Carnegie Mellon

15-826: Multimedia (Databases) and Data Mining

Lecture #10: Fractals - case studies

C. Faloutsos

Must-read Material - I

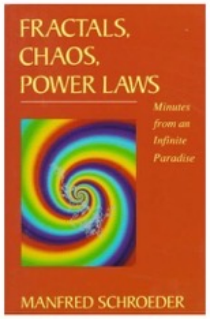
- Christos Faloutsos and Ibrahim Kamel, *Beyond Uniformity and Independence: Analysis of R-trees Using the Concept of Fractal Dimension*, Proc. ACM SIGACT-SIGMOD-SIGART PODS, May 1994, pp. 4-13, Minneapolis, MN.

Must-read Material - II

- Bernd-Uwe Pagel, Flip Korn and Christos Faloutsos, *Deflating the Dimensionality Curse using Multiple Fractal Dimensions*, ICDE 2000, San Diego, CA, Feb. 2000.

Optional Material

Optional, but **very** useful: Manfred Schroeder *Fractals, Chaos, Power Laws: Minutes from an Infinite Paradise* W.H. Freeman and Company, 1991



Reminder

- Code at

www.cs.cmu.edu/~christos/SRC/fdnq_h.zip

Also, in ‘R’

> library(fdim);

Outline

Goal: 'Find similar / interesting things'

- Intro to DB
- Indexing - similarity search
- Data Mining

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Indexing - Detailed outline

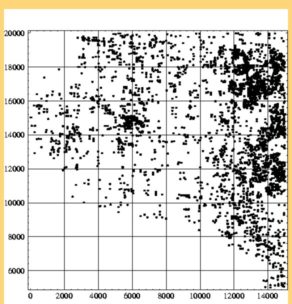
- primary key indexing
- secondary key / multi-key indexing
- spatial access methods
  - z-ordering
  - R-trees
  - misc
- fractals
  - intro
  - applications
- text
- ...

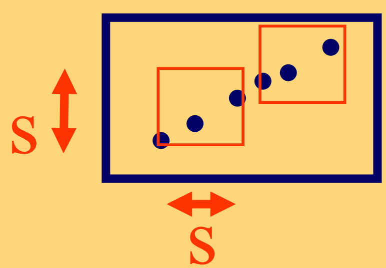
Indexing - Detailed outline

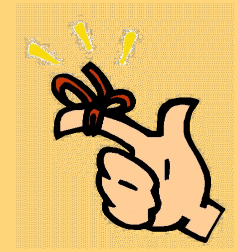
- fractals
  - intro
  - applications
    - disk accesses for R-trees (range queries)
    - dim. curse revisited
    - nearest neighbors estimation

Problem:

- Selectivity of a range query in R-trees?



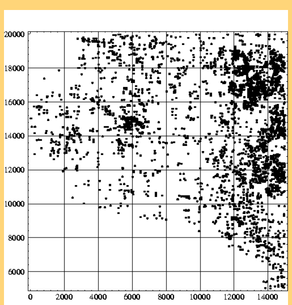


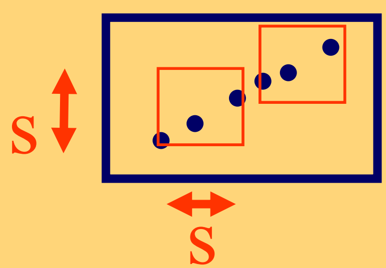


Solution:

- Selectivity of a range query in R-trees?
- Depends on *fractal* dimension

$$s = (C/N)^{1/D0}$$

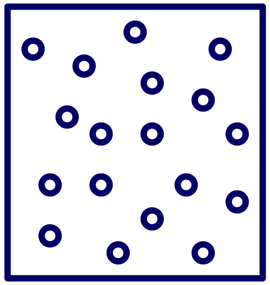




Case study#1: R-tree performance

Problem

- Given
  - N points in E-dim space

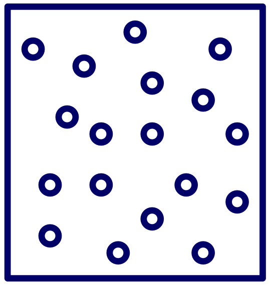


- Estimate # disk accesses for a range query
  $(q1 \times \ldots \times q_E)$

(assume: 'good' R-tree, with tight, cube-like MBRs)

Case study#1: R-tree performance

Problem



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Typically, in DB Q-opt?

Case study#1: R-tree performance

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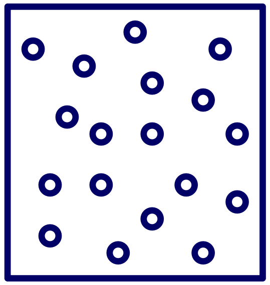

- Estimate # disk accesses for a range query

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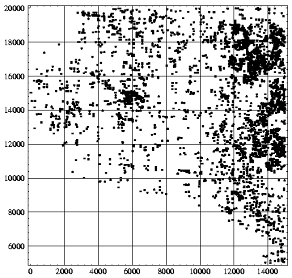
Case study#1: R-tree performance

Problem



- Given
  - N points in E-dim space
  - with fractal dimension D
- Estimate # disk accesses for a range query
  ($q1$ x ... x q_E)

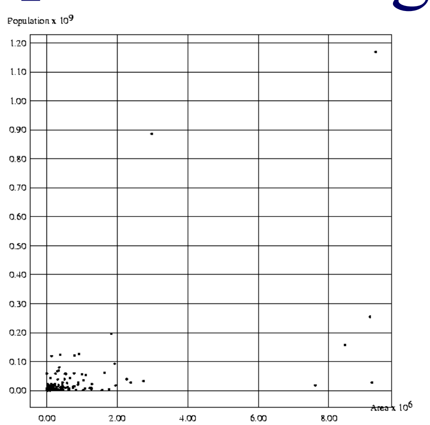
(assume: ‘good’ R-tree, with tight, cube-like MBRs)

Typically, in DB Q-opt: uniformity + independence

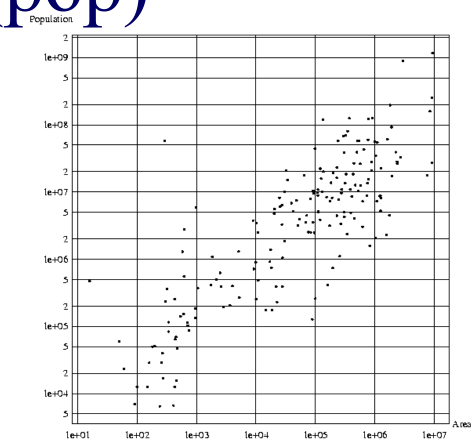
Examples:World's countries

- BUT: area vs population for ~200 countries (1991 CIA fact-book).

pop



log(pop)



area

log(area)

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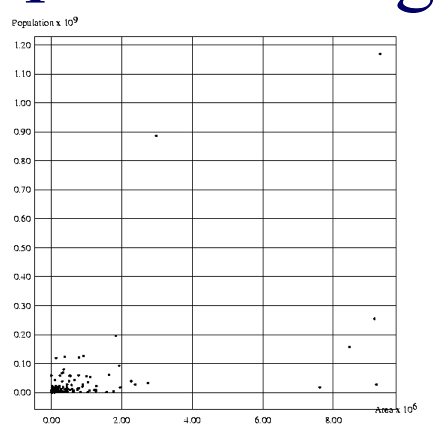
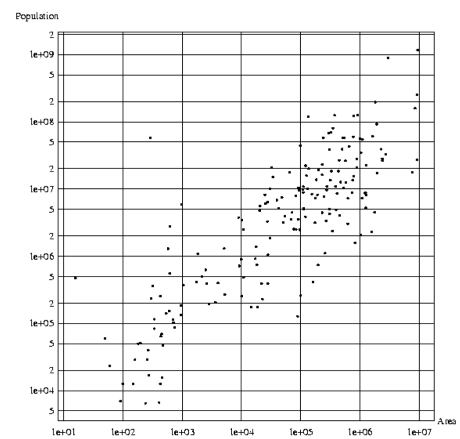
Examples: World’s countries

- neither uniform, nor independent!

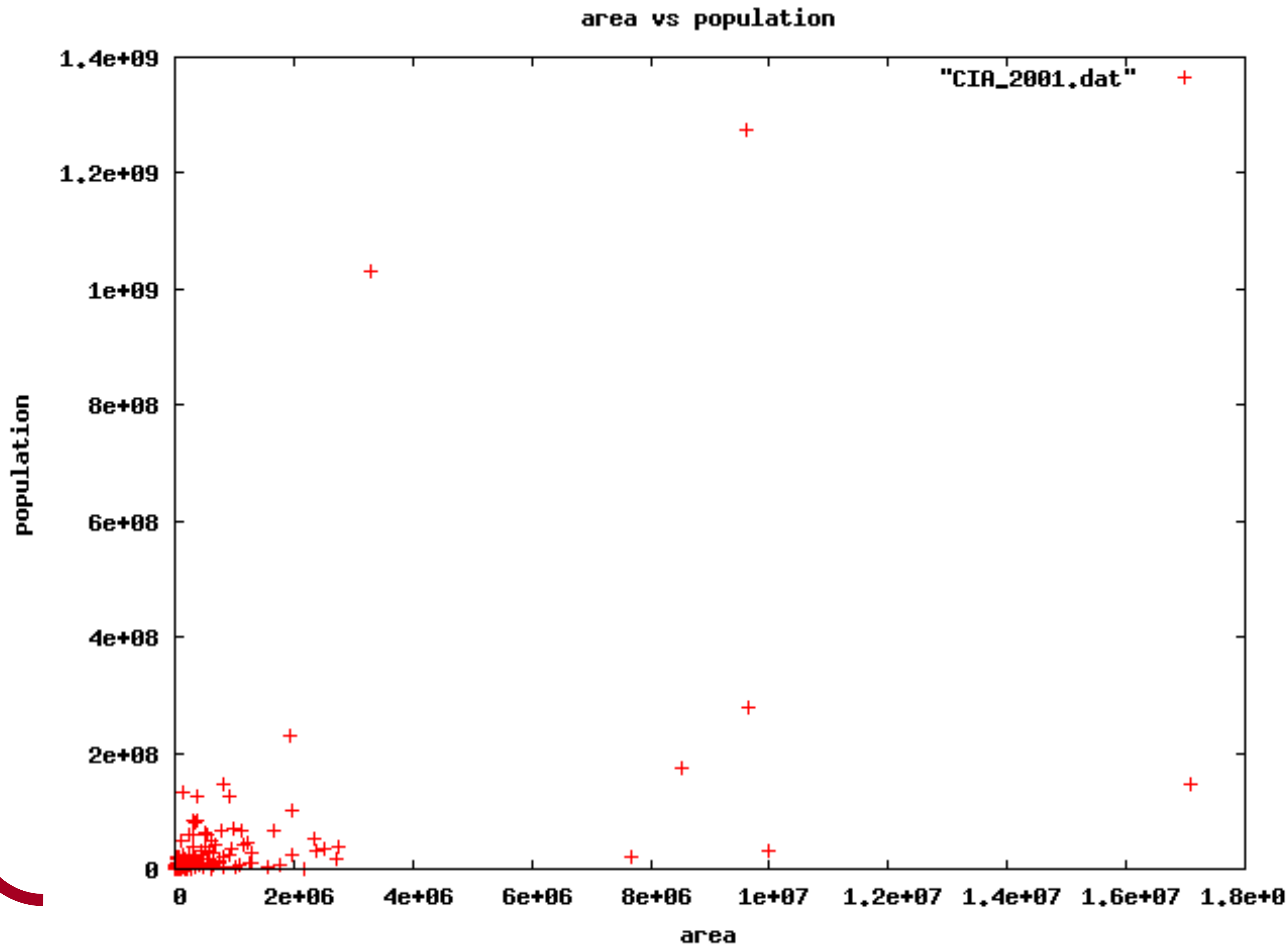
pop log(pop)





area log(area)

For fun: identification



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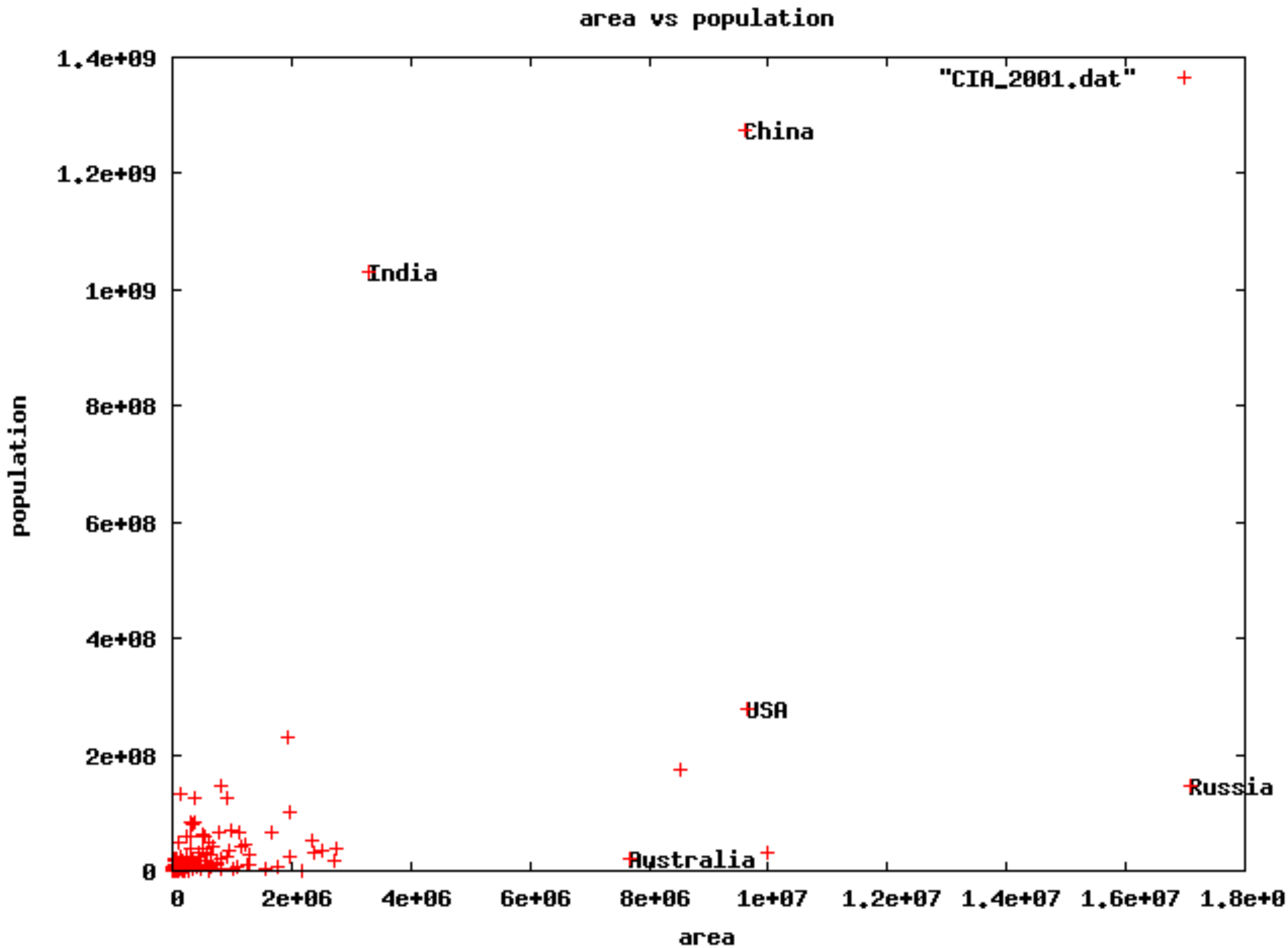

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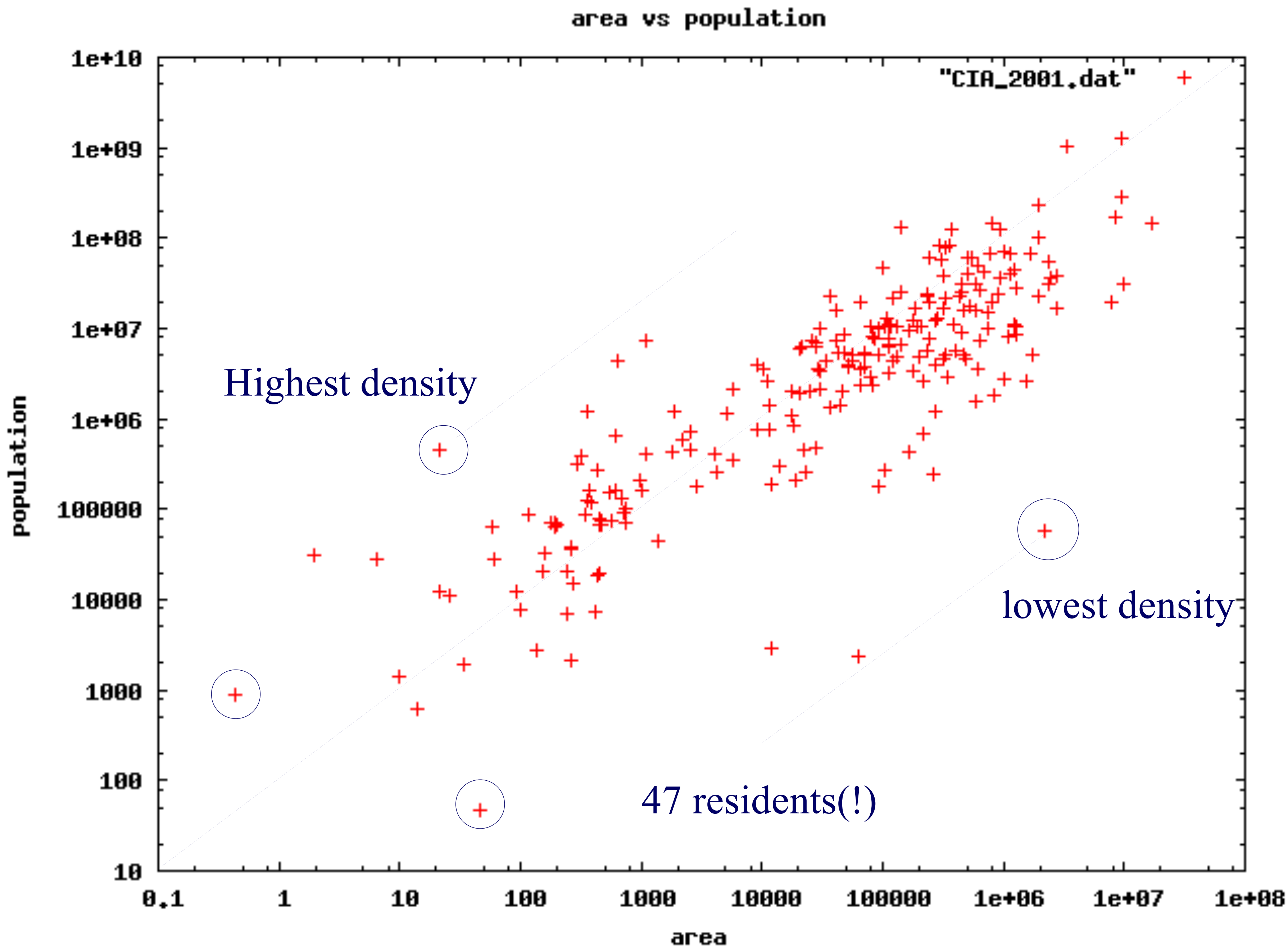

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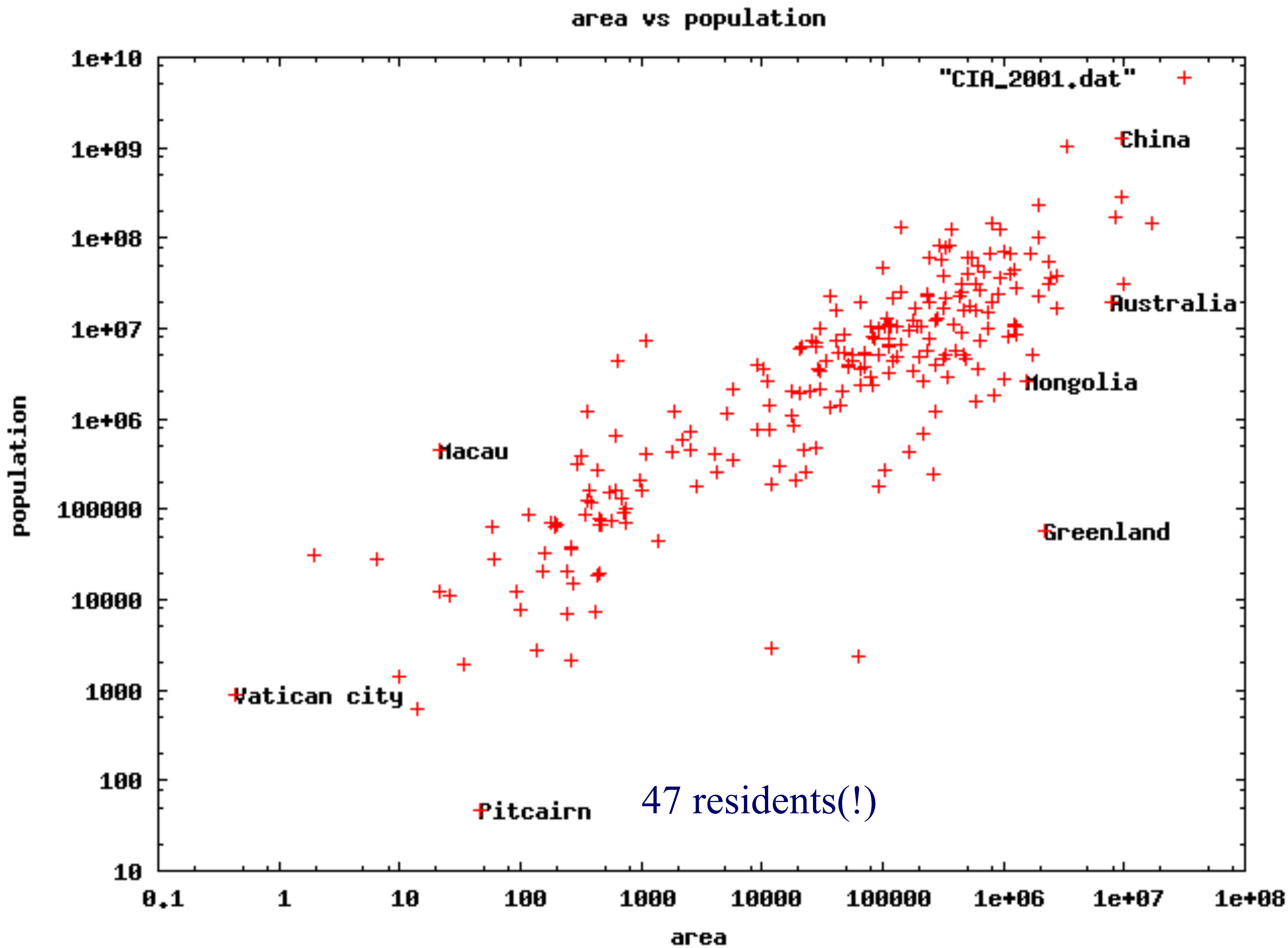
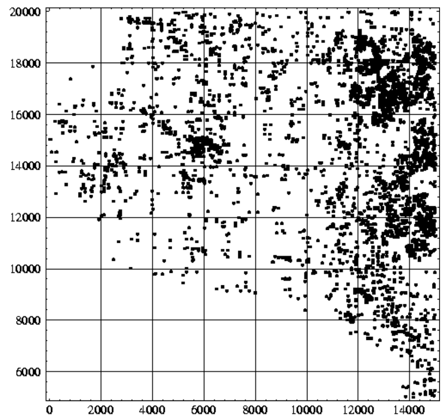


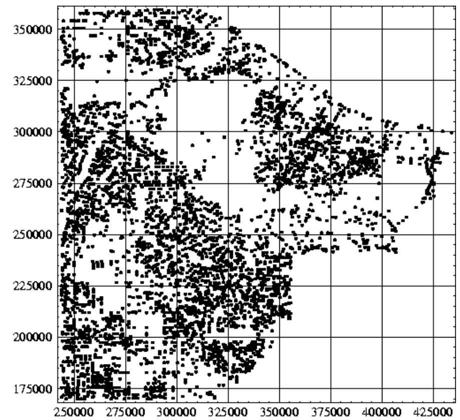
Examples: TIGER files

- neither uniform, nor independent!

MG county



LB county



How to proceed?

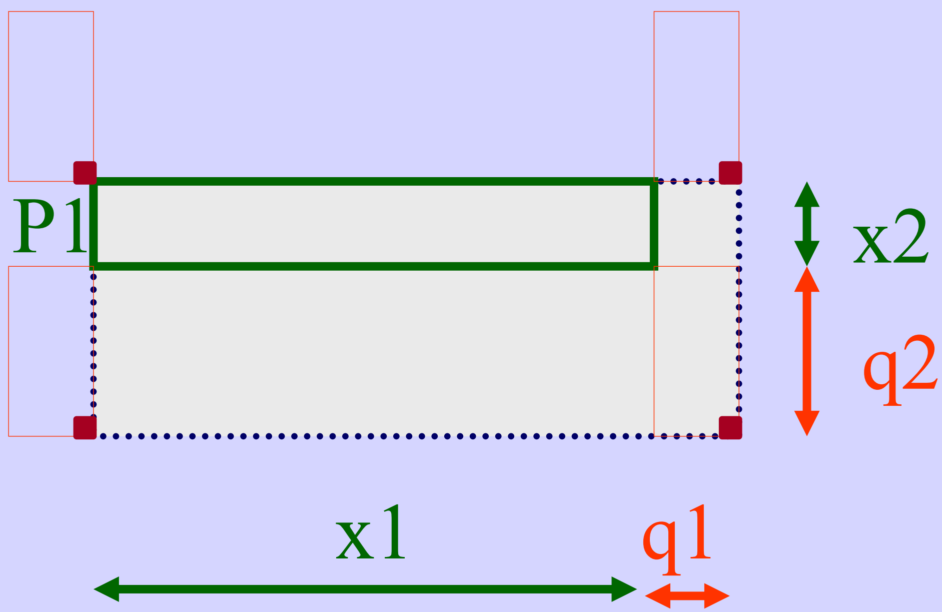
- recall the [Pagel+] formula, for range queries of size q1 x q2

$$\#DiskAccesses(q1,q2) = \\ sum\ (x_{i,1} + q1) * (x_{i,2} + q2)$$

Proof

R-trees - performance analysis

- How many times will P1 be retrieved (unif. queries of size q1xq2)?



How to proceed?

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$$\#DiskAccesses(q1,q2) = sum\ (x_{i,1} + q1) * (x_{i,2} + q2)$$

But:

formula needs to know the $x_{i,j}$ sizes of MBRs!

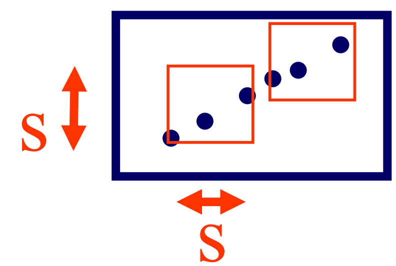
How to proceed?

But:

formula needs to know the $x_{i,j}$ sizes of MBRs!

Answer (jumping ahead):

$$\boxed{s = (C/N)^{1/D0}}$$



How to proceed?

But:

formula needs to know the $x_{i,j}$ sizes of MBRs!

Answer (jumping ahead):

$$s = (C/N)^{1/D0}$$

Hausdorff fd

side of (parent) MBR

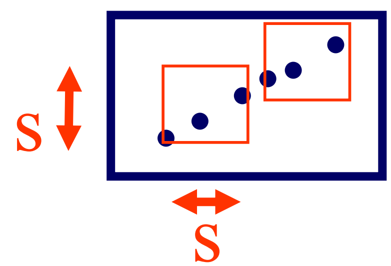
page capacity

# of data points

'smell' tests:

- C $\nearrow$ s $\nearrow$ $\searrow$
- N $\nearrow$ s $\nearrow$ $\searrow$
- D0 $\nearrow$ s $\nearrow$ $\searrow$

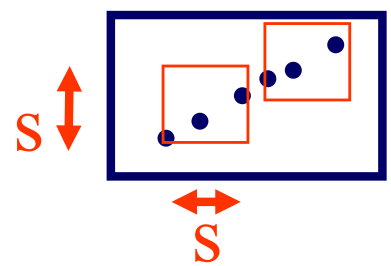
$$\boxed{s = (C/N)^{1/D0}}$$



‘smell’ tests:

- C $\nearrow$ s $\nearrow$
- N $\nearrow$ s $\searrow$
- D0 $\nearrow$ s $\nearrow$

$$\boxed{s = (C/N)^{1/D0}}$$



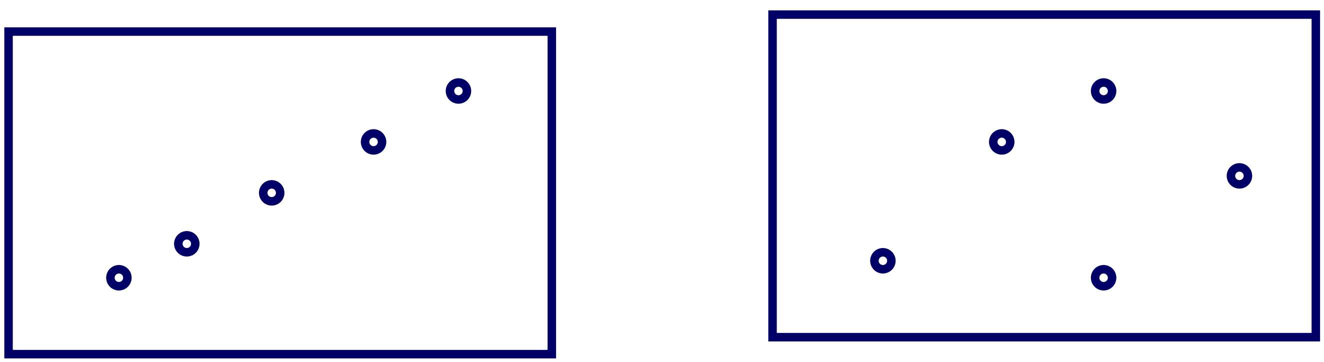
PROOF

R-trees - performance analysis

I.e: for range queries - how many disk accesses, if we just now that we have

- *N* points in *E*-d space?

A: can not tell! need to know distribution



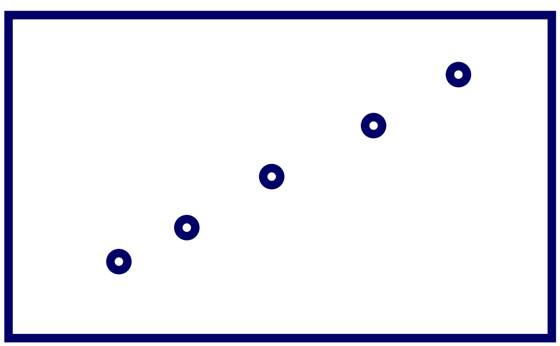
PROOF

R-trees - performance analysis

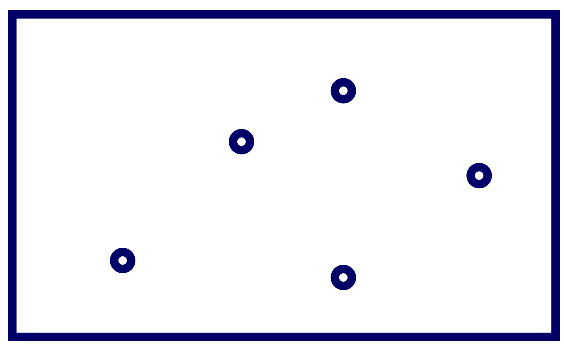
Q: OK - so we are told that the **Hausdorff** fractal dim. = D0 - Next step?

(also know that there are at most *C* points per page)

D0=1



D0=2



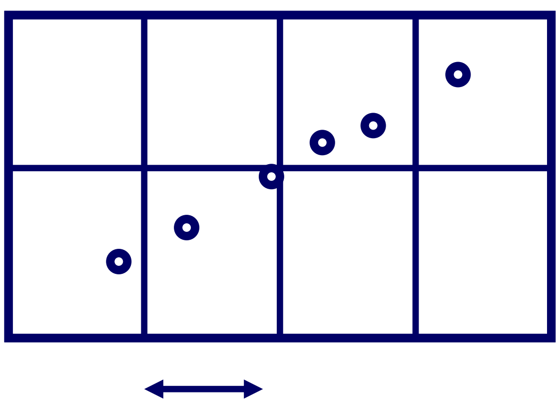
R-trees - performance analysis

Assumption1: square-like parents (s*s)

Assumption2: fully packed (C points each)

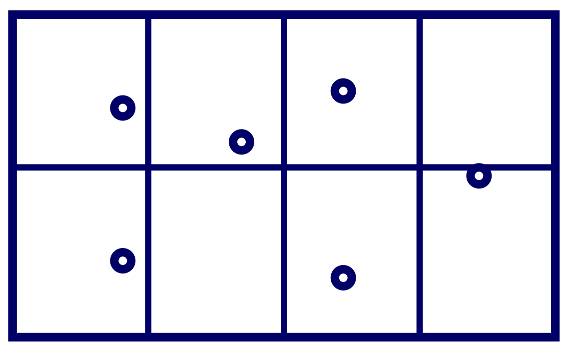
Assumption3: non-overlapping

D0=1



s1=s2=s

D0=2



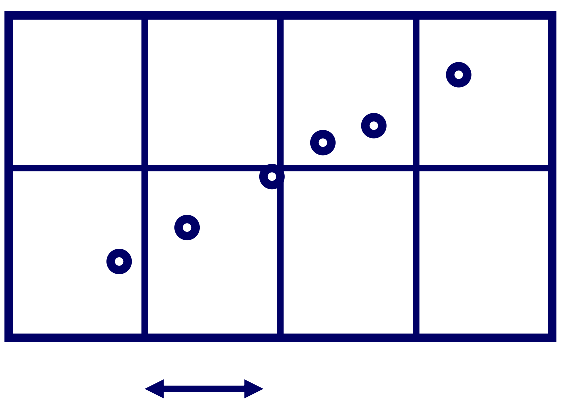
PROOF

R-trees - performance analysis

Assumption1: square-like parents (s*s)

Assumption2: fully packed (N/C non-empty)

Assumption3: non-overlapping



R-trees - performance analysis

Hint: dfn of Hausdorff f.d.:

Felix Hausdorff (1868-1942)

PROOF

Reminder: Hausdorff or box-counting fd:

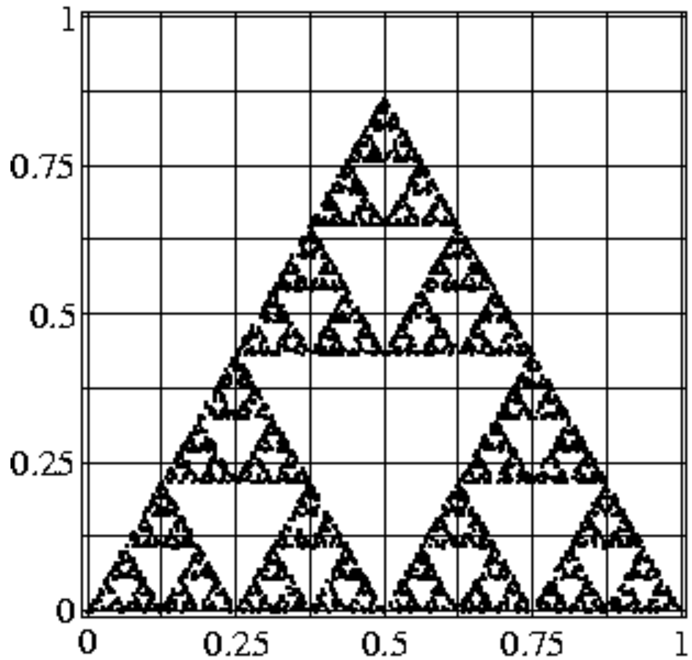
- Box counting plot: Log(N (r)) vs Log (r)
- r: grid side
- N (r): count of non-empty cells
- (Hausdorff) fractal dimension D0:

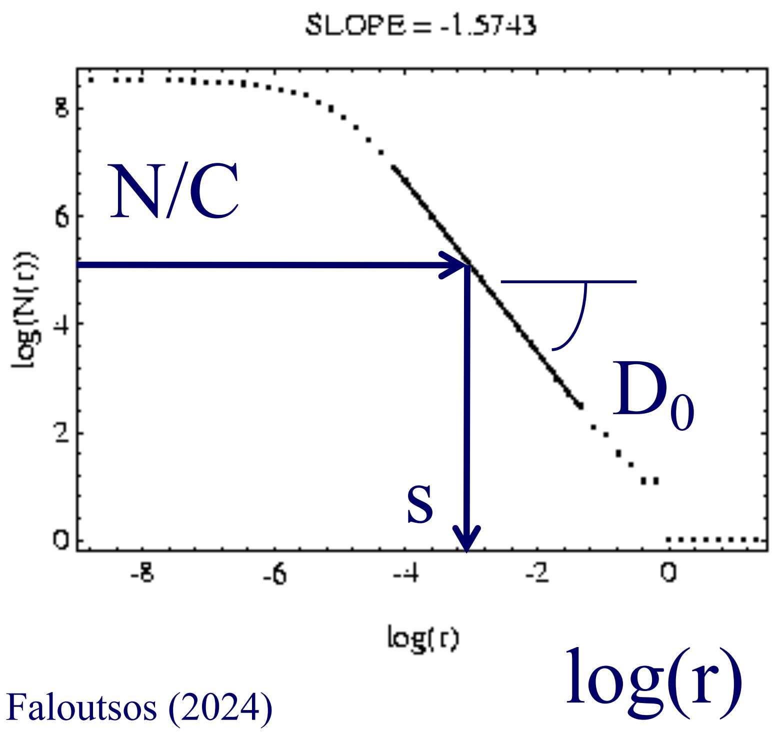
$$D_0 = -\frac{\partial \log(N(r))}{\partial \log(r)}$$

Reminder

- Hausdorff fd:

r — log(#non-empty cells)





PROOF

Reminder

- dfn of Hausdorff fd implies that

$$N(r) \sim r^{(-D0)}$$

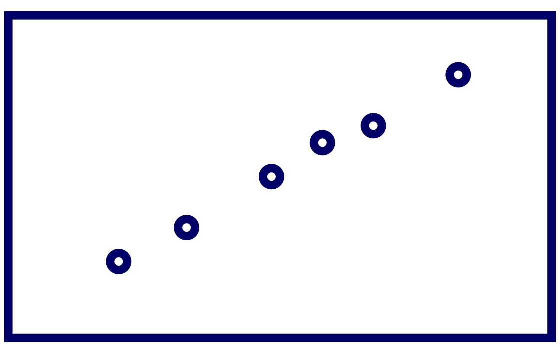
\# non-empty cells of side r

PROOF

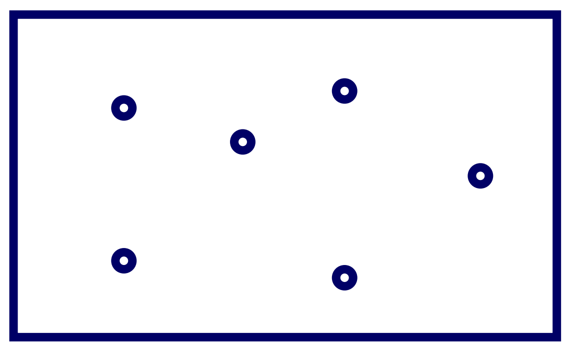
R-trees - performance analysis

Q (rephrased): what is the side s1, s2, ... of parent nodes, given *N* data points, packed by *C*, with f.d. = *D0*

D0=1



D0=2



PROOF

R-trees - performance analysis

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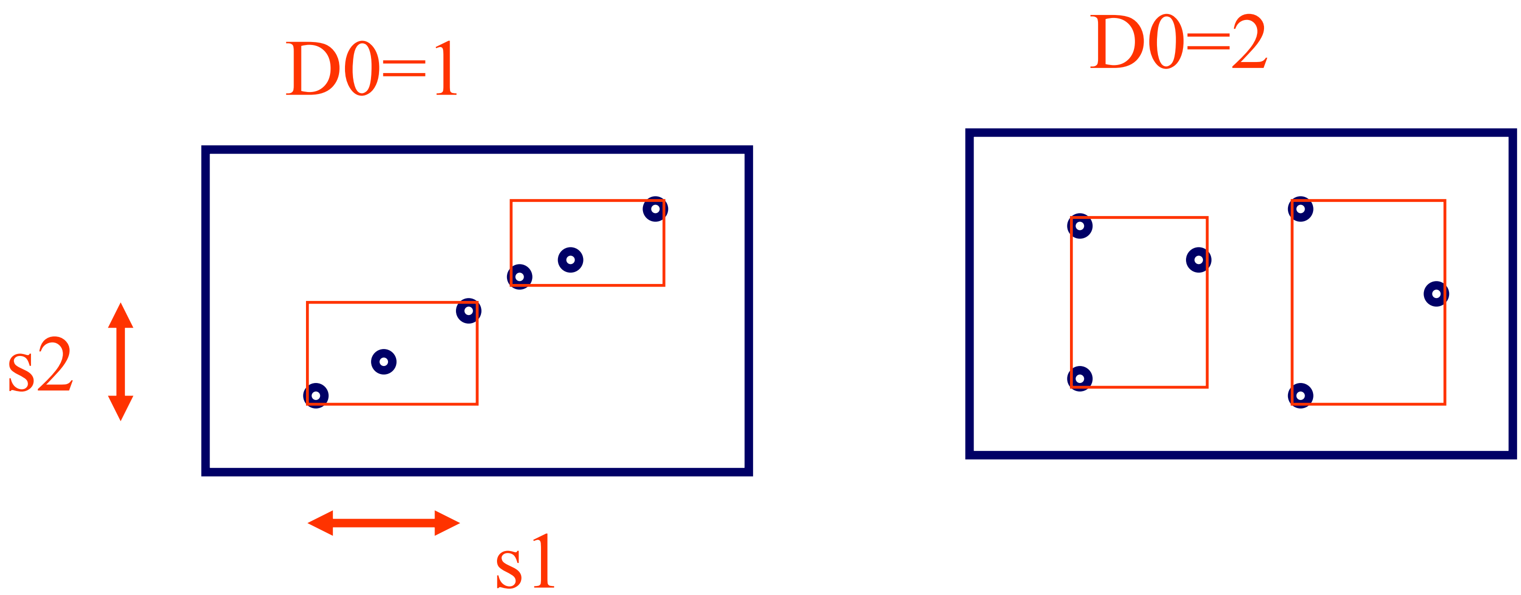

PROOF

R-trees - performance analysis

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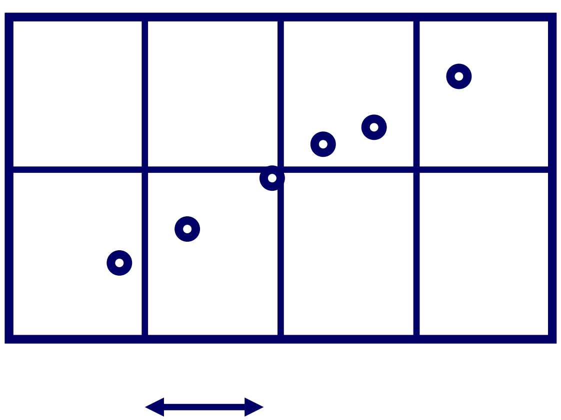
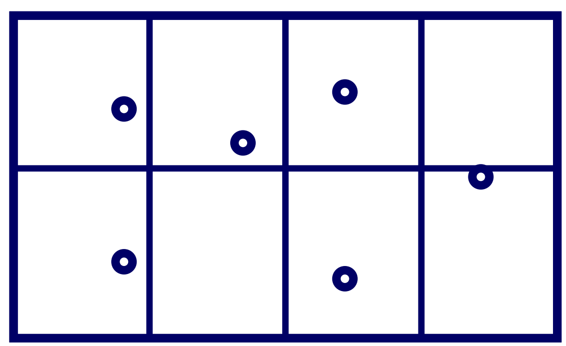


s1=s2=s

D0=2



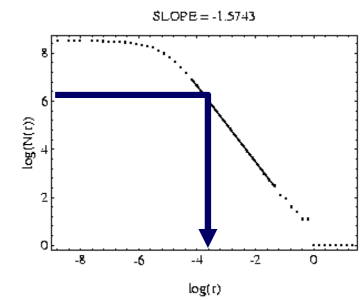
PROOF

R-trees - performance analysis

A: (educated guess)

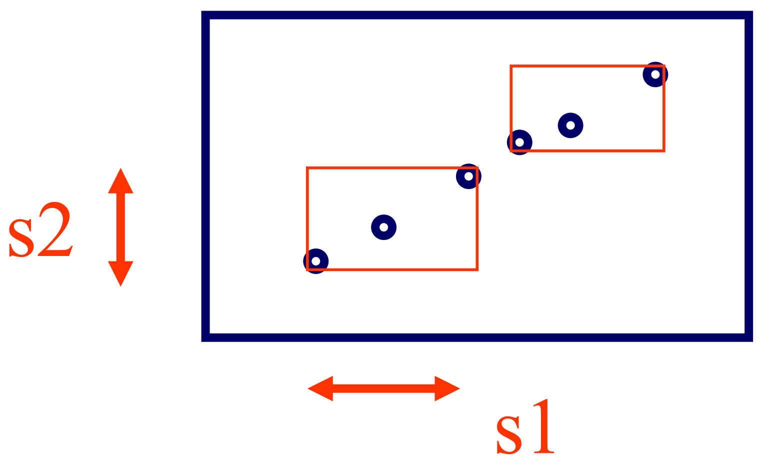
- s=s1=s2 (= ...) - square-like MBRs
- N/C non-empty cells = K * $s^{(-D0)}$

log(#cells)



log(s)

D0=1



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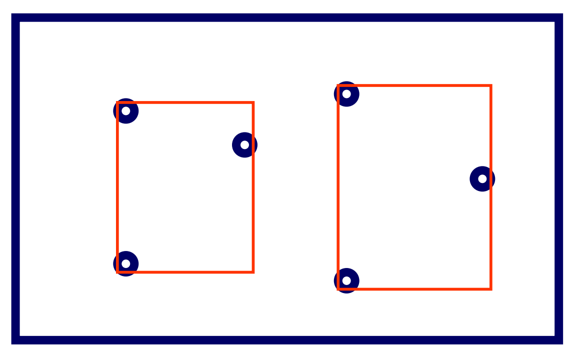
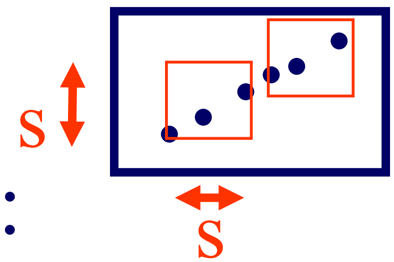


R-trees - performance analysis



PROOF of derivations: in [PODS 94].

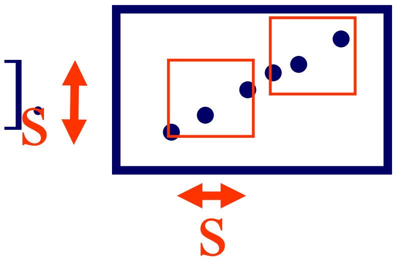
Finally, expected side s of parent MBRs:

$$\boxed{s = (C/N)^{1/D0}}$$

Q: sanity check: how does s change with $D0$?

A:

R-trees - performance analysis



PROOF of derivations: in [Kamel+, PODS 94]

Finally, expected side *s* of parent MBRs:

$$s = (C/N)^{1/D0}$$

Q: sanity check: how does *s* change with *D0?*

A: *s* grows with *D0*

Q: does it make sense?

Q: does it suffer from (intrinsic) dim. curse?

PROOF

R-trees - performance analysis

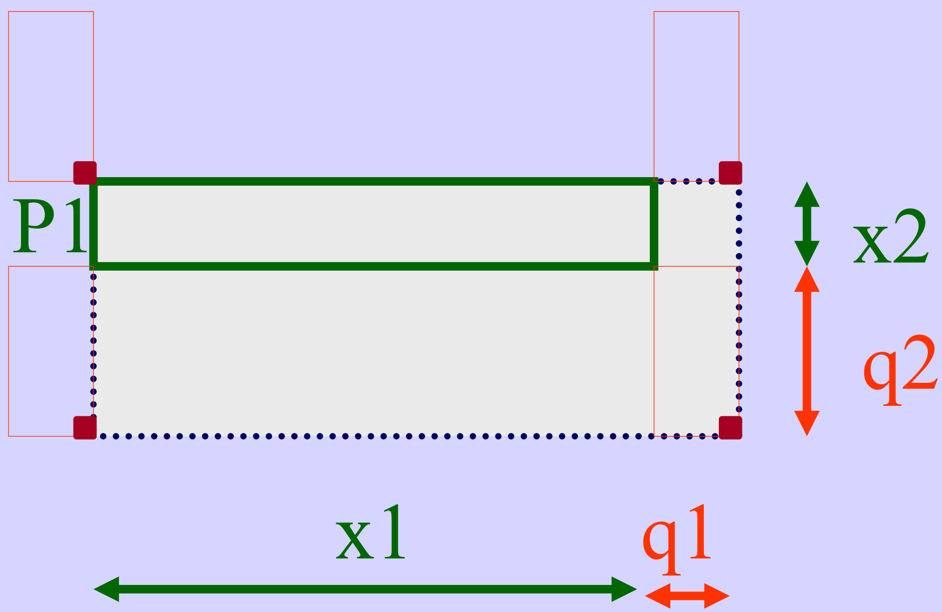
Q: Final-final formula (# disk accesses for range queries $q1$ x $q2$ x ...):

A:

Proof

R-trees - performance analysis

- How many times will P1 be retrieved (unif. queries of size q1xq2)?



PROOF

R-trees - performance analysis

Q: Final-final formula (# disk accesses for range queries $q1$ x $q2$ x ...):

A: # of parent-node accesses:

$$N/C * (s + q1) * (s + q2) * \ldots (s + q_E)$$

A: # of grand-parent node accesses

PROOF

R-trees - performance analysis

Q: Final-final formula (# disk accesses for range queries *q1* x *q2* x ...):

A: # of parent-node accesses:

$$N/C * (s + q1) * (s + q2) * \ldots (s + q_E)$$

A: # of grand-parent node accesses

$$N/(C^2) * (s' + q1) * (s' + q2) * \ldots (s' + q_E)$$

$$s' = ??$$

PROOF

R-trees - performance analysis

Q: Final-final formula (# disk accesses for range queries *q1* x *q2* x ...):

A: # of parent-node accesses:

$$N/C * (s + q1) * (s + q2) * \dots (s + q_E)$$

A: # of grand-parent node accesses

$$N/(C^\wedge 2) * (s' + q1) * (s' + q2) * \dots (s' + q_E)$$

$$s' = (C^\wedge 2/N)^{1/D0}$$

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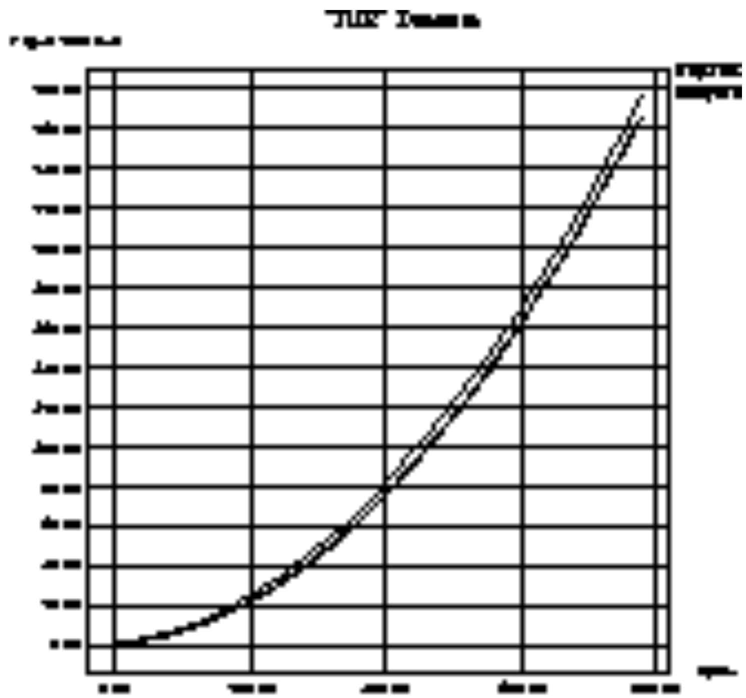
R-trees - performance analysis

Results:

IUE (x-y star coordinates)

leaf accesses



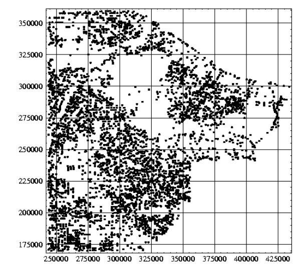
(a) IUE - Leaf accesses vs. query s

query side

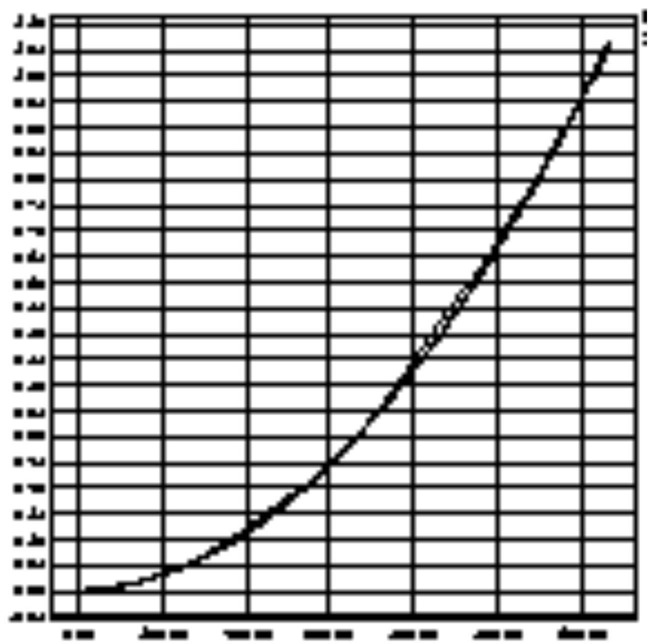
R-trees - performance analysis

Results:

LB County



leaf accesses



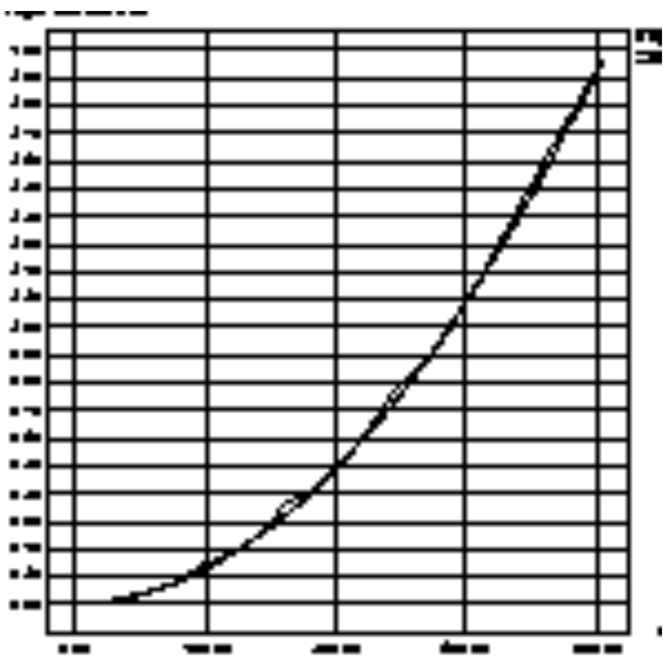
query side

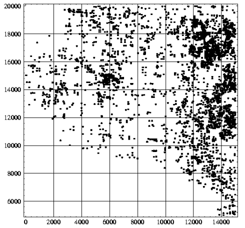
R-trees - performance analysis

Results:

MG-county

leaf accesses



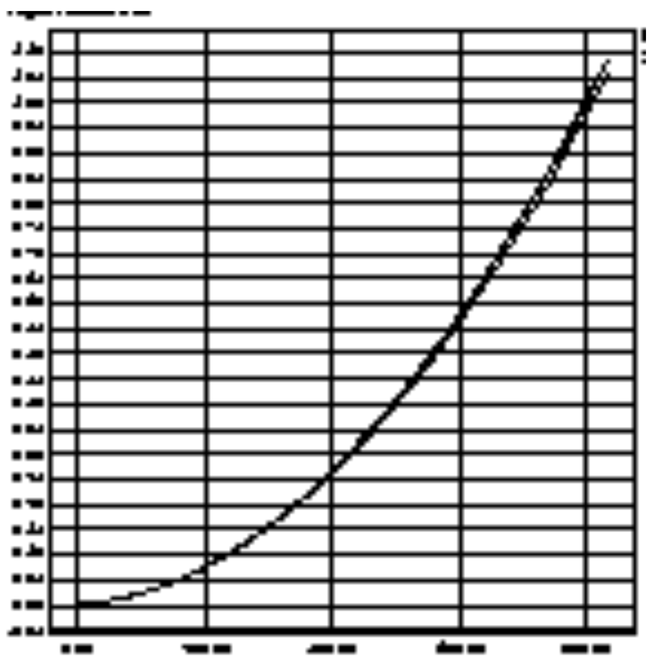


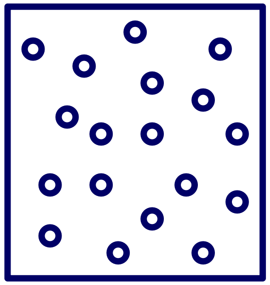
query side

R-trees - performance analysis

Results: 2D- uniform

leaf accesses





query side

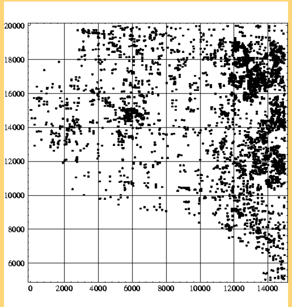
R-trees - performance analysis

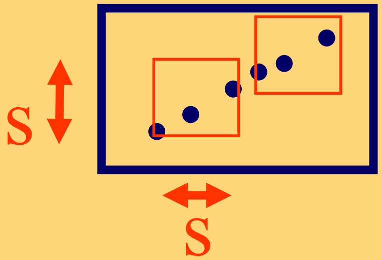
Conclusions: usually, <5% relative error, for range queries

Solution:

- Selectivity of a range query in R-trees?
- Depends on *fractal* dimension

$$s = (C/N)^{1/D0}$$





Indexing - Detailed outline

- primary key indexing
- secondary key / multi-key indexing
- spatial access methods
  - z-ordering
  - R-trees
  - misc
- fractals
  - intro
  - applications
- text
- ...

Indexing - Detailed outline

- fractals
  - intro
  - applications
    - disk accesses for R-trees (range queries)
    - dim. curse revisited
    - nearest neighbors estimation

Must-read Material

- Bernd-Uwe Pagel, Flip Korn and Christos Faloutsos, *Deflating the Dimensionality Curse using Multiple Fractal Dimensions*, ICDE 2000, San Diego, CA, Feb. 2000.

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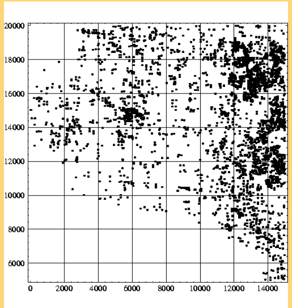
Problem:

- Q: Do all S.A.M. suffer in high dimensions?

- Q: what to do?



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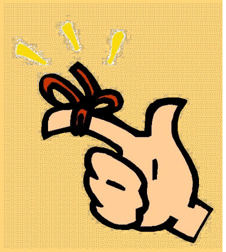
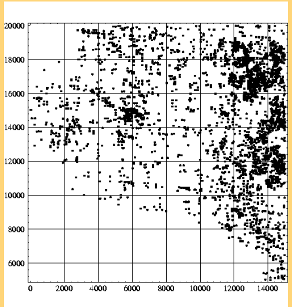


Solutions:

- Q: Do all S.A.M. suffer in high dimensions?
- A: Only in high *fractal* dimensions
- Q: what to do?
- A: dim-reduction; approximate knn; etc



$$P_{all}^{L\infty}(k) \approx \sum_{j=0}^{h} \left\{ \frac{1}{C^{h-j}} + \left[1 + \left(\frac{k}{C^{h-j}} \right)^{1/D} \right]^{D} \right\}$$

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Dimensionality ‘curse’

- Q: What is the problem in high-d?

Dimensionality 'curse'

- Q: What is the problem in high-d?
- A: indices do not seem to help, for many queries (eg., k-nn)
  - in high-d (& uniform distributions), most points are equidistant -> k-nn retrieves too many near-neighbors
  - [Yao & Yao, '85]: search effort ~ O($N^{(1-1/d)}$)

- Yao, A. C. and F. F. Yao (May 6-8, 1985). A General Approach to d-Dimensional Geometric Queries. Proc. of the 17th Annual ACM Symposium on Theory of Computing (STOC), Providence, RI.

Dimensionality 'curse'

- (counter-intuitive, for db mentality)
- Q: What to do, then?

Dimensionality 'curse'

- A1: switch to seq. scanning
- A2: dim. reduction
- A3: consider the 'intrinsic' /fractal dimensionality
- A4: find *approximate* nn

Dimensionality ‘curse’

- A1: switch to seq. scanning
  - X-trees [Kriegel+, VLDB 96]
  - VA-files [Schek+, VLDB 98], ‘test of time’ award

Dimensionality ‘curse’

- A1: switch to seq. scanning
- A2: dim. reduction
- A3: consider the ‘intrinsic’/fractal dimensionality
- A4: find approximate nn

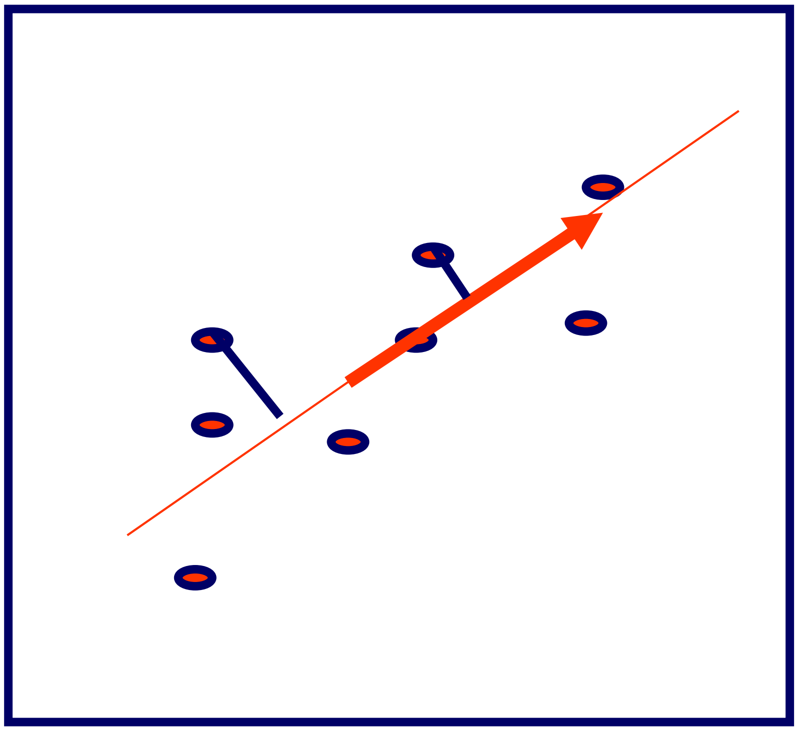
Dim. reduction

a.k.a. feature selection/extraction:

- SVD (optimal, to preserve Euclidean distances)
- random projections
- using the fractal dimension [Traina+ SBBD2000]

Singular Value Decomposition (SVD)

- SVD (~LSI ~ KL ~ PCA ~ spectral analysis...)



LSI: S. Dumais; M. Berry

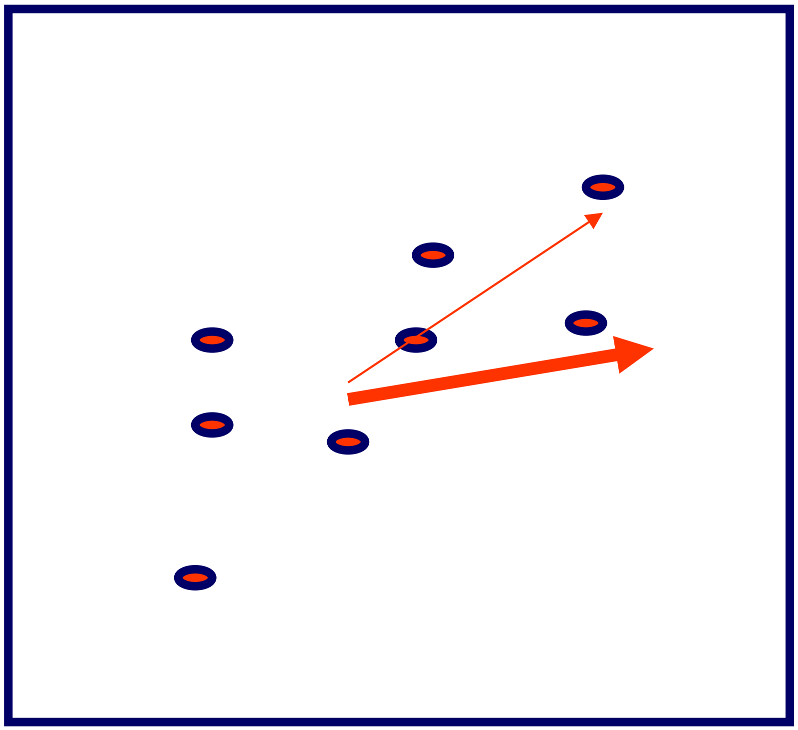
KL: eg, Duda+Hart

PCA: eg., Jolliffe

MANY more PROOF: soon

Random projections

- random projections(Johnson-Lindenstrauss thm [Papadimitriou+ pods98])



Random projections

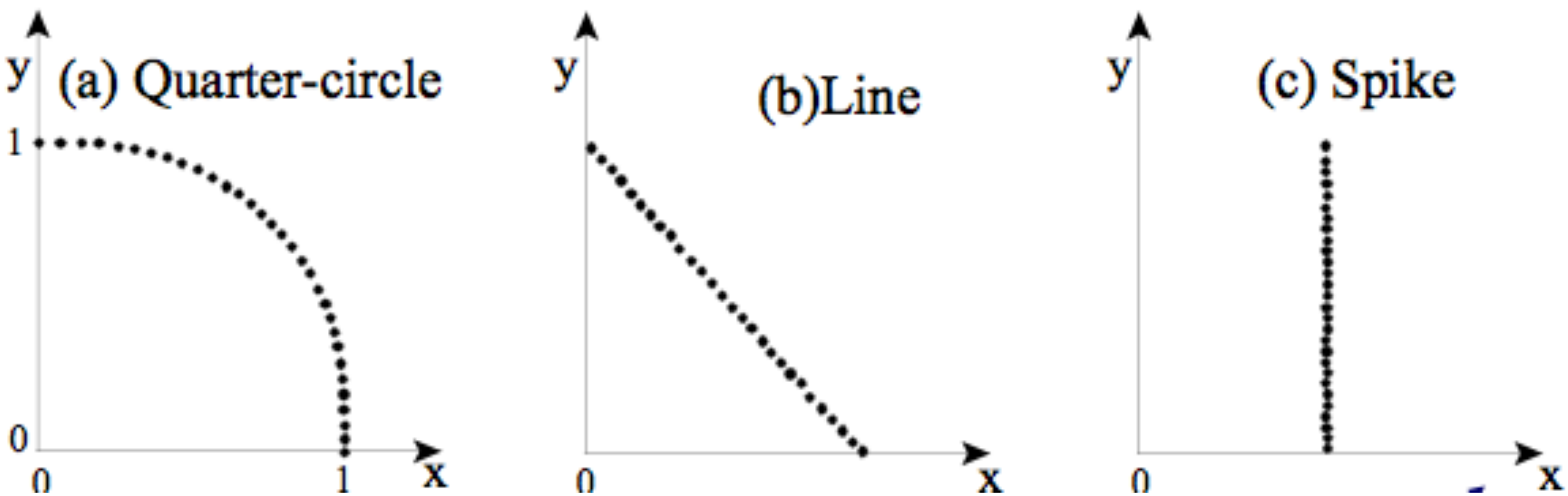
- pick 'enough' random directions (will be ~orthogonal, in high-d!!)
- distances are preserved probabilistically, within epsilon
- (also, use as a pre-processing step for SVD [Papadimitriou+ PODS98]

Dim. reduction - w/ fractals

- Main idea: drop those attributes that don't affect the intrinsic ('fractal') dimensionality [Traina+, SBBD 2000]

Dim. reduction - w/ fractals

global FD=1



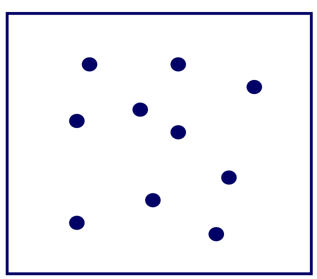
Dimensionality 'curse'

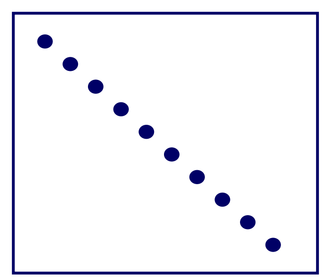
- A1: switch to seq. scanning
- A2: dim. reduction
- A3: consider the 'intrinsic'/fractal dimensionality
- A4: find **approximate** nn

Intrinsic dimensionality

- before we give up, compute the intrinsic dim.:
- the lower, the better... [Pagel+, ICDE 2000]
- more PROOF: in a few foils

intr. d = 2





intr. d = 1

Dimensionality 'curse'

- A1: switch to seq. scanning
- A2: dim. reduction
- A3: consider the 'intrinsic' /fractal dimensionality
- A4: find approximate nn

Approximate nn

- [Arya + Mount, SODA93], [Patella+ ICDE 2000]
- Idea: find k neighbors, such that the distance of the k-th one is guaranteed to be within epsilon of the actual.

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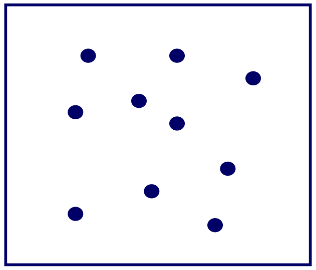
Estimation of knn effort

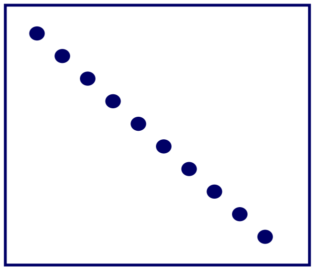
- (Q: how serious is the dim. curse, e.g.:)
- Q: what is the search effort for k-nn?
  - given N points, in E dimensions, in an R-tree, with k-nn queries (‘biased’ model)

[Pagel, Korn + ICDE 2000]


(Overview of proofs)

- assume that your points are uniformly distributed in a *d*-dimensional manifold (= hyper-plane)
- derive the formulas
- substitute *d* for the fractal dimension

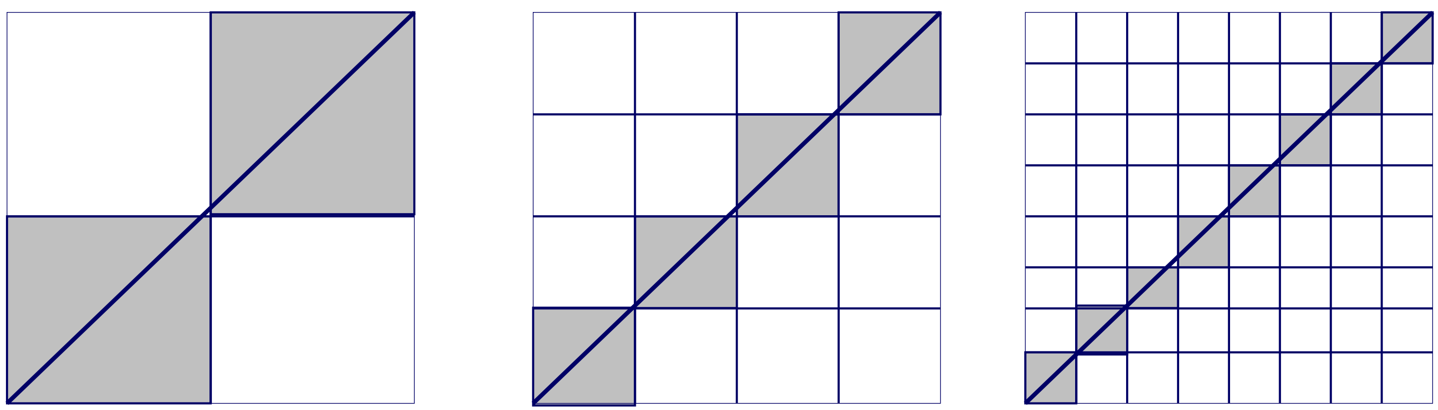




PROOF

Reminder: Hausdorff Dimension (D_0)

- r = side length (each dimension)
- $B(r)$ = # boxes containing points $\propto r^{D0}$



$r = 1/2 \quad B = 2$

$\log r = -1$

$\log B = 1$

$r = 1/4 \quad B = 4$

$\log r = -2$

$\log B = 2$

$r = 1/8 \quad B = 8$

$\log r = -3$

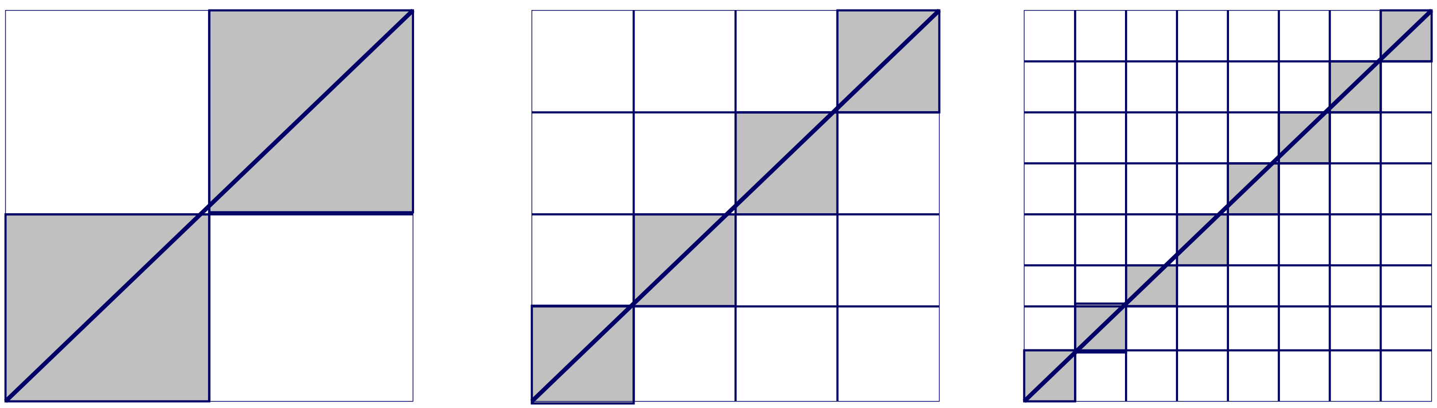
$\log B = 3$

PROOF

Reminder: Correlation Dimension (D_2)

- $S(r) = \sum p_i^2$ (squared % pts in box) $\propto r^{D2}$

  $\propto$ #pairs(within <= r)



$r = 1/2 \quad S = 1/2$ | $r = 1/4 \quad S = 1/4$ | $r = 1/8 \quad S = 1/8$

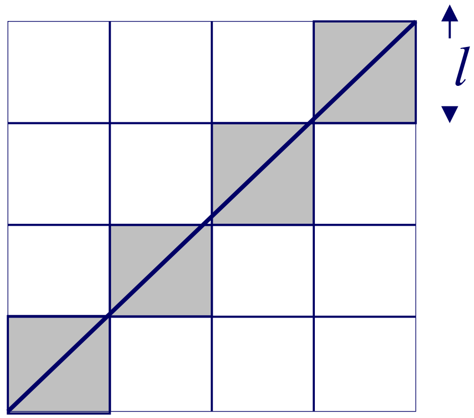
$\log r = -1$ | $\log r = -2$ | $\log r = -3$

$\log S = -1$ | $\log S = -2$ | $\log S = -3$

PROOF

Observation #1

- How to determine avg MBR side l?
  - N = #pts, C = MBR capacity



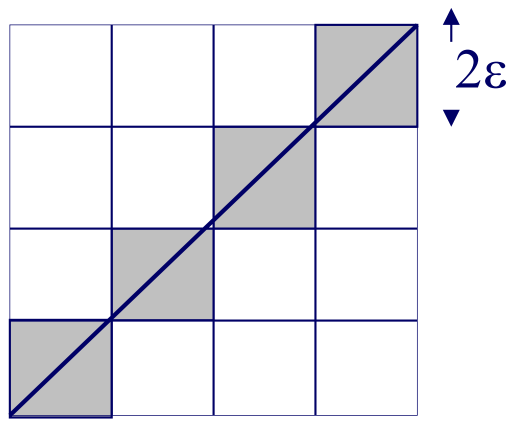
Hausdorff dimension: $B(r) \propto r^{D0}$

$$B(l) = N/C = l^{-D0} \Rightarrow \boxed{l = (N/C)^{-1/D0}}$$

PROOF

Observation #2

- *k*-NN query → ε-range query
  - For *k* pts, what radius ε do we expect?



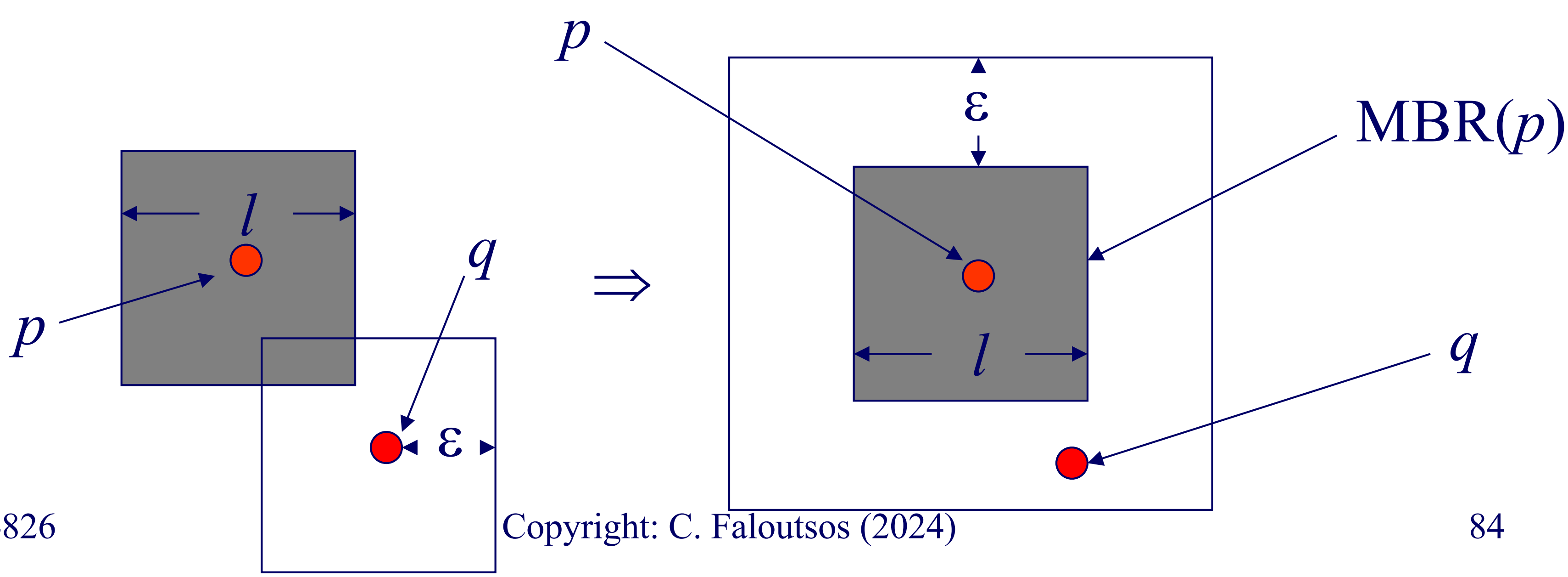
Correlation dimension: $S(r) \propto r^{D2}$

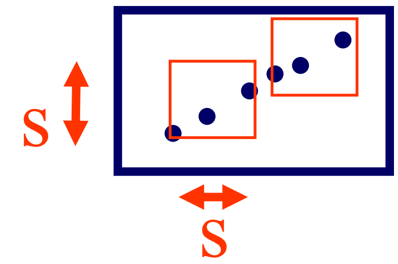
$$S(\varepsilon) = \frac{k}{N-1} = (2\varepsilon)^{D2}$$

PROOF

Observation #3

- Estimate avg # query-sensitive anchors:
  - How many **expected** q will touch **avg** page?
  - Page touch: q stabs ε-dilated MBR(p)

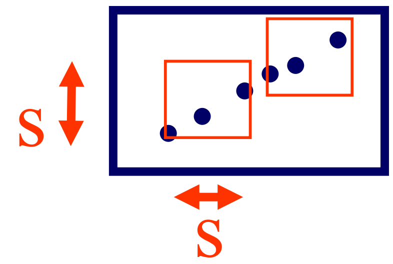




Asymptotic Formula

- k-NN page accesses as N → ∞
  - C = page capacity
  - D = fractal dimension (=$D0$ ~ $D2$)
  - h = height of tree

$$P_{all}^{L\infty}(k) \approx \sum_{j=0}^{h} \left\{ \frac{1}{C^{h-j}} + \left[1 + \left(\frac{k}{C^{h-j}} \right)^{1/D} \right]^{D} \right\}$$



Asymptotic Formula

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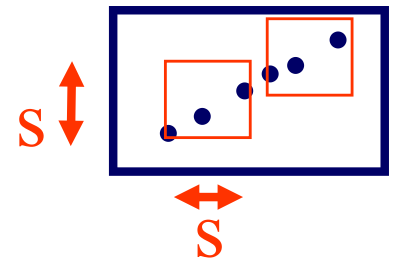
- Observations?



Asymptotic Formula

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- NO mention of the embedding dimensionality!!
- Still have <u>dim. curse, but on f.d. *D*</u>

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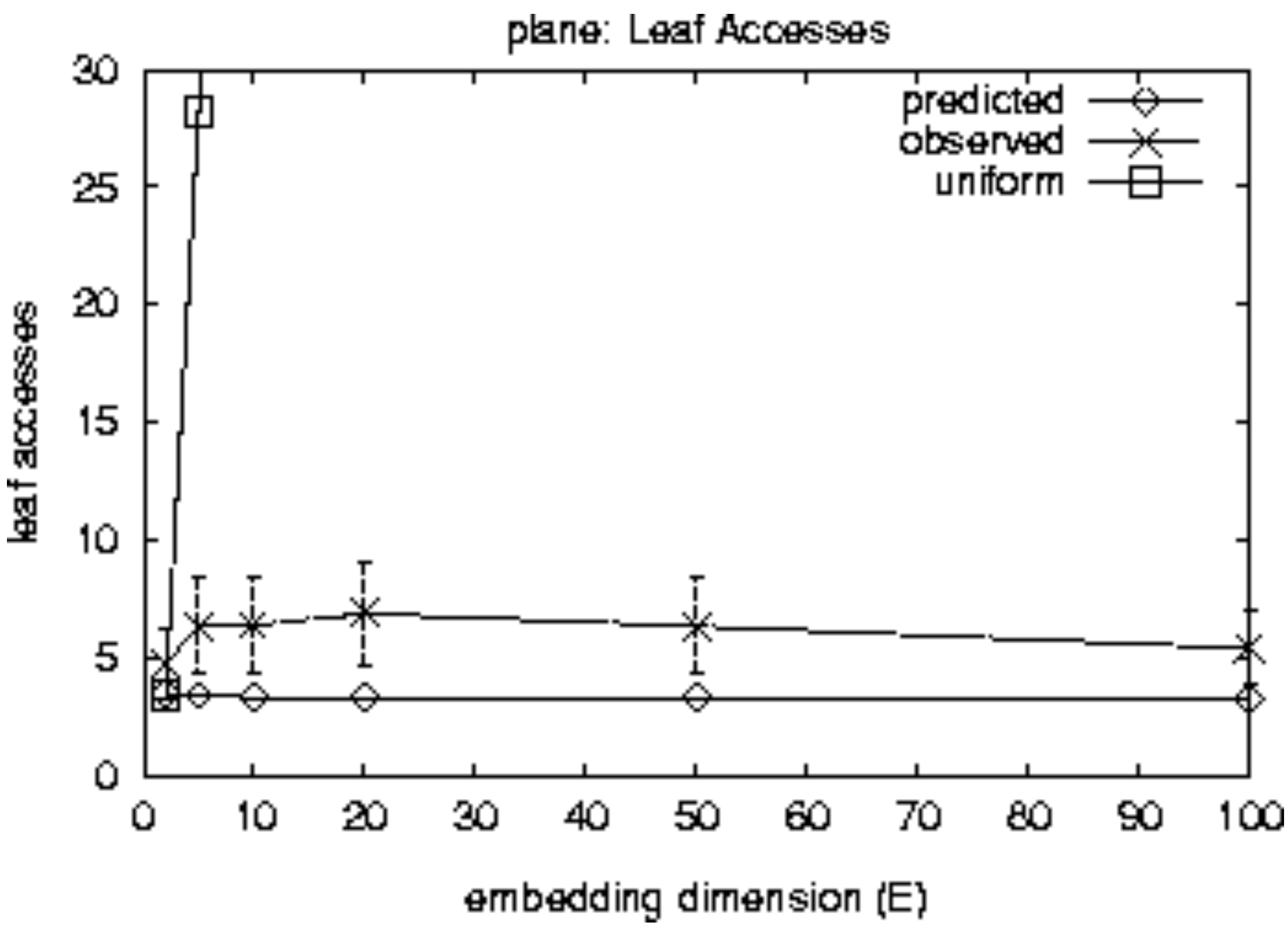
Embedding Dimension



E	*unif/ind*	*fractal*	*leaf*
2	3.49	3.49	4.75
5	28.26	3.45	6.40
10	847.26	3.34	6.42
20	All	3.36	6.9
50	All	3.32	6.37
100	All	3.32	5.43

`plane`

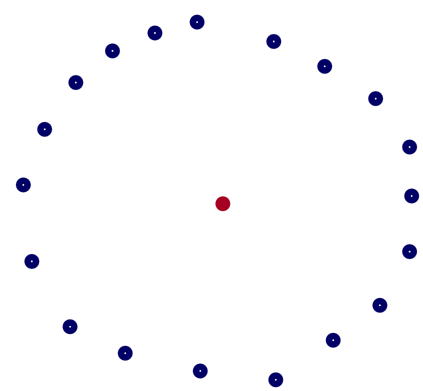
$k = 50$

L_∞ dist

A word of caution:

Nearest neighbors: may be meaningless!

Norio Katayama, Shin'ichi Satoh:
Distinctiveness-Sensitive Nearest Neighbor Search for Efficient Similarity Retrieval of Multimedia Information.
ICDE 2001: 493-502



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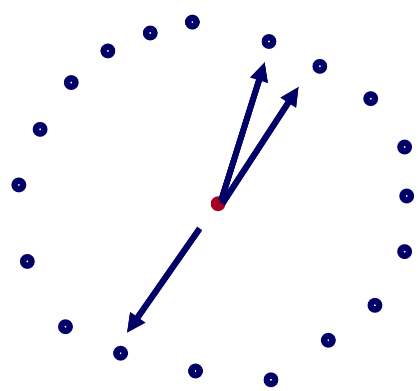
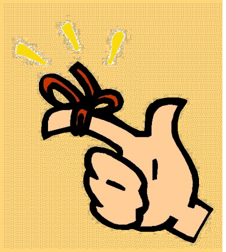


Conclusions

- Dimensionality ‘curse’:
  - for high-d, indices slow down to ~O(N)
- If the **intrinsic** dim. is low, there is hope
- otherwise, do seq. scan, or sacrifice accuracy (approximate nn)

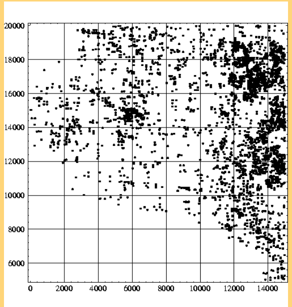
Conclusions – cont' d

- Worst-case theory is **over-pessimistic**
- High dimensional data can exhibit good performance if **correlated, non-uniform**
- Many real data sets are **self-similar**
- Determinant is **intrinsic** dimensionality
  - multiple fractal dimensions (D_0 and D_2)
  - indication of how far one can go



Solutions:

- Q: Do all S.A.M. suffer in high dimensions?
- A: Only in high *fractal* dimensions
- Q: what to do?
- A: dim-reduction; approximate knn; etc



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References

- Sunil Arya, David M. Mount: *Approximate Nearest Neighbor Queries in Fixed Dimensions*. SODA 1993: 271-280

  ANN library:

  http://www.cs.umd.edu/~mount/ANN/

References

- Berchtold, S., D. A. Keim, et al. (1996). The X-tree : An Index Structure for High-Dimensional Data. VLDB, Mumbai (Bombay), India.

References cnt'd

- Pagel, B.-U., F. Korn, et al. (2000). *Deflating the Dimensionality Curse Using Multiple Fractal Dimensions*. ICDE, San Diego, CA.
- Papadimitriou, C. H., P. Raghavan, et al. (1998). Latent Semantic Indexing: A Probabilistic Analysis. PODS, Seattle, WA.

References cnt' d

- Traina, C., A. J. M. Traina, et al. (2000). *Distance Exponent: A New Concept for Selectivity Estimation in Metric Trees*. ICDE, San Diego, CA.
- Weber, R., H.-J. Schek, et al. (1998). A Quantitative Analysis and Performance Study for Similarity-Search Methods in high-dimensional spaces. VLDB, New York, NY.

References cnt' d

- Yao, A. C. and F. F. Yao (May 6-8, 1985). A General Approach to d-Dimensional Geometric Queries. Proc. of the 17th Annual ACM Symposium on Theory of Computing (STOC), Providence, RI.