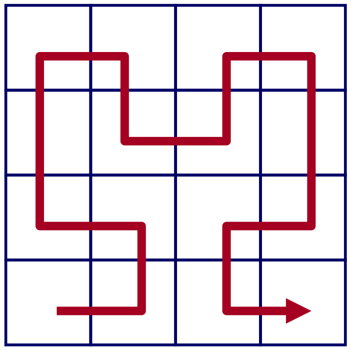
A: NO! approximation with 1-3 pieces/z-values is best [Orenstein90]



z-ordering - analysis

Q: how to measure the ‘goodness’ of a curve?

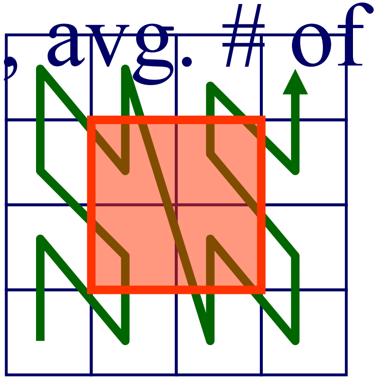


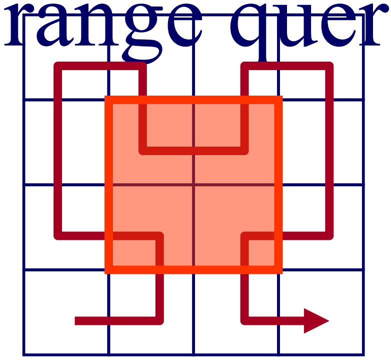


z-ordering - analysis

Q: how to measure the ‘goodness’ of a curve?

A: e.g., avg. # of runs, for range queries





4 runs 3 runs

(#runs ~ #disk accesses on B-tree)

z-ordering - analysis

Q: So, is Hilbert really better?

A: 27% fewer runs, for 2-d (similar for 3-d)

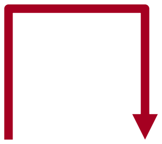
Q: are there formulas for #runs, #of quadtree blocks etc?

A: Yes ([Jagadish; Moon+ etc] see textbook)

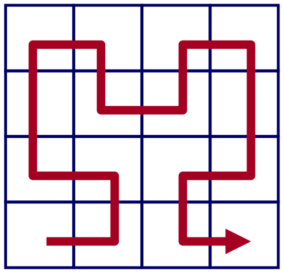
z-ordering - fun observations

Hilbert and z-ordering curves: "space filling curves": eventually, they visit every point

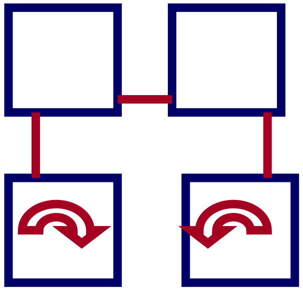
in n-d space - therefore:



order-1



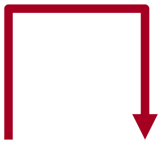
order-2



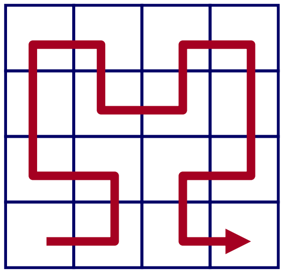
... order (n+1)

z-ordering - fun observations

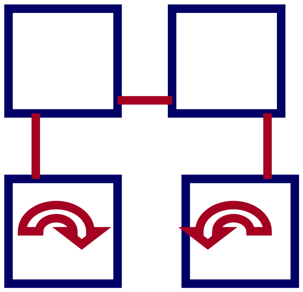
... they show that the plane has as many points as a line (-> headaches for 1900’s mathematics/topology). (fractals, again!)



order-1



order-2

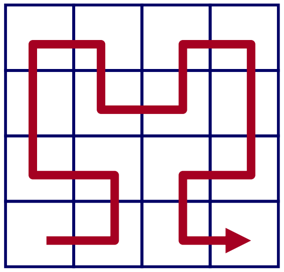


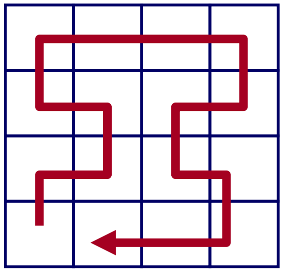
... order (n+1)

z-ordering - fun observations

Observation #2: Hilbert (like) curve for video encoding [Y. Matias+, CRYPTO '87]:

Given a frame, visit its pixels in randomized hilbert order; compress; and transmit



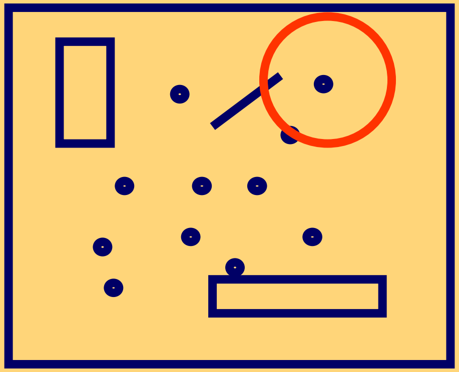


z-ordering - fun observations

In general, Hilbert curve is great for preserving distances, clustering, vector quantization etc

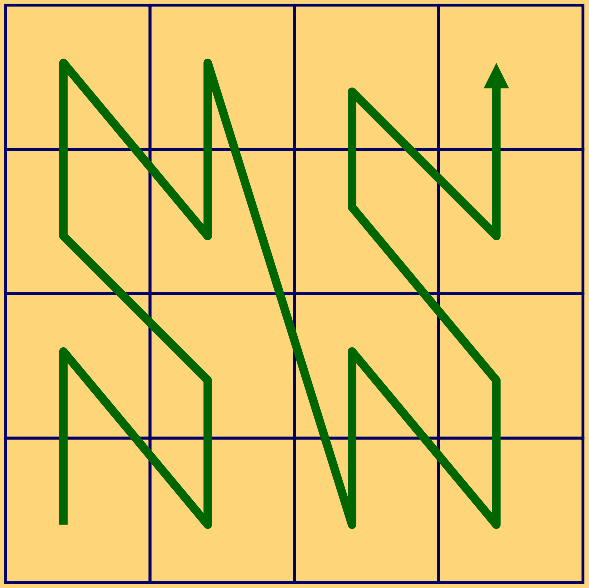
Spatial Access Methods - problem

- Given a collection of geometric objects (points, lines, polygons, ...)
- Find cities within 100mi from Pittsburgh



Solution#1: z-ordering

A: z-ordering/bit-shuffling/linear-quadtrees



Conclusions

- z-ordering is a great idea (n-d points -> 1-d points; feed to B-trees)
- used by TIGER system

  http://www.census.gov/geo/www/tiger/
- and (most probably) by other GIS products
- works great with low-dim points