15-826: Multimedia (Databases) and Data Mining

Lecture #6: Spatial Access Methods

Part III: R-trees

C. Faloutsos

Must-read material

- MM-Textbook, Chapter 5.2
- Ramakrinshan+Gehrke, Chapter 28.6
- Guttman, A. (June 1984). *R-Trees: A Dynamic Index Structure for Spatial Searching*. Proc. ACM SIGMOD, Boston, Mass.

R-trees – impact:

- Popular method; like multi-d B-trees
- guaranteed utilization; fast search (low dim's)
- Used in practice:
  - Oracle spatial (R-tree)
  - Postgres: `create index … using gist`
  - Databricks (R-trees and z-order)
  - Sqlite3: www.sqlite.org/rtree.html
  - Python: `(pip install rtree)`

Outline

Goal: 'Find similar / interesting things'

- Intro to DB
- Indexing - similarity search
- Data Mining

Indexing - Detailed outline

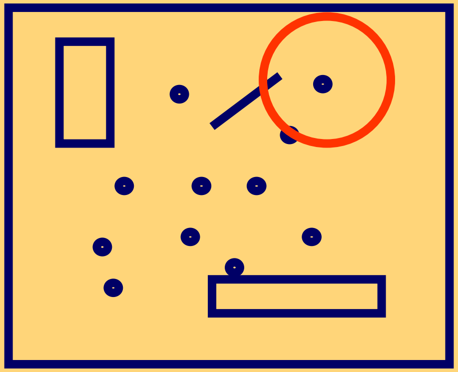
- primary key indexing
- secondary key / multi-key indexing
- spatial access methods
  - problem dfn
  - z-ordering
  - R-trees
  - ...
- text
- ...

Indexing - more detailed outline

- R-trees
  - main idea; file structure
  - algorithms: insertion/split
  - deletion
  - search: range, nn, spatial joins
  - performance analysis
  - variations (packed; hilbert;...)

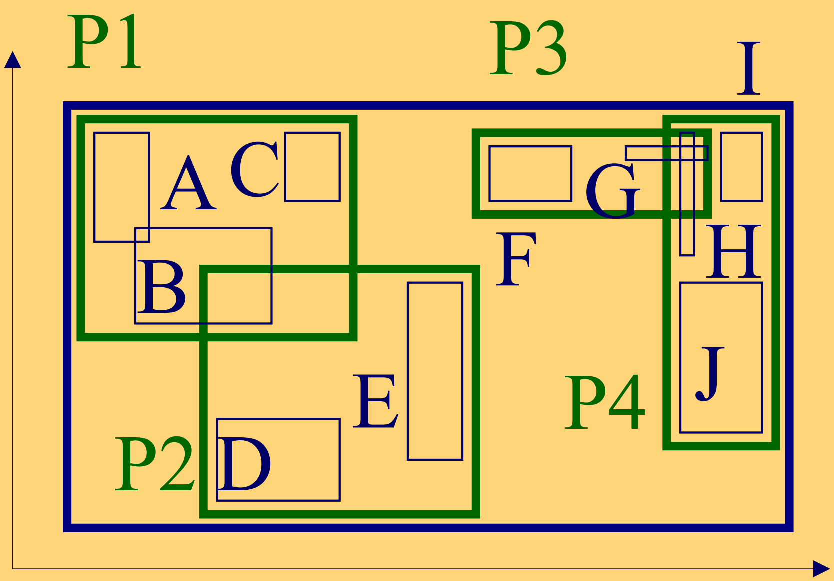
Spatial Access Methods - problem

- Given a collection of geometric objects (points, lines, polygons, ...)
- Find cities within 100mi from Pittsburgh



Solution#2: R-trees

- multi-dim trees
- Allow nodes to overlap
- Guaranteed 50% utilization



R-trees

- z-ordering: cuts regions to pieces -> dup. elim.
- how could we avoid that?
- Idea: try to extend/merge B-trees and k-d trees

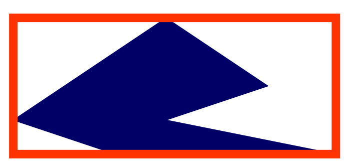
R-trees

- [Guttman 84] Main idea: allow parents to overlap!

Antonin Guttman
[https://dblp.org/pid/81/3404.html]

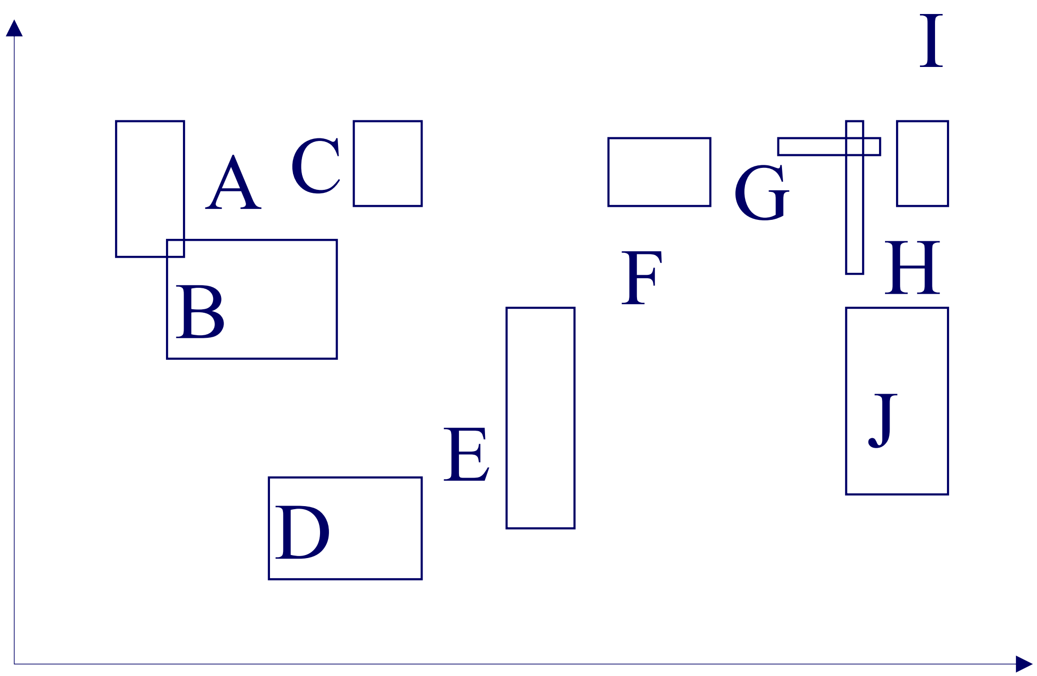
R-trees

- [Guttman 84] Main idea: allow parents to overlap!
  - => guaranteed 50% utilization
  - => easier insertion/split algorithms.
  - (only deal with Minimum Bounding Rectangles - **MBR**s)



R-trees

- eg., w/ fanout 4: group nearby rectangles to parent MBRs; each group -> disk page



R-trees

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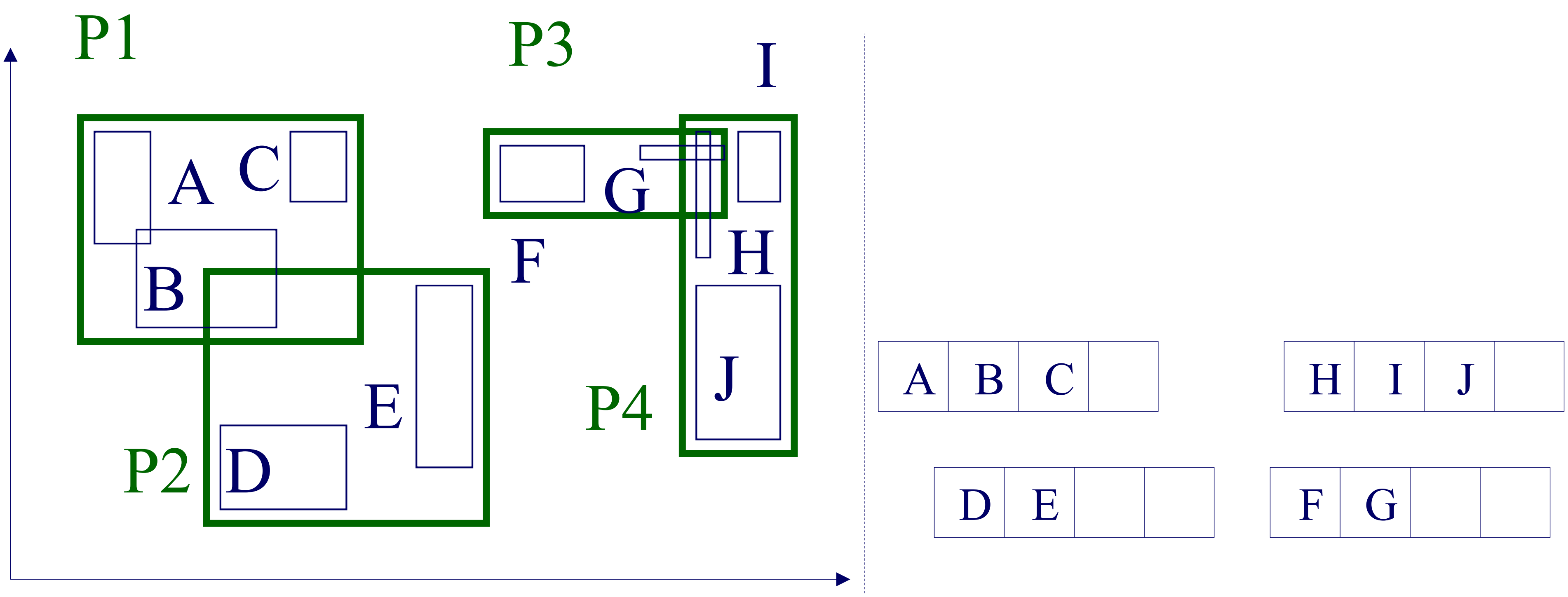

R-trees

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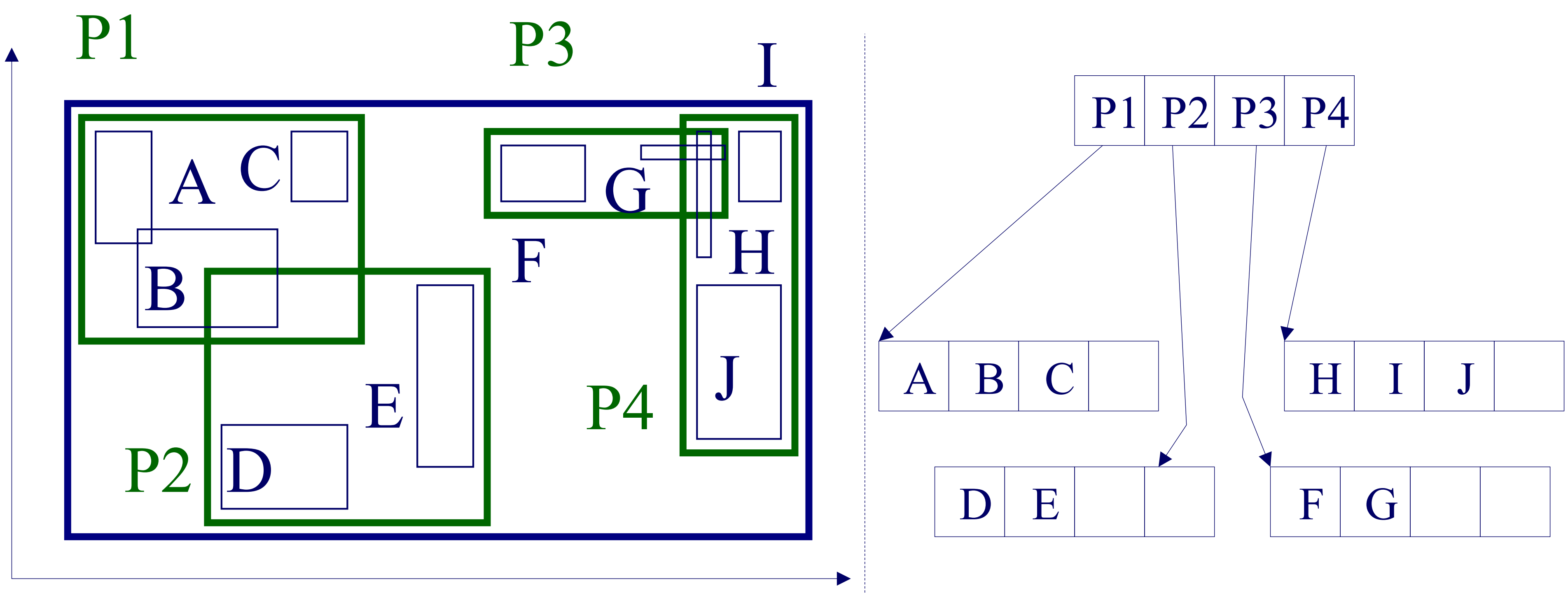
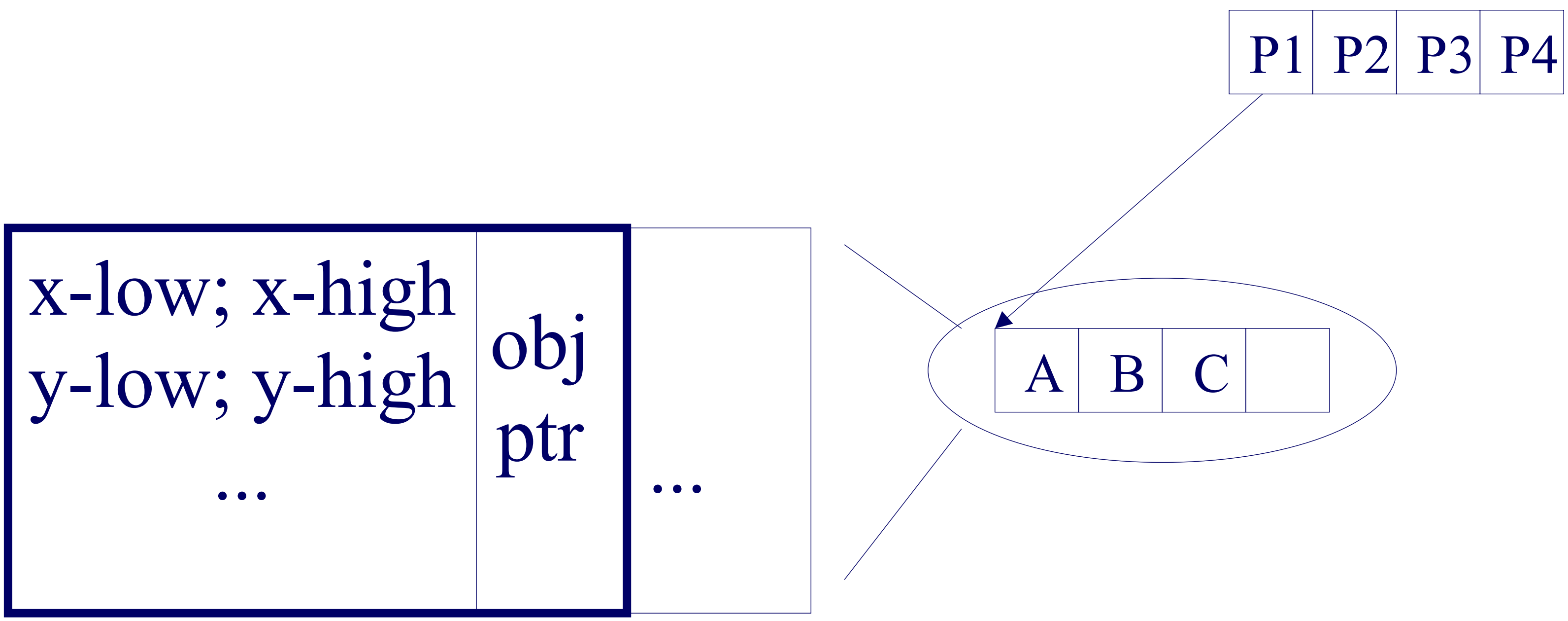


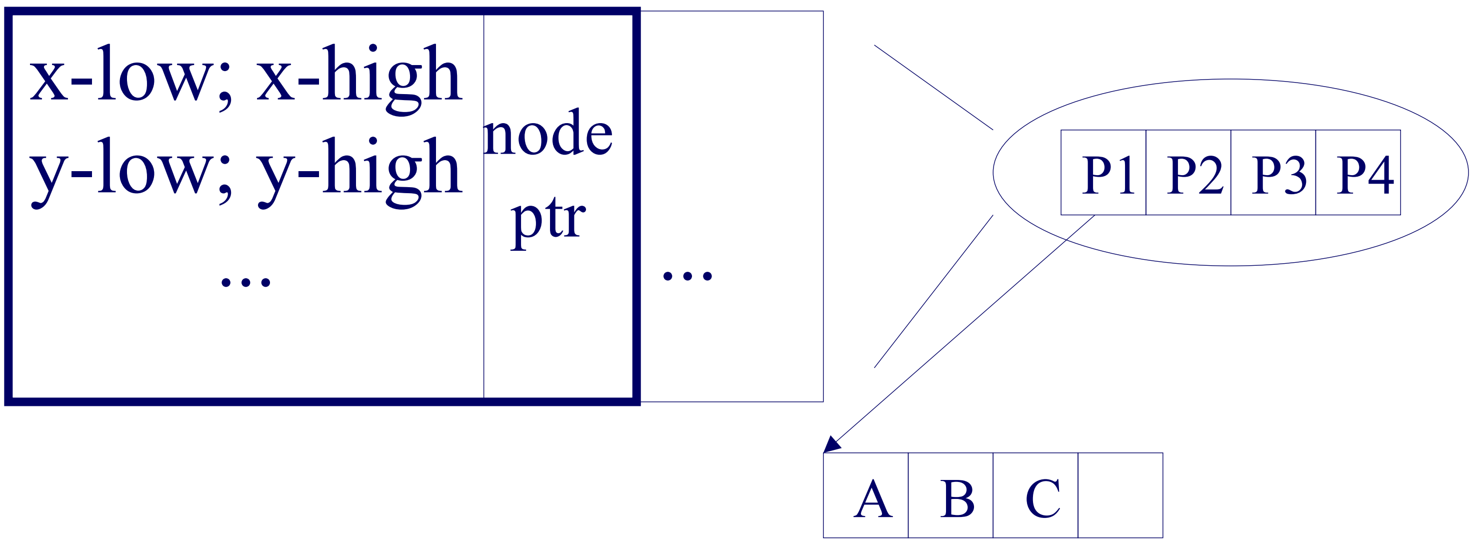
R-trees - format of nodes

- {(MBR; obj-ptr)} for leaf nodes

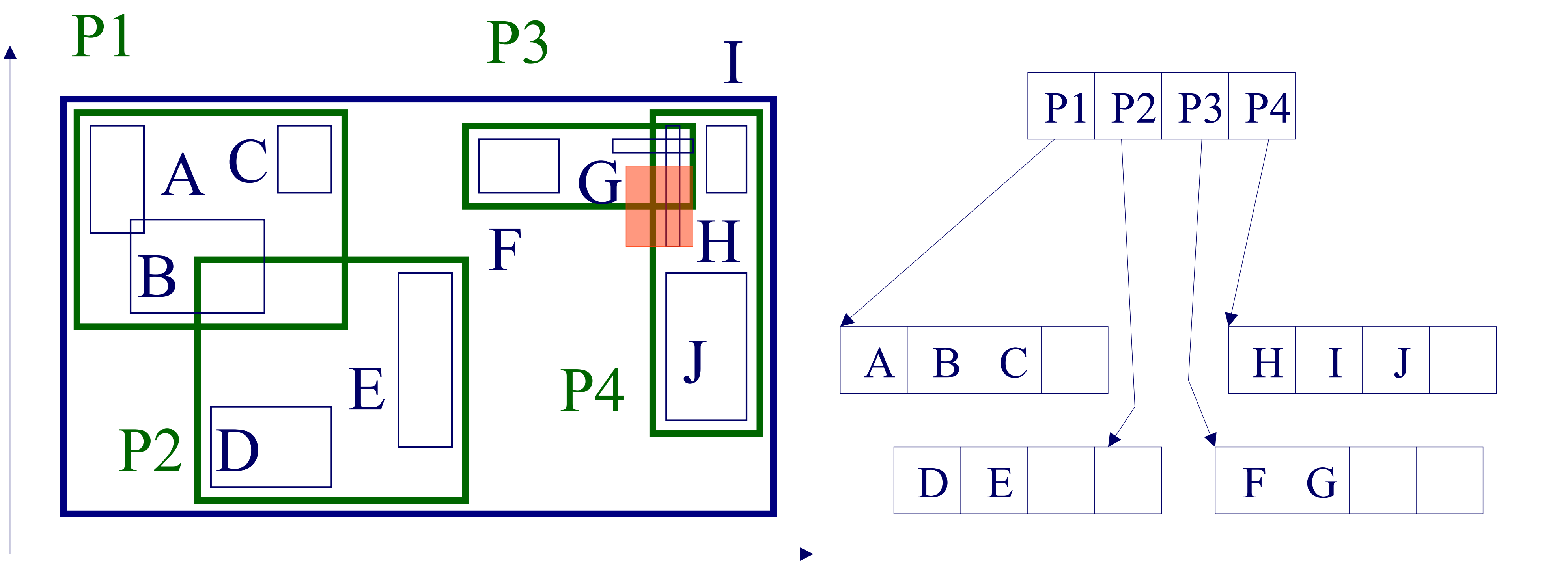


R-trees - format of nodes

- {(MBR; node-ptr)} for non-leaf nodes



R-trees - range search?



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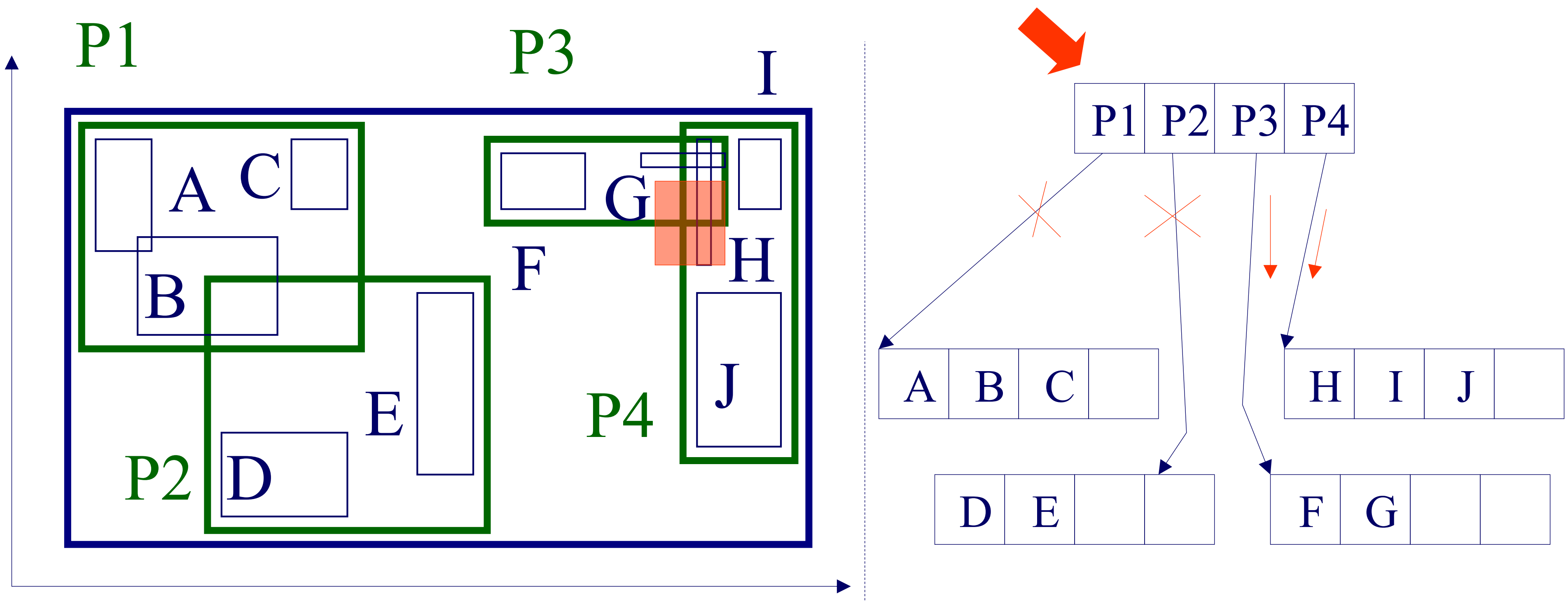

R-trees - range search

Observations:

- every parent node completely covers its 'children'
- a child MBR may be covered by more than one parent - it is stored under ONLY ONE of them. (ie., no need for dup. elim.)

R-trees - range search

Observations - cont’d

- a point query may follow multiple branches.
- everything works for **any** dimensionality

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DETAILS

Indexing - more detailed outline

- R-trees
  - main idea; file structure
  - algorithms: insertion/split
  - deletion
  - search: range, nn, spatial joins
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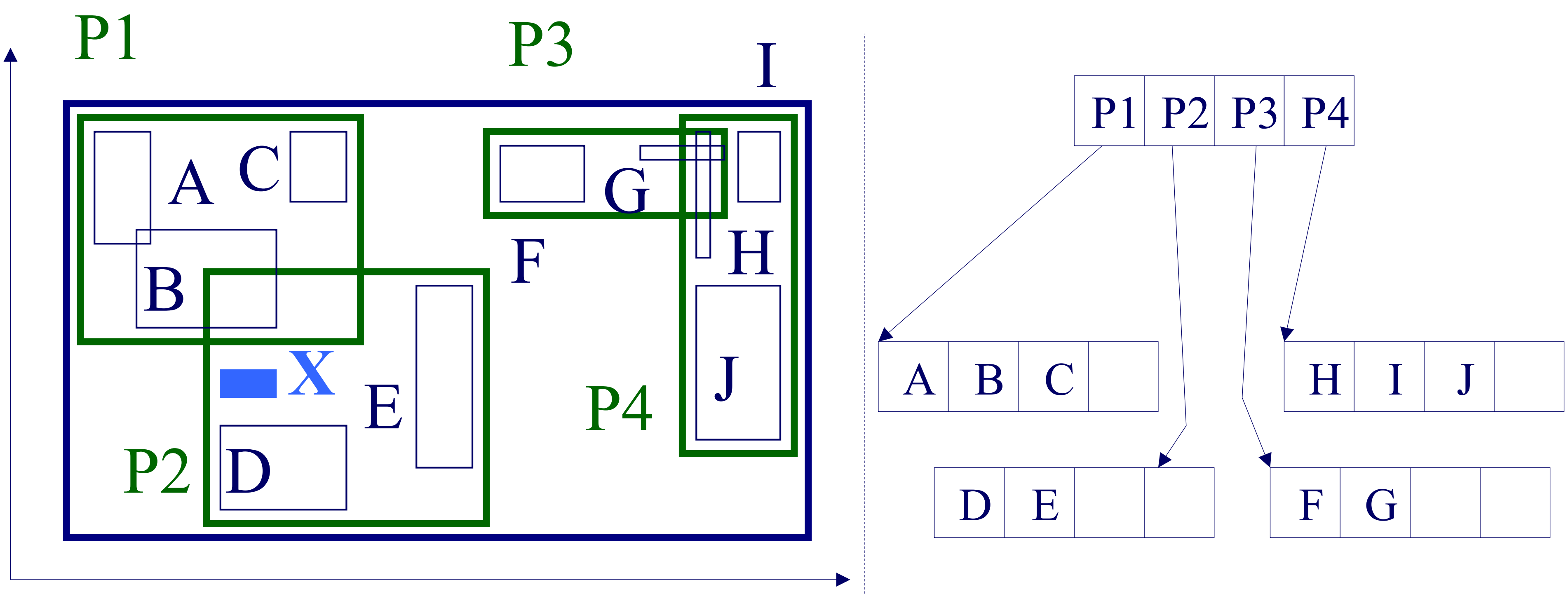
  - variations (packed; hilbert;...)

DETAILS

R-trees - insertion

- eg., rectangle 'X'



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DETAILS

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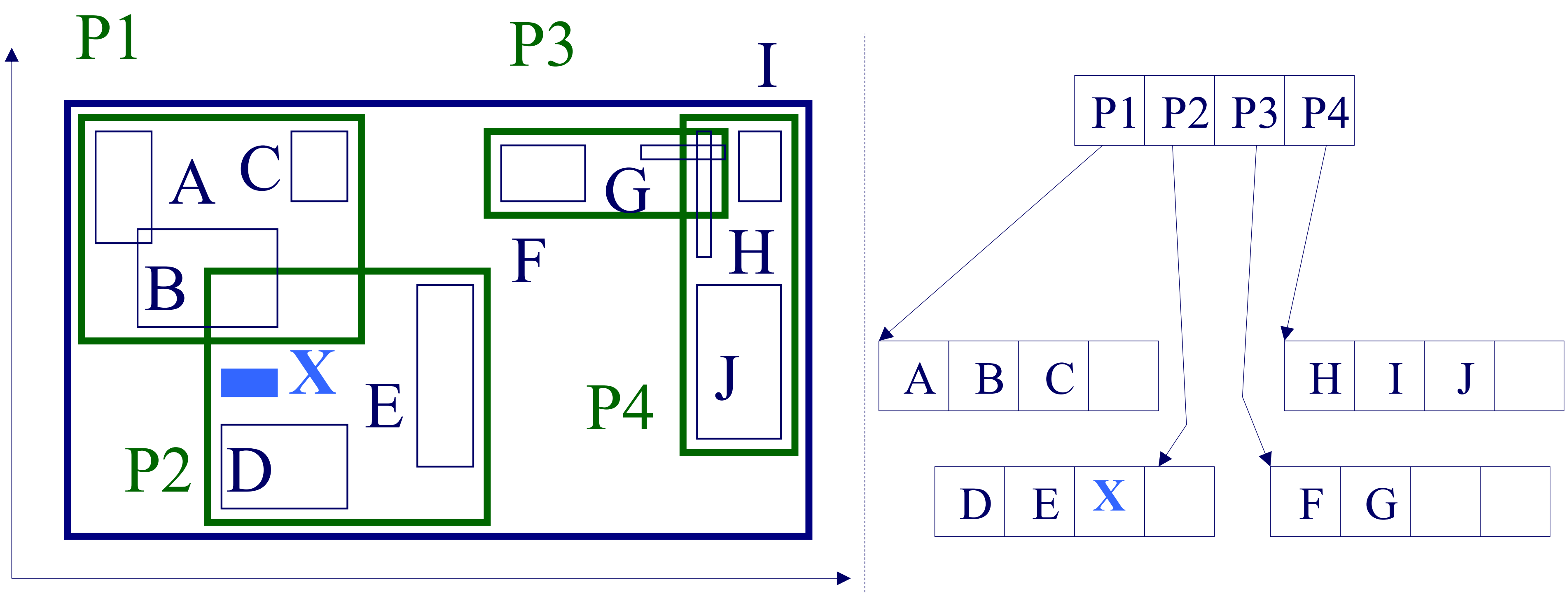

R-trees - insertion

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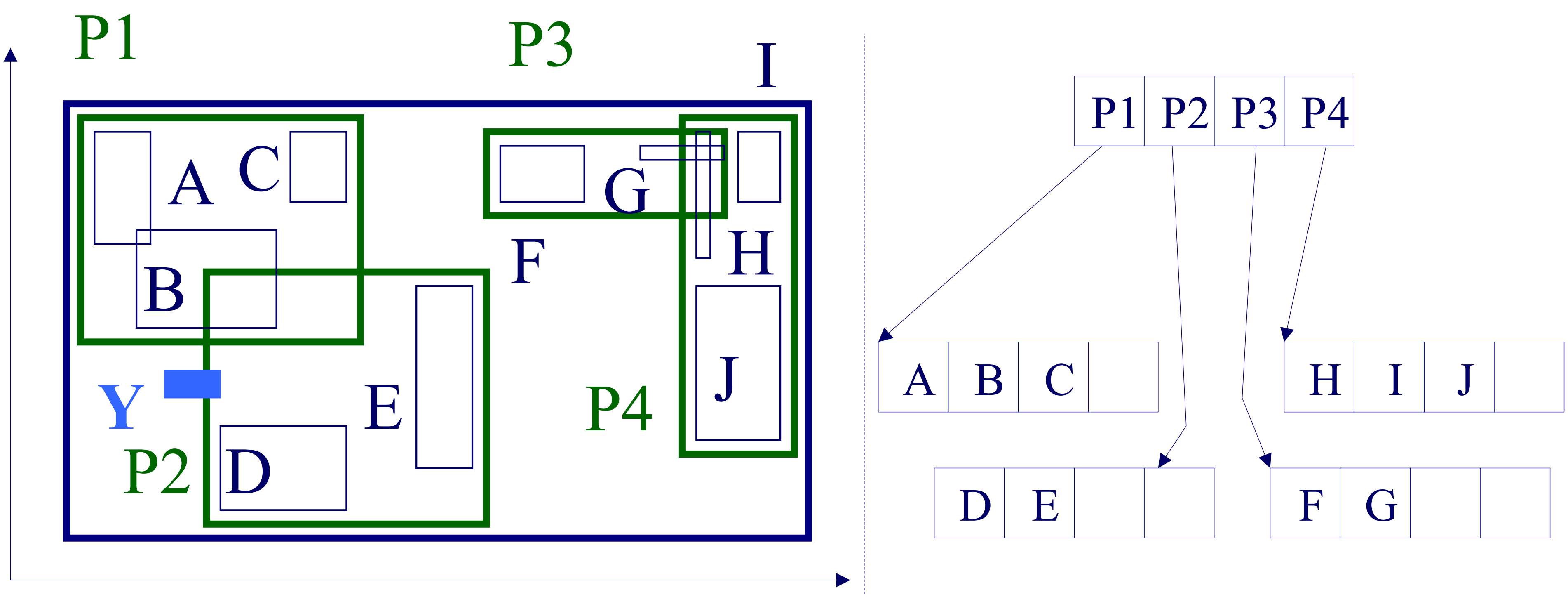
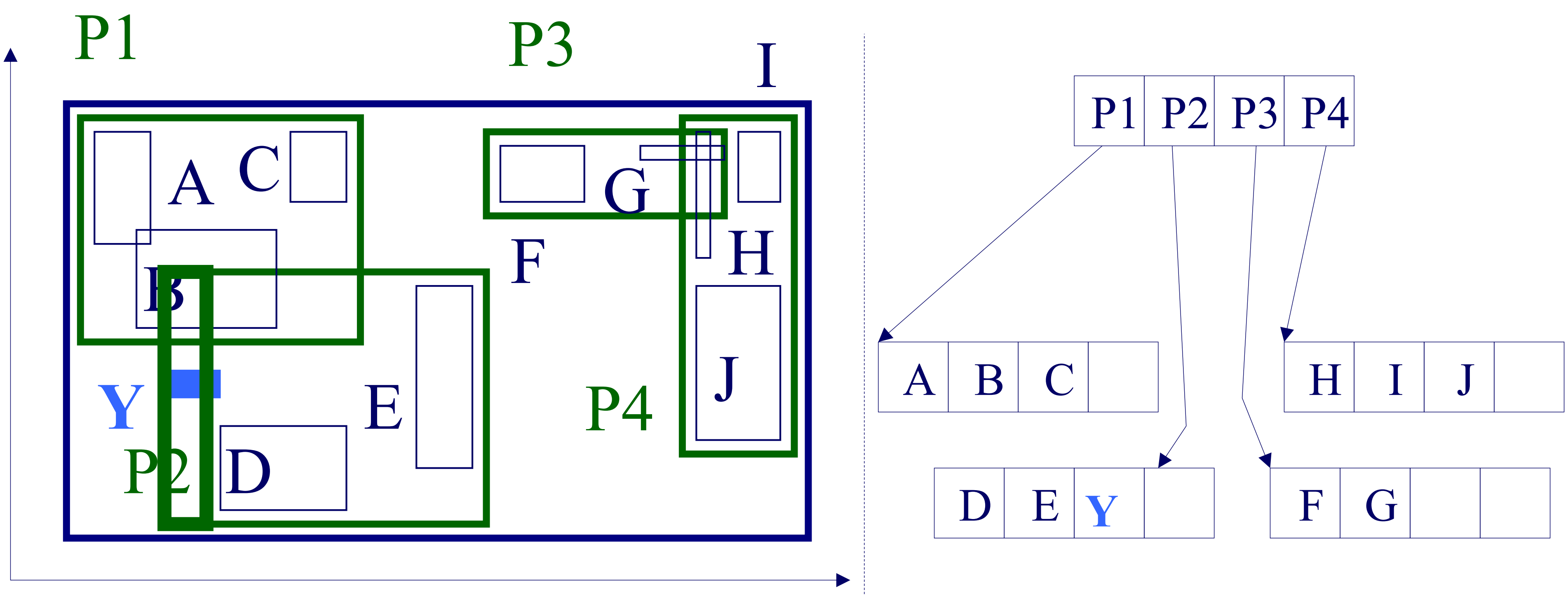


DETAILS

R-trees - insertion

- eg., rectangle 'Y' : extend suitable parent.



DETAILS

R-trees - insertion

- eg., rectangle 'Y' : extend suitable parent.
- Q: how to measure 'suitability' ?

DETAILS

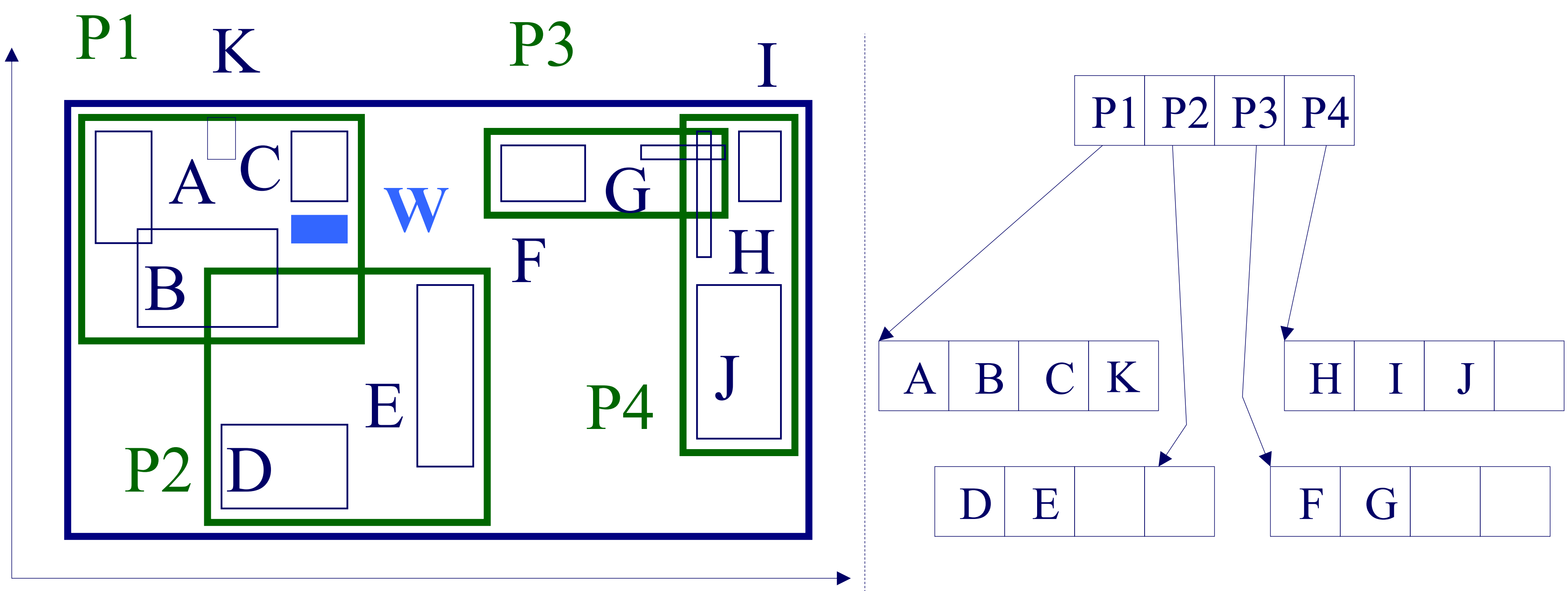
R-trees - insertion

- eg., rectangle 'Y': extend suitable parent.
- Q: how to measure 'suitability'?
- A: by increase in area (volume) (more details: later, under 'performance analysis')
- Q: what if there is no room? how to split?

DETAILS

R-trees - insertion

- eg., rectangle 'W'



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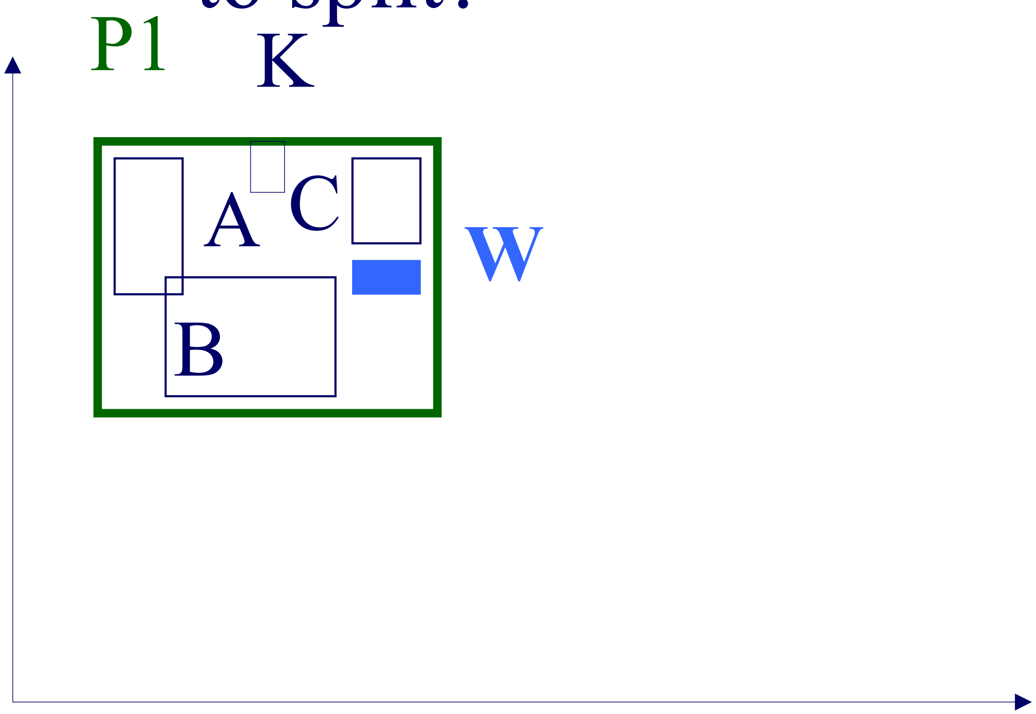
DETAILS

R-trees - insertion

- eg., rectangle 'W' - focus on 'P1' - how to split?



DETAILS

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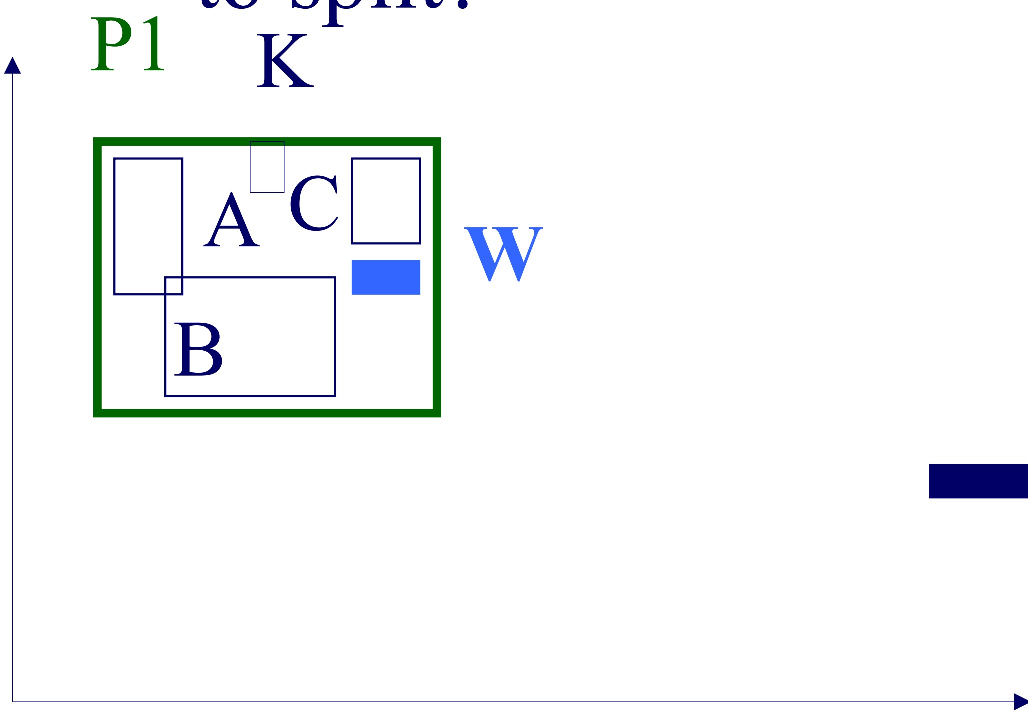
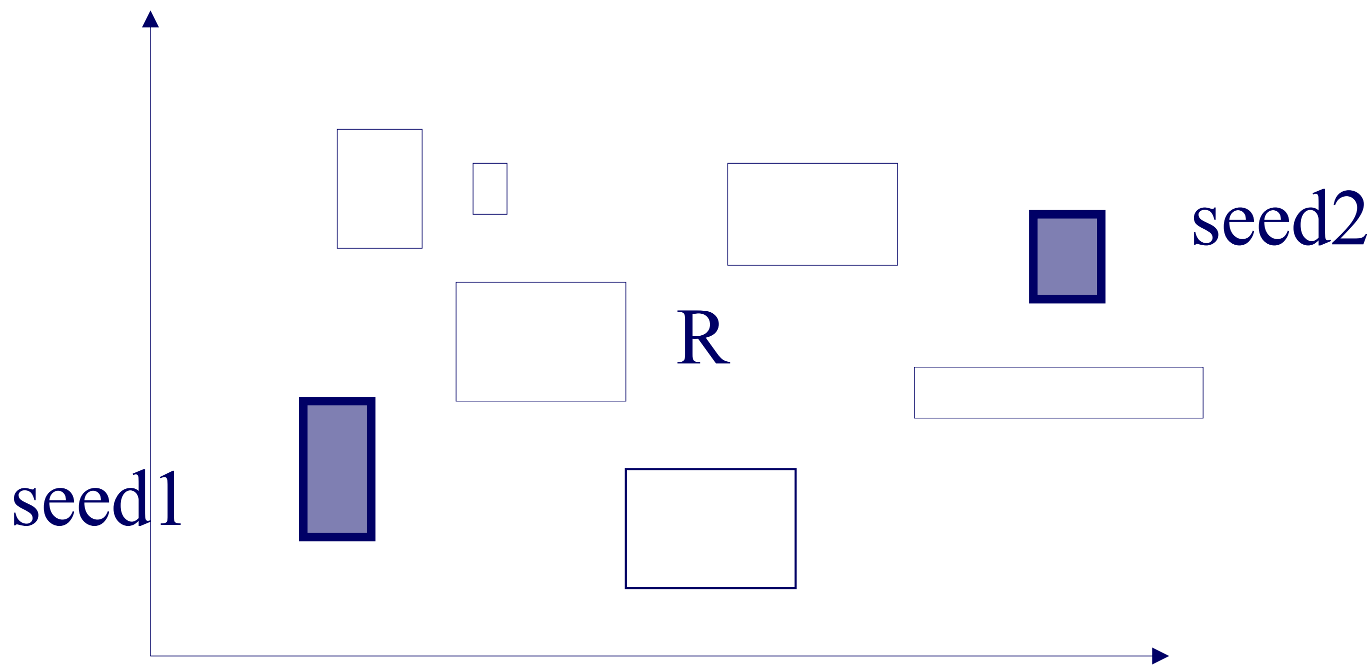


- (A1: plane sweep,
  until 50% of rectangles)
- A2: ‘linear’ split
- A3: quadratic split
- A4: exponential split

DETAILS

R-trees - insertion & split

- pick two rectangles as ‘seeds’;
- assign each rectangle ‘R’ to the ‘closest’ ‘seed’



DETAILS

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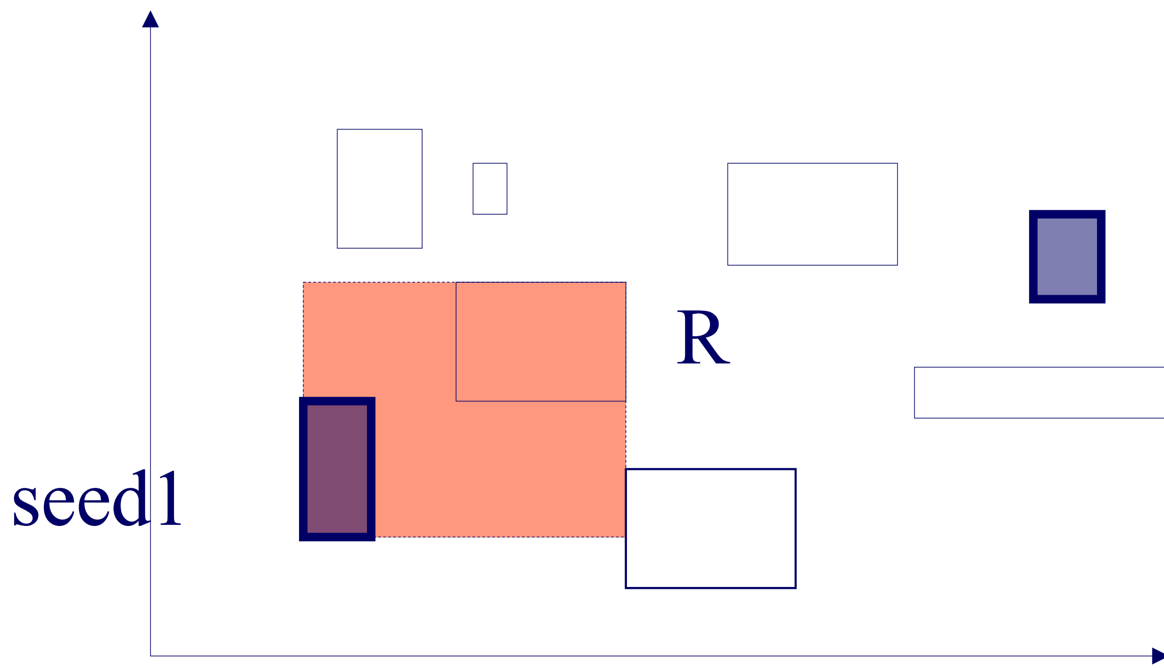

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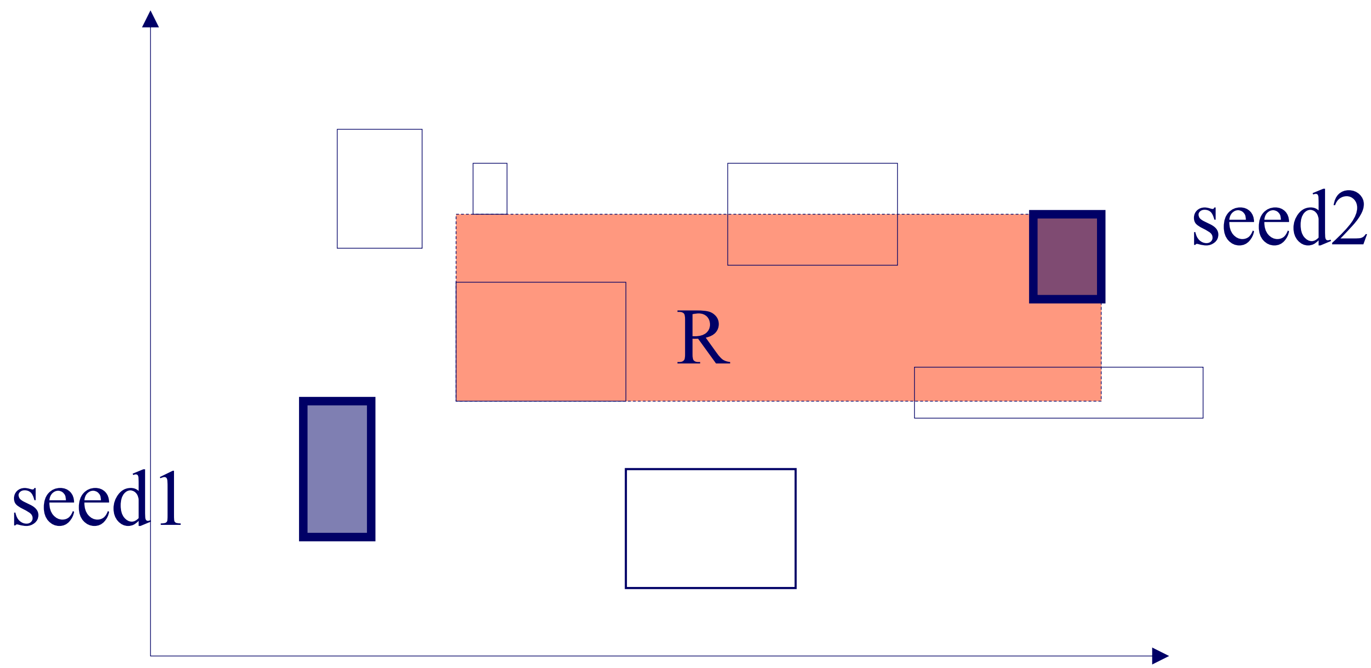

DETAILS

R-trees - insertion & split

- pick two rectangles as 'seeds';
- assign each rectangle 'R' to the 'closest' 'seed'
- smart idea: pre-sort rectangles according to delta of closeness (ie., schedule easiest choices first!)

DETAILS

R-trees - insertion - pseudocode

- decide which parent to put new rectangle into ('closest' parent)
- if overflow, split to two, using (say,) the quadratic split algorithm
  - propagate the split upwards, if necessary
- update the MBRs of the affected parents.

DETAILS

R-trees - insertion - observations

- **many** more split algorithms exist (see refs)

DETAILS

Indexing - more detailed outline

- R-trees
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  - performance analysis
  - variations (packed; hilbert;...)

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DETAILS

R-trees - deletion

- delete rectangle
- if underflow
  - ??

DETAILS

R-trees - deletion

- delete rectangle
- if underflow
  - temporarily delete all siblings (!);
  - delete the parent node and
  - re-insert them

Indexing - more detailed outline

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R-trees - range search

pseudocode:

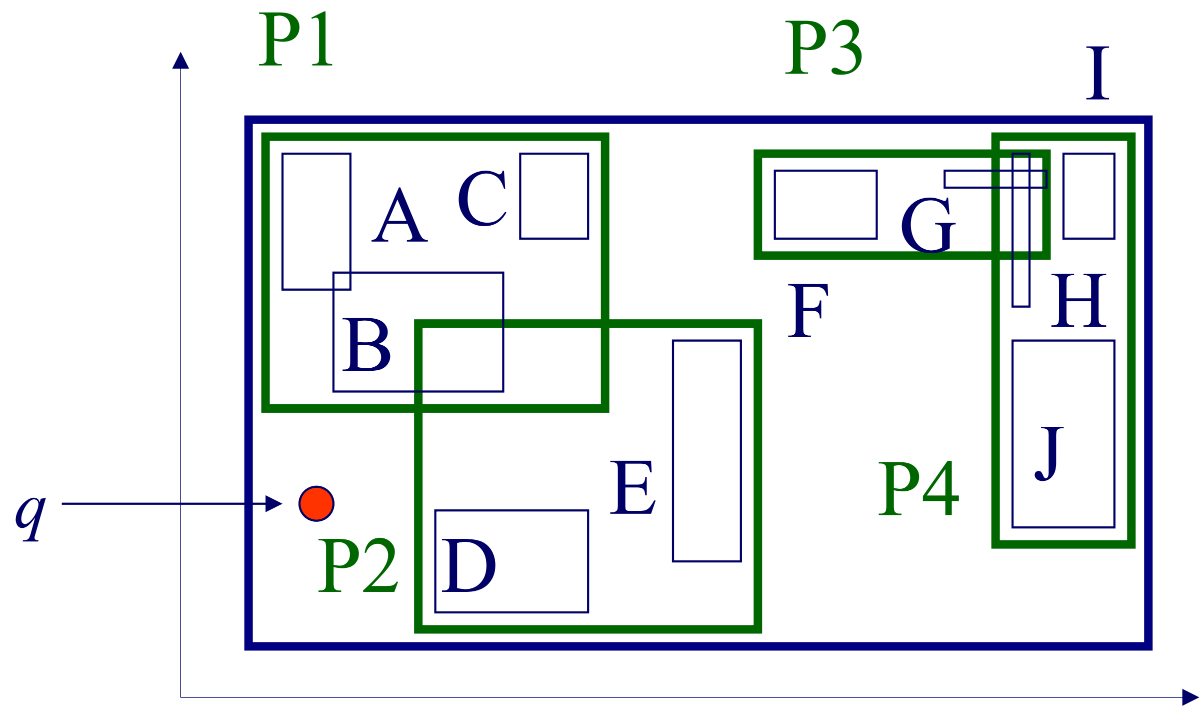
check the root

for each branch,

if its MBR intersects the query rectangle

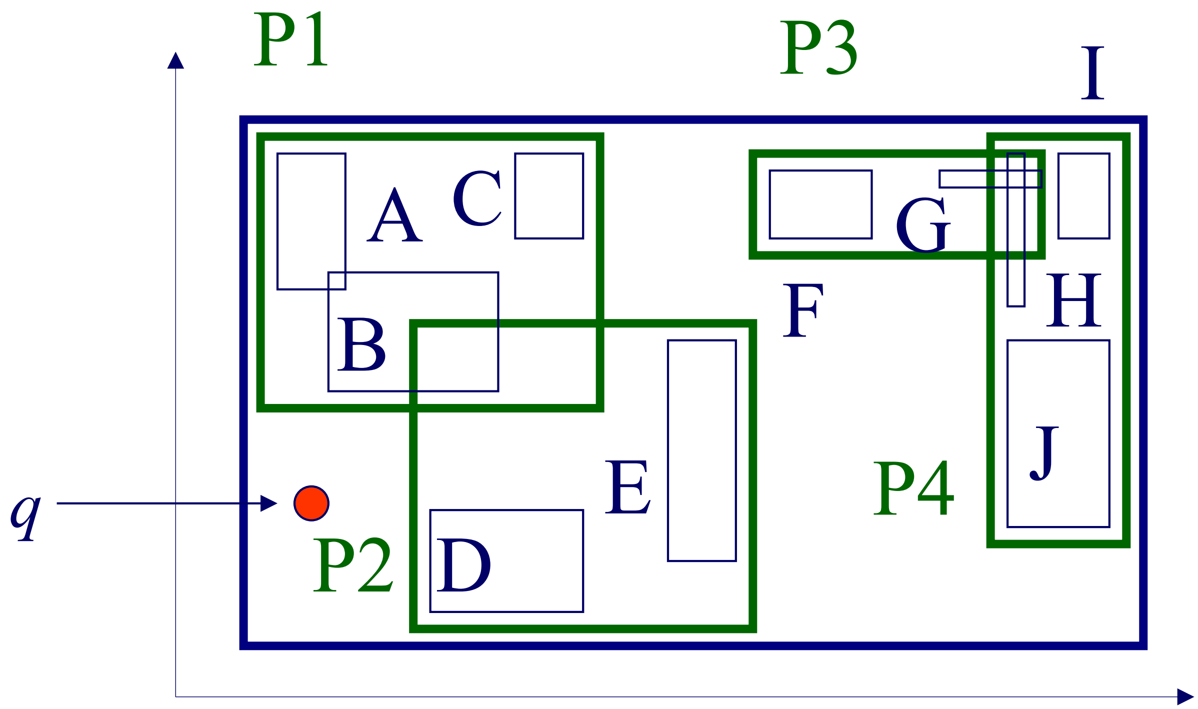
apply range-search (or print out, if this is a leaf)

R-trees - nn search



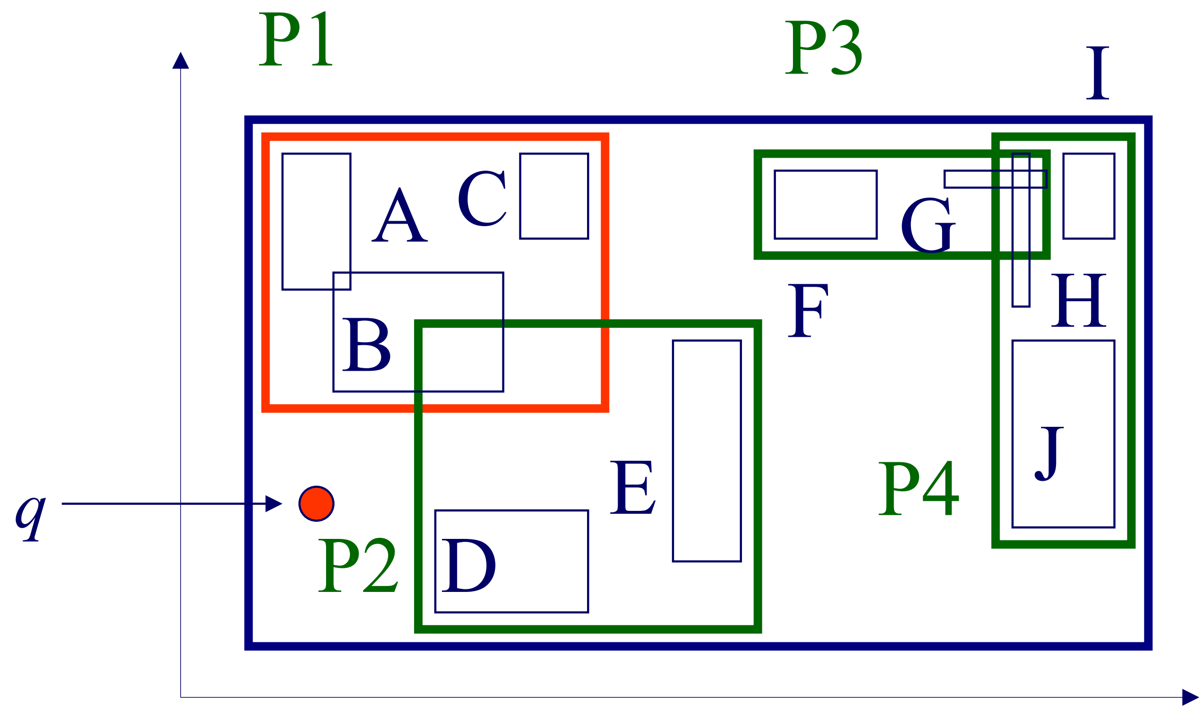
R-trees - nn search

- Q: How? (find near neighbor; refine...)



R-trees - nn search

- A1: depth-first search; then, range query



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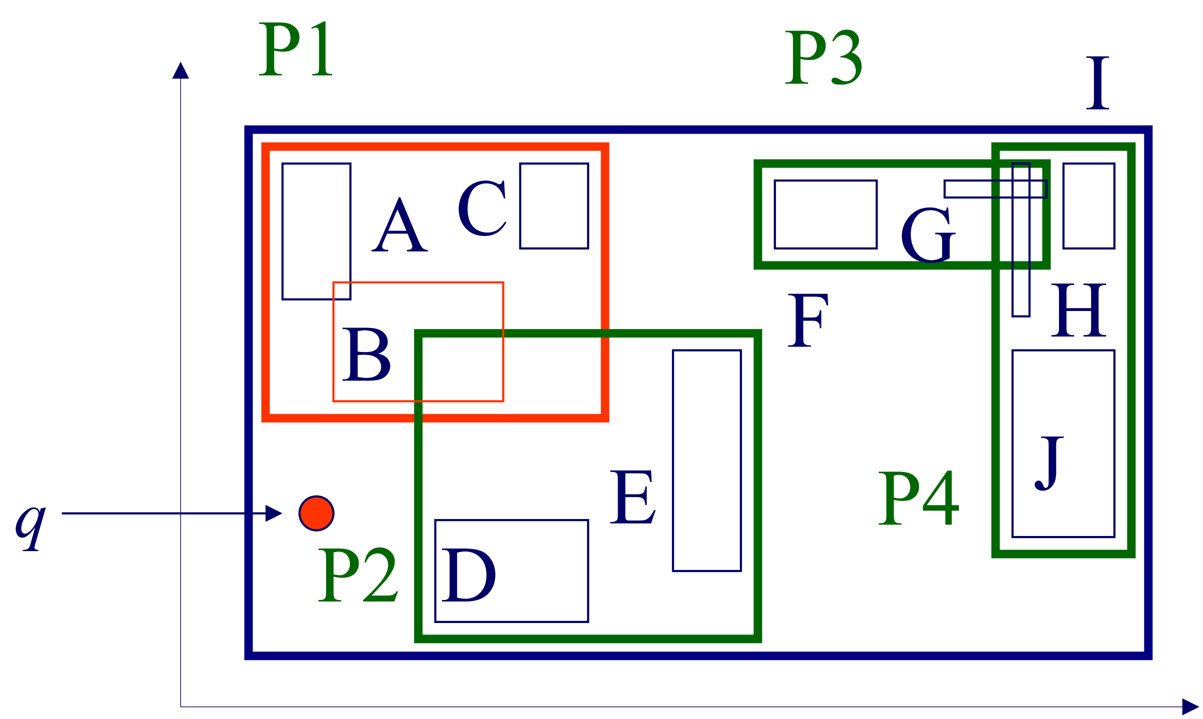

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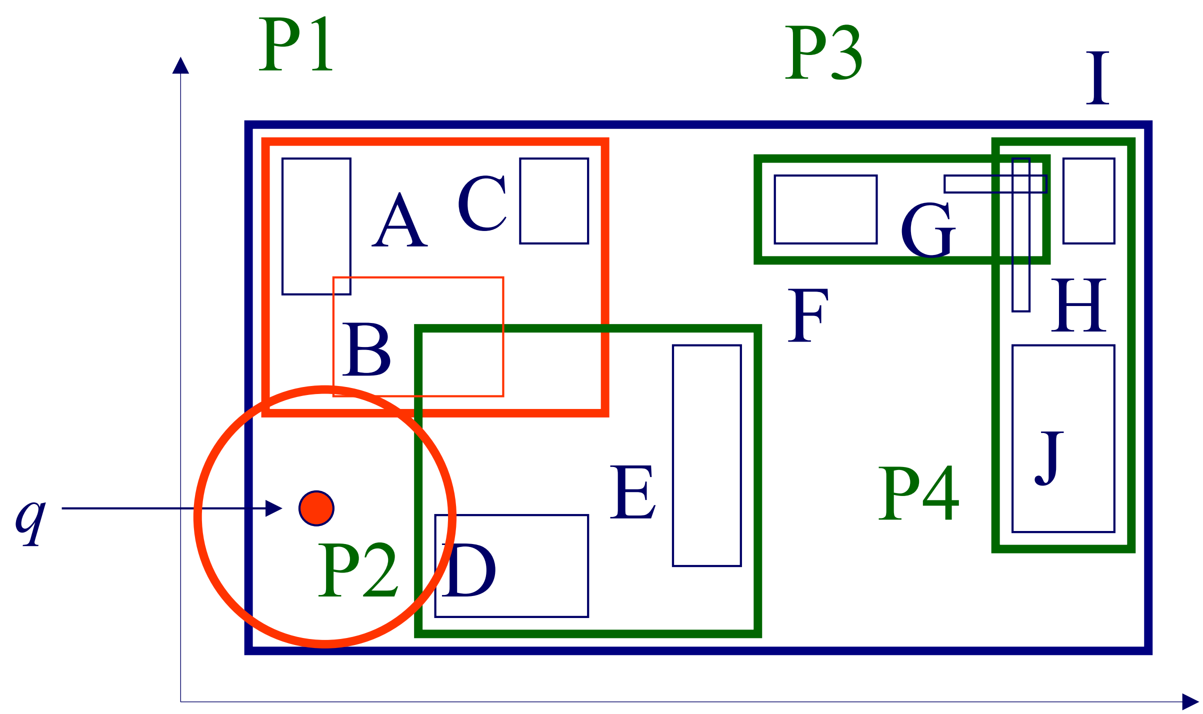

Indexing - more detailed outline

- R-trees
  - main idea; file structure
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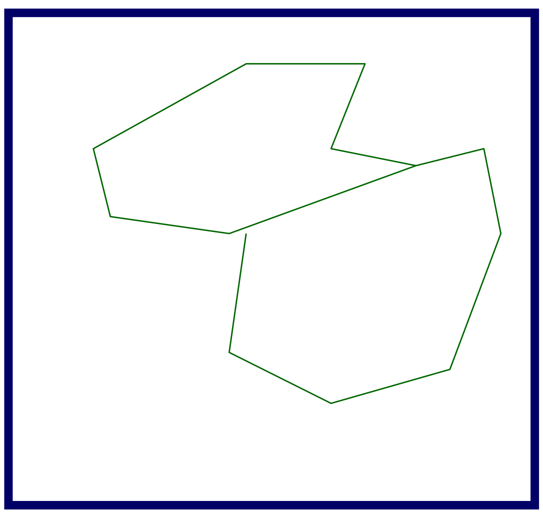
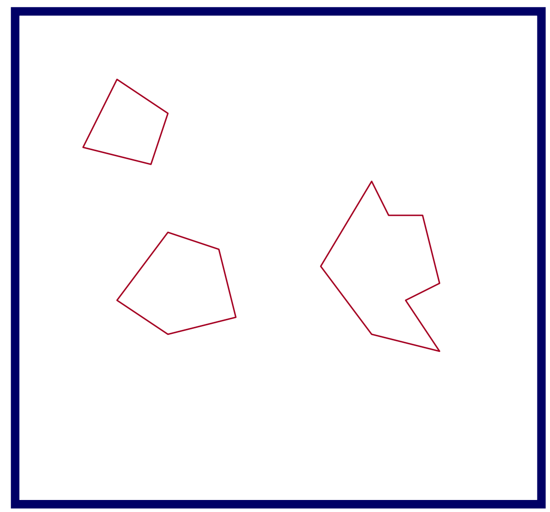
  - search: range, nn, **spatial joins**
  - performance analysis
  - variations (packed; hilbert;...)

R-trees - spatial joins

Spatial joins: find (quickly) all

counties intersecting lakes





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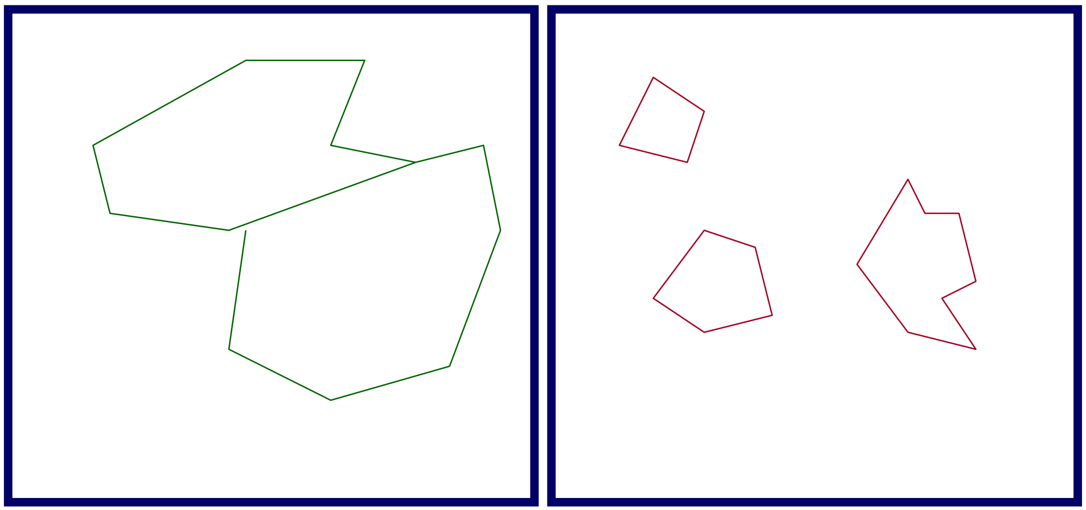

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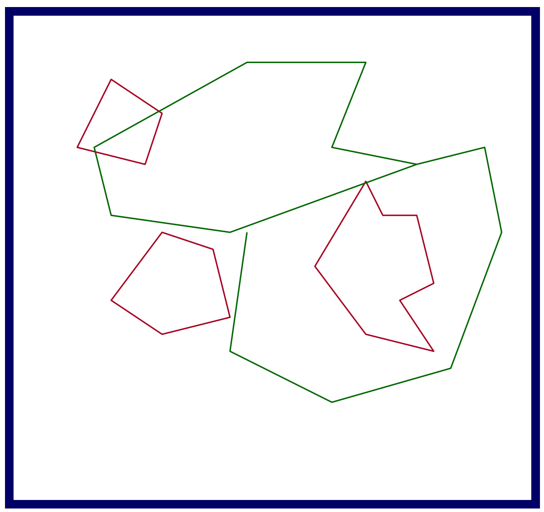

R-trees - spatial joins

Assume that they are both organized in R-trees:

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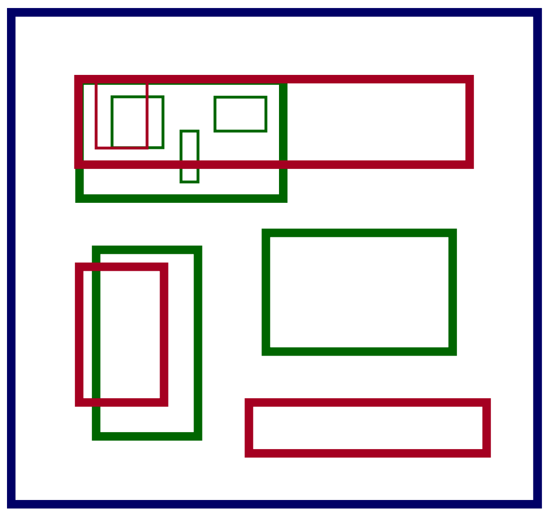

R-trees - spatial joins

for each parent P1 of tree T1
    for each parent P2 of tree T2
        if their MBRs intersect,
            process them recursively (ie., check their children)

DETAILS

R-trees - spatial joins

Improvements - variations:

- [Seeger+, sigmod 92]: do some pre-filtering; do plane-sweeping to avoid *N1* * *N2* tests for intersection
- [Lo & Ravishankar, sigmod 94]: 'seeded' R-trees

(FYI, many more papers on spatial joins, without R-trees: [Koudas+ Sevcik], e.t.c.)

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R-trees - performance analysis

- How many disk (=node) accesses we' ll need for
  - range
  - nn
  - spatial joins
- why does it matter?

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- why does it matter?
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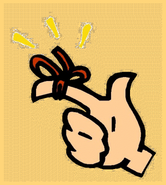
R-trees - performance analysis

- motivating question: on, e.g., split, should we try to minimize the area (volume)? the perimeter? the overlap? or a weighted combination? why?



R-trees - performance analysis

- Thus, given a tree with N nodes (i=1, ... N) we expect

#DiskAccesses(q1,q2) =

$$\text{sum} (x_{i,1} + q1) * (x_{i,2} + q2)$$

$$= \text{sum} (x_{i,1} * x_{i,2}) +$$

$$q2 * \text{sum} (x_{i,1}) +$$

$$q1 * \text{sum} (x_{i,2})$$

$$q1 * q2 * N$$

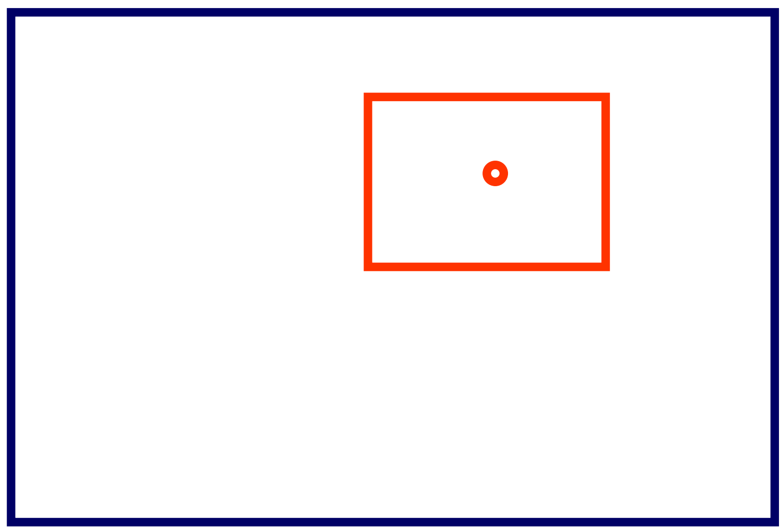
'volume'

surface area

count

R-trees - performance analysis

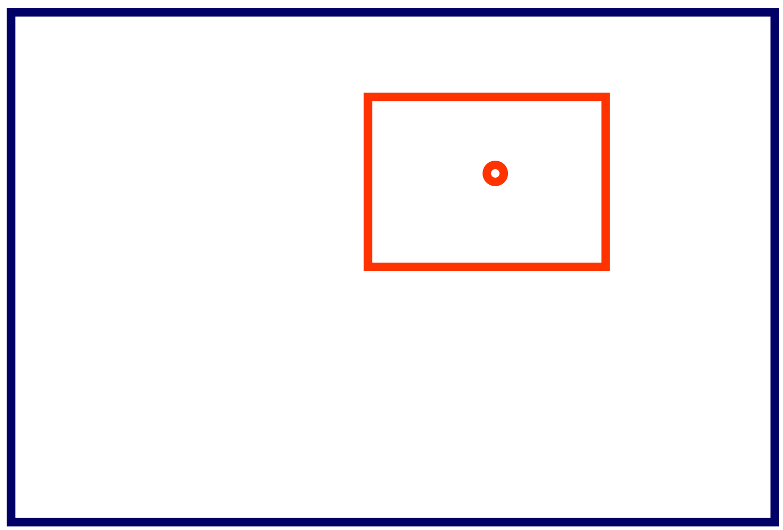
- How many disk accesses for range queries?
  - query distribution wrt location?
  - " " wrt size?



Proof

R-trees - performance analysis

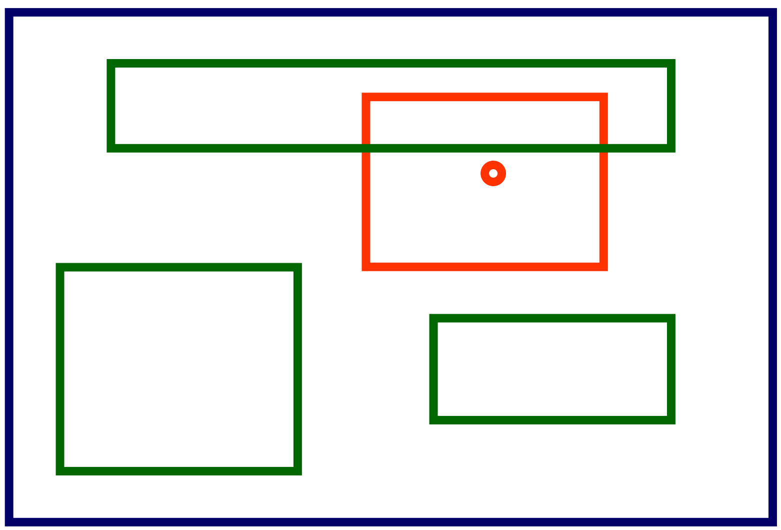
- How many disk accesses for range queries?
  - query distribution wrt location? uniform; (biased)
  - “ “ wrt size? uniform



Proof

R-trees - performance analysis

- easier case: we know the positions of parent MBRs, eg:



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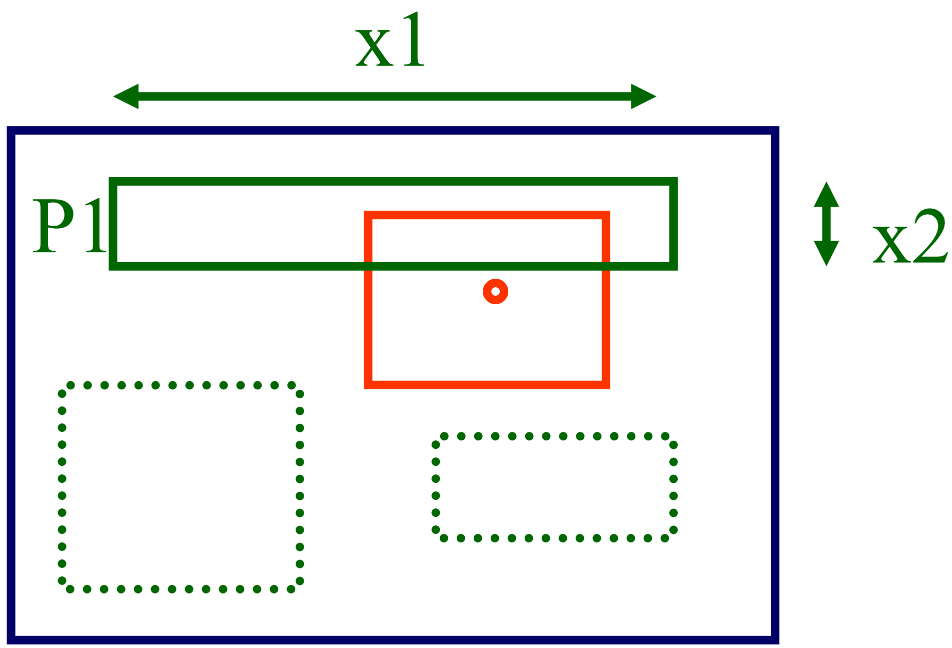
R-trees - performance analysis

- How many times will P1 be retrieved (unif. queries)?



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R-trees - performance analysis

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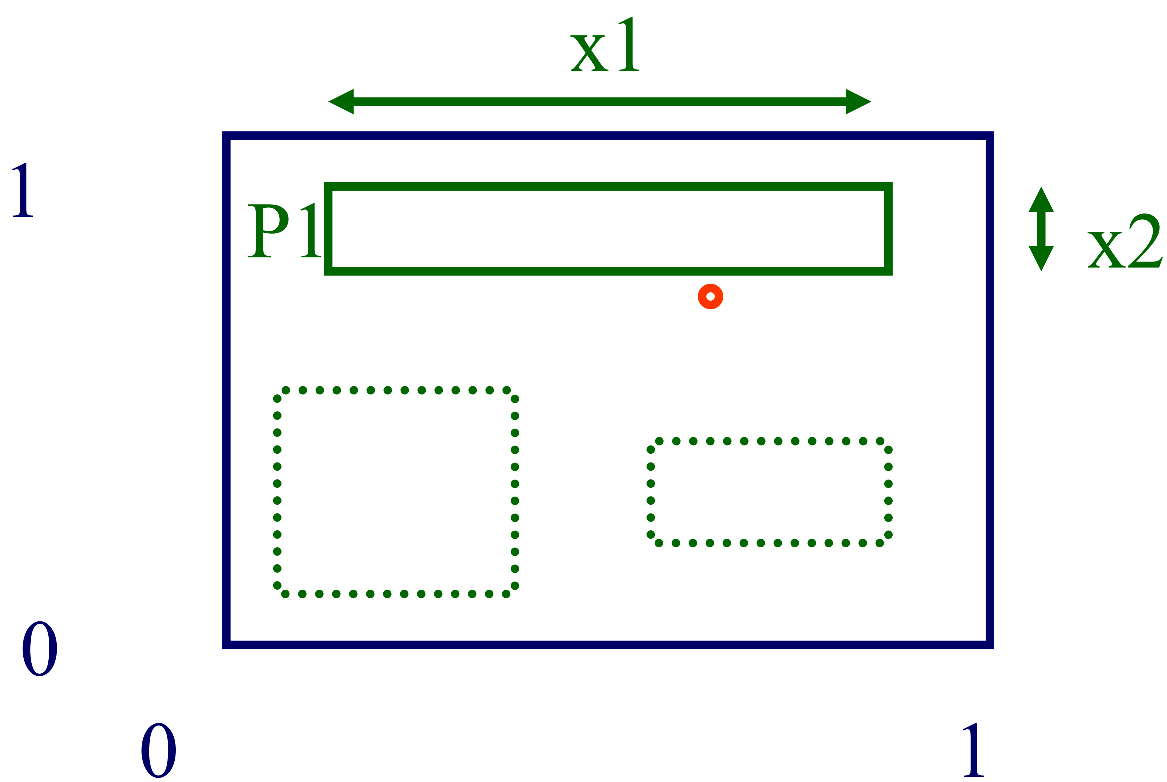
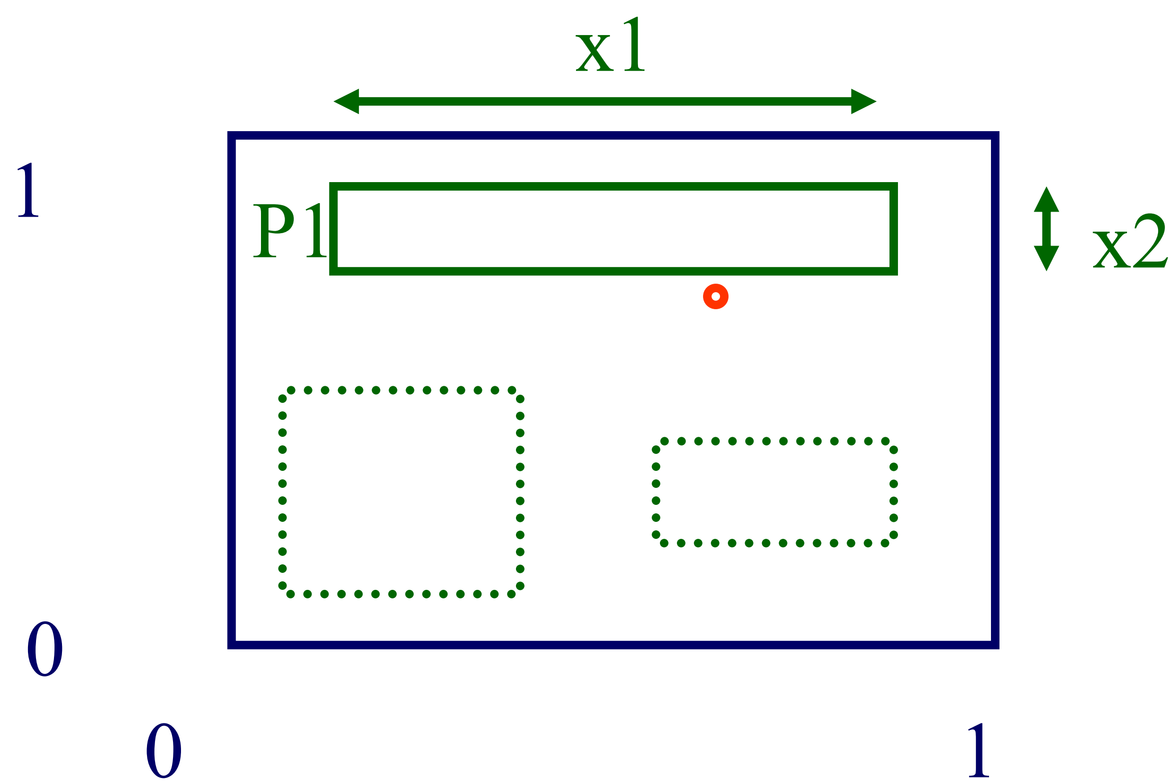


Proof

R-trees - performance analysis

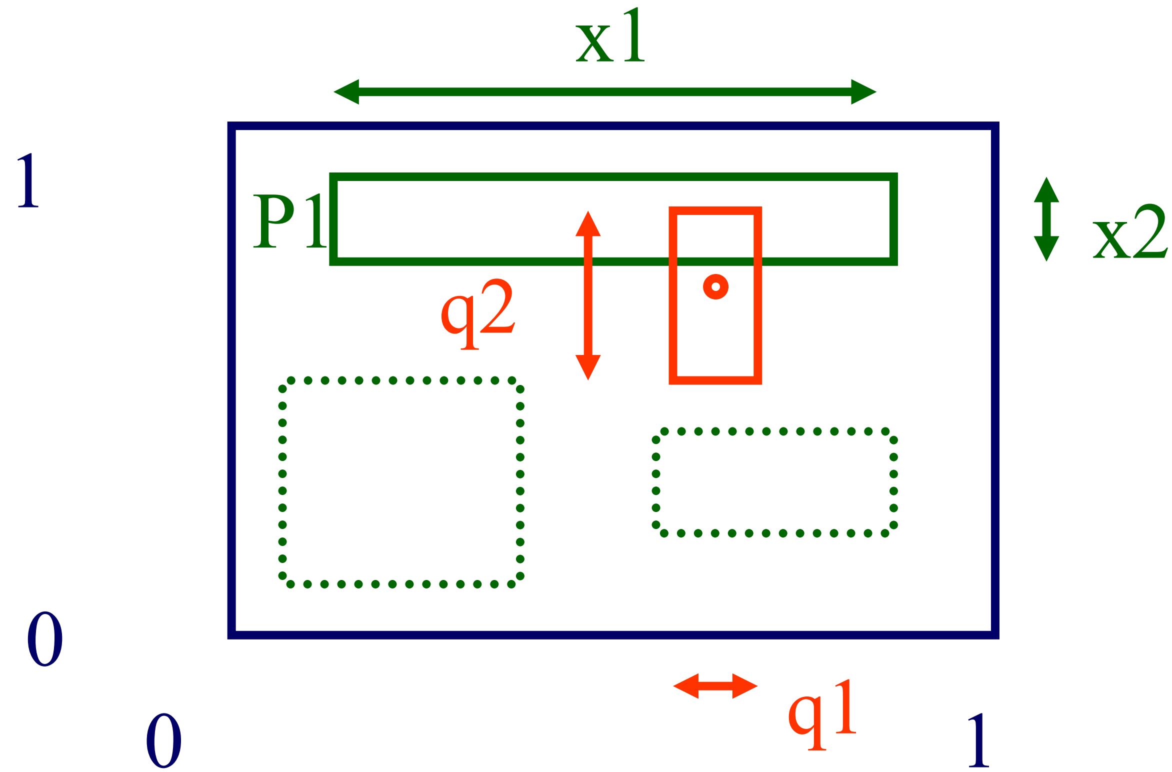
- How many times will P1 be retrieved (unif. POINT queries)? A: x1*x2



Proof

R-trees - performance analysis

- How many times will P1 be retrieved (unif. queries of size q1xq2)?



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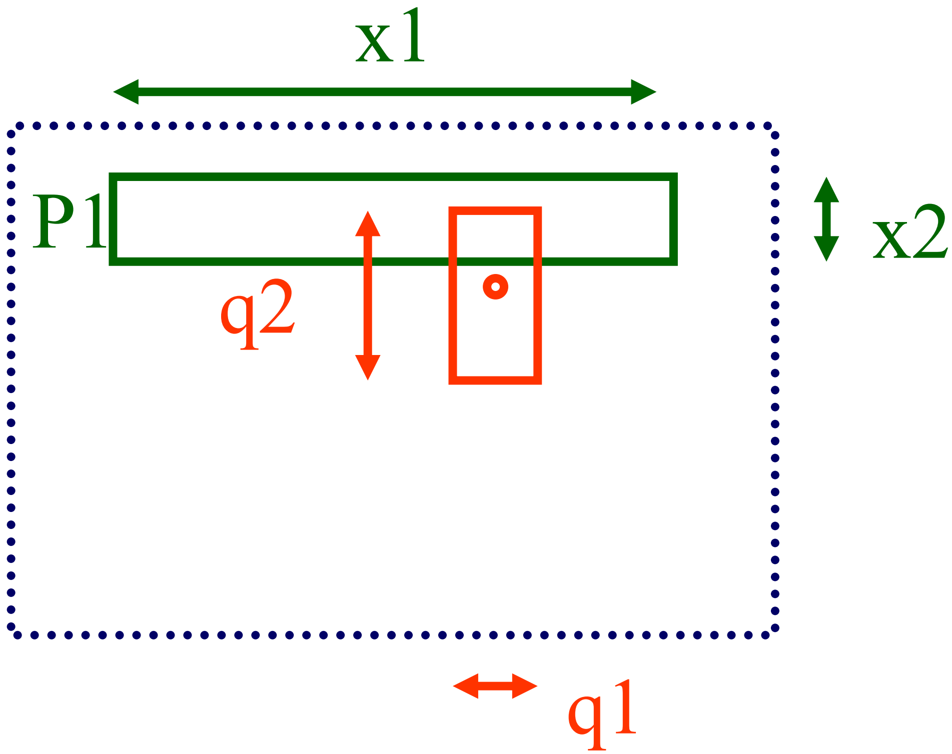

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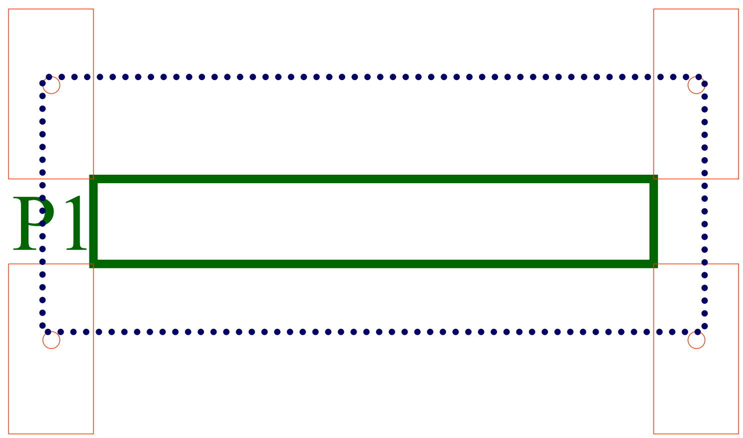

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Proof

R-trees - performance analysis

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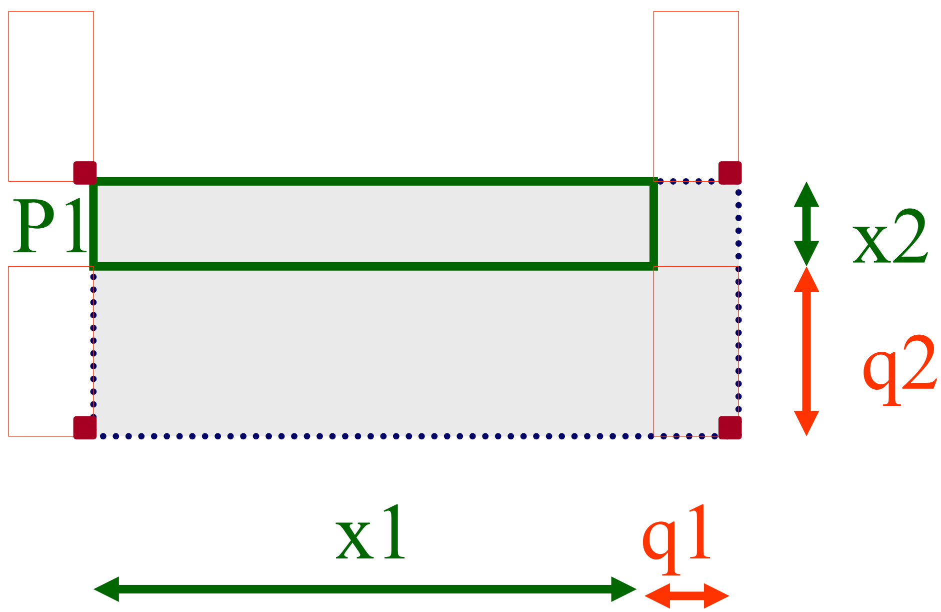
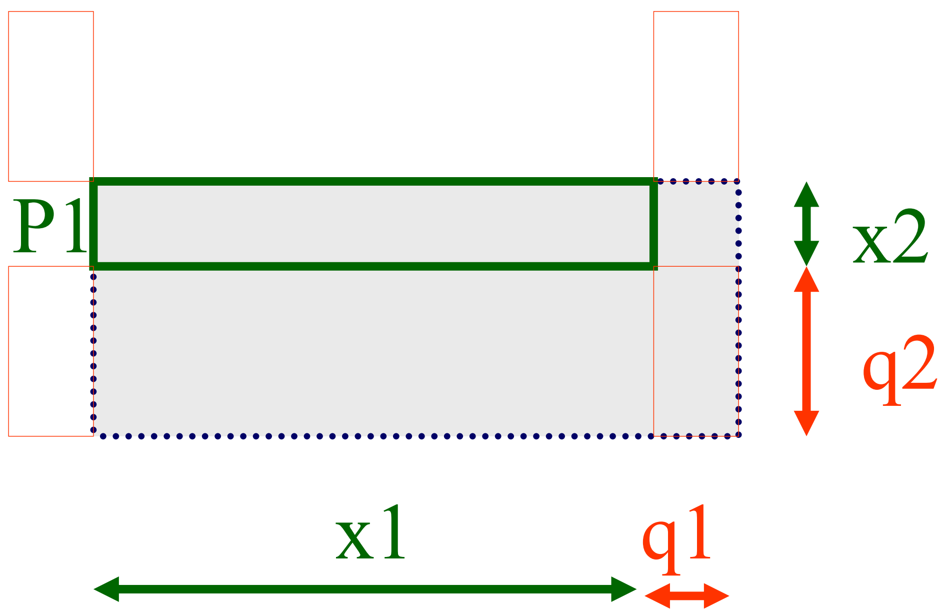


Proof

R-trees - performance analysis

- How many times will P1 be retrieved (unif. queries of size q1xq2)? A: (x1+q1)*(x2+q2)



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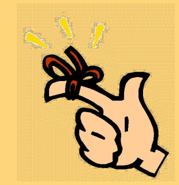

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$$q1 * q2 * N$$

'volume'

surface area

count

R-trees - performance analysis

Observations:

- for point queries: only volume matters
- for horizontal-line queries: (q2=0): vertical length matters
- for large queries (q1, q2 >> 0): the count N matters

R-trees - performance analysis

Observations (cont' ed)

- overlap: does not seem to matter
- formula: easily extendible to *n* dimensions
- (for even more details: [Pagel +, PODS93], [Kamel+, CIKM93])

Berndt-Uwe Pagel

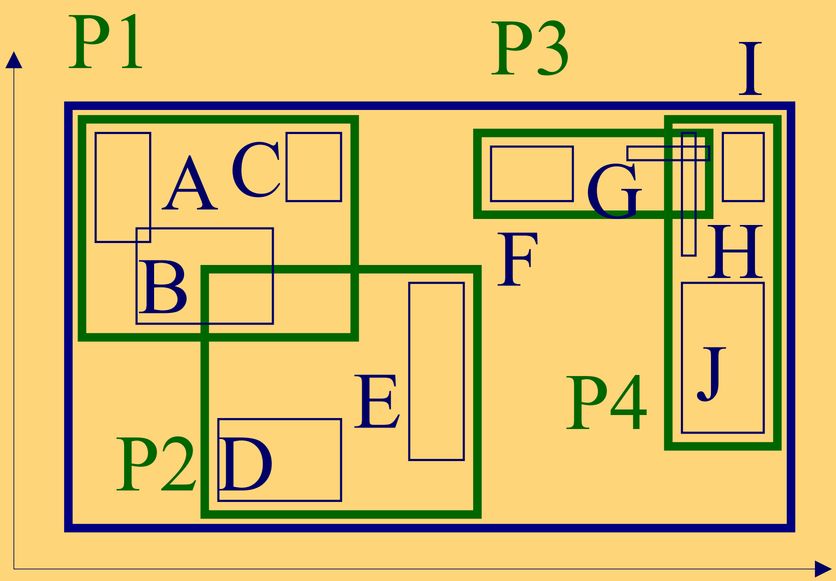
R-trees - performance analysis

Conclusions:

- splits should try to minimize area and perimeter
- ie., we want **few**, **small**, **square-like** parent MBRs
- rule of thumb: shoot for queries with q1=q2 = 0.1 (or =0.5 or so).

Solution#2: R-trees

- multi-dim trees
- Allow nodes to overlap
- Guaranteed 50% utilization – fast search (in low dim's)



R-trees – conclusions:

- Used in practice:
  - Oracle spatial (R-tree)
  - Postgres: `create index … using gist`
  - Databricks (R-trees and z-order)
  - Sqlite3: `www.sqlite.org/rtree.html`
  - Python: `(pip install rtree)`

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- Jagadish, H. V. (May 23-25, 1990). Linear Clustering of Objects with Multiple Attributes. ACM SIGMOD Conf., Atlantic City, NJ.
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