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15-826: Multimedia (Databases) and Data Mining

Lecture #27: Graph mining - Generators & tools

Christos Faloutsos

NOT in the final exam

Sit back and enjoy the show ☺

Must-read material (1 of 2)

Fully Automatic Cross-Associations, by D. Chakrabarti, S. Papadimitriou, D. Modha and C. Faloutsos, in KDD 2004 (pages 79-88), Washington, USA

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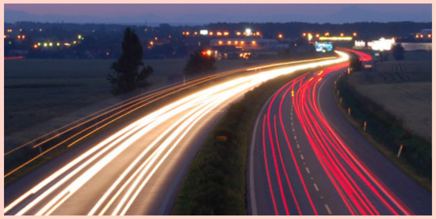
Must-read material (2 of 2)

J. Leskovec, D. Chakrabarti, J. Kleinberg, and C. Faloutsos, *Realistic, Mathematically Tractable Graph Generation and Evolution, Using Kronecker Multiplication*, in PKDD 2005, Porto, Portugal

Main outline



- Introduction
- Indexing
- Mining
  - Graphs – patterns
  - Graphs – generators and tools
  - Association rules
  - …

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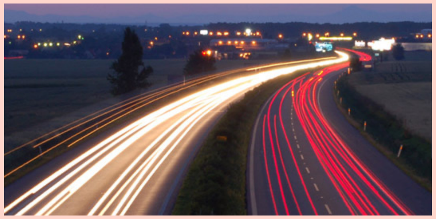
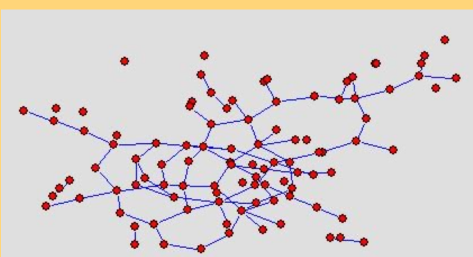


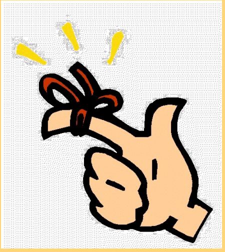
Detailed outline

- Graphs – generators
  - Erdos-Renyi
  - Other generators
  - Kronecker
- Graphs - tools

Problem

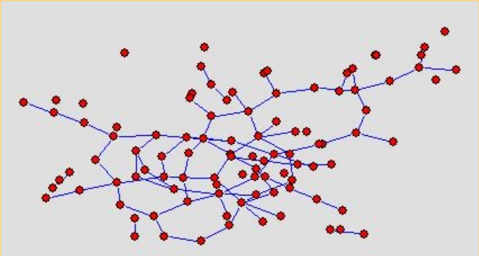
- Q: How to generate realistic graphs?

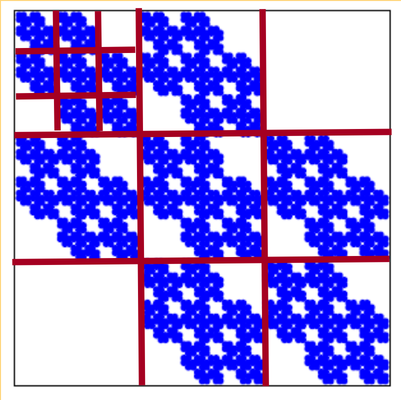


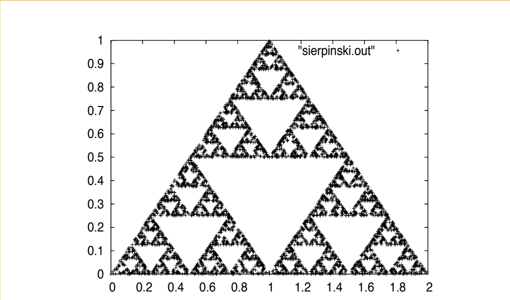


Answer:

- Q: How to generate realistic graphs?
- A: self-similarity – ‘Kronecker’ graphs





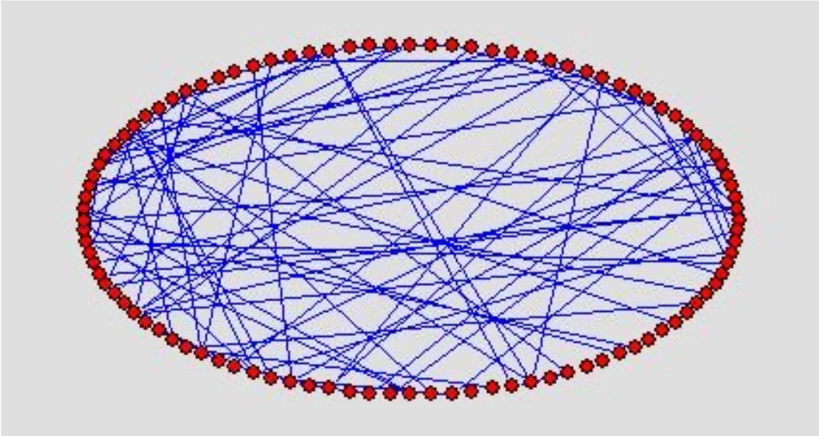


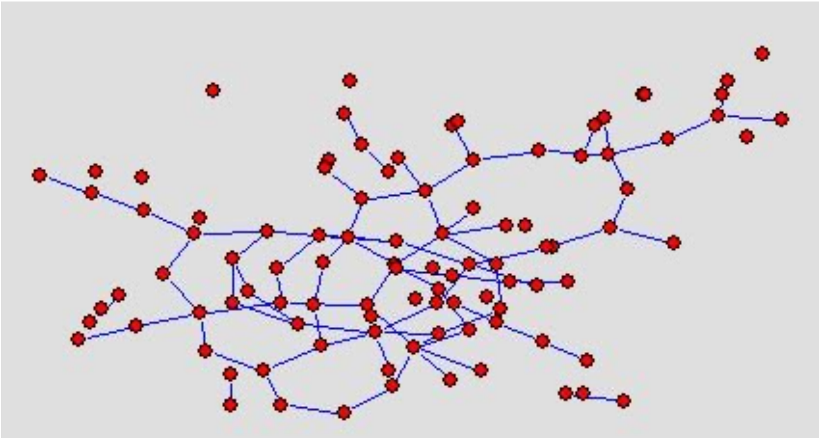
Generators

- How to generate random, realistic graphs?
  - Erdos-Renyi model: beautiful, but unrealistic
  - degree-based generators
  - process-based generators
  - recursive/self-similar generators

Erdos-Renyi

- random graph – 100 nodes, avg degree = 2
- Fascinating properties (phase transition)
- But: unrealistic (Poisson degree distribution != power law)

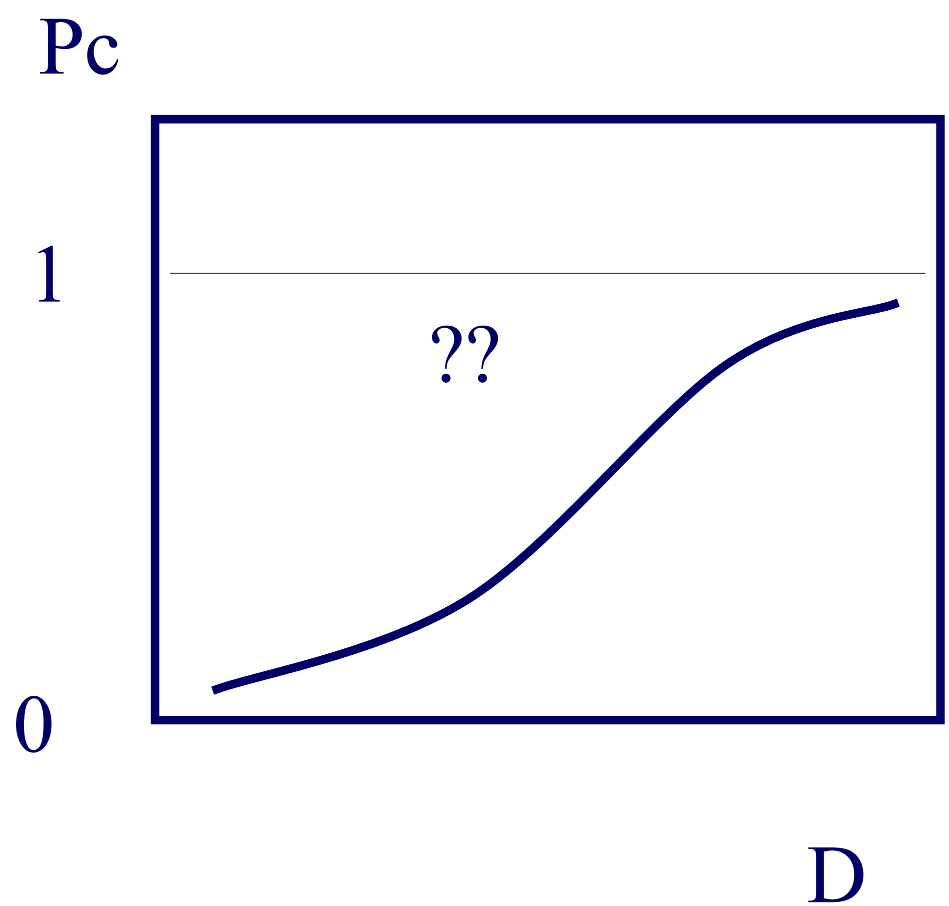




E-R model & Phase transition

- vary avg degree D
- watch Pc =

  Prob(there is a giant
  connected component)
- How do you expect it to be?



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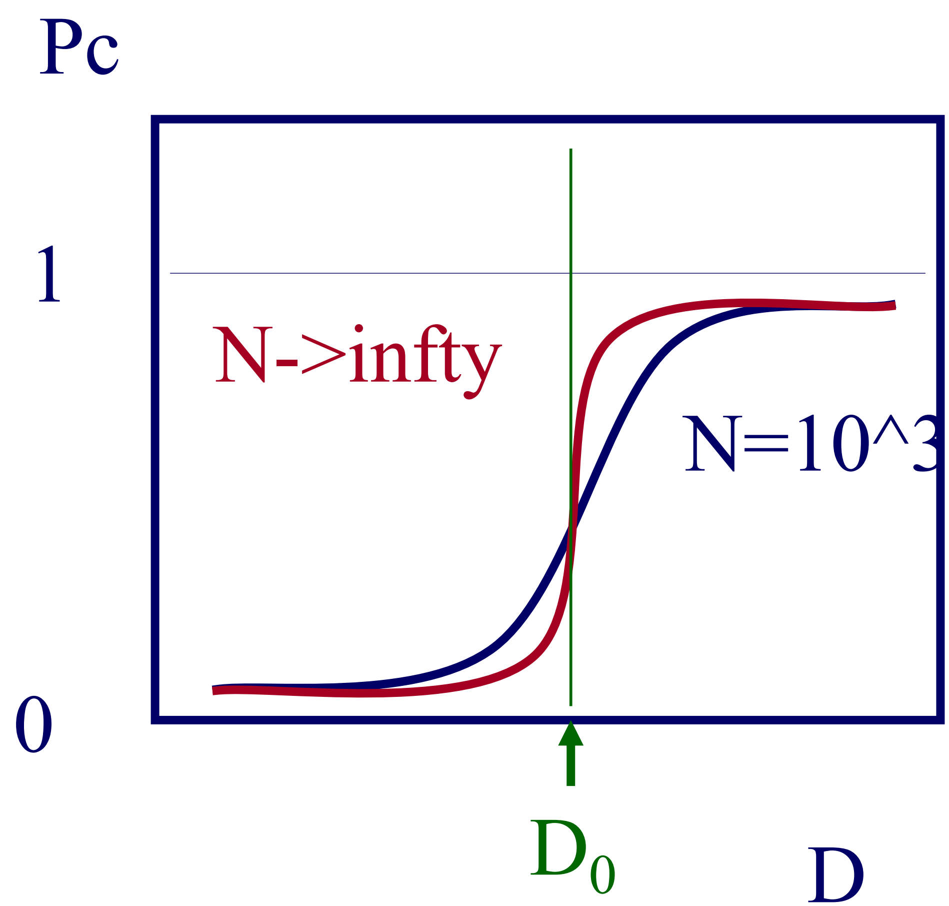


Degree-based

- Figure out the degree distribution (eg., ‘Zipf’)
- Assign degrees to nodes
- Put edges, so that they match the original degree distribution

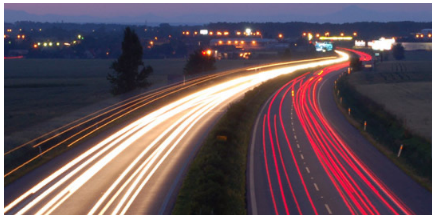
Process-based

- Barabasi; Barabasi-Albert: Preferential attachment -> power-law tails!
  - ‘rich get richer’
- [Kumar+]: preferential attachment + mimick
  - Create ‘communities’

Process-based (cont'd)

- [Fabrikant+, '02]: H.O.T.: connect to closest, high connectivity neighbor
- [Pennock+, '02]: Winner does NOT take all

Detailed outline



- Graphs – generators
  - Erdos-Renyi
  - Other generators
  - ➡ Kronecker
- Graphs - tools

Recursive generators

- (RMAT [Chakrabarti+,'04])
- Kronecker product

Wish list for a generator:

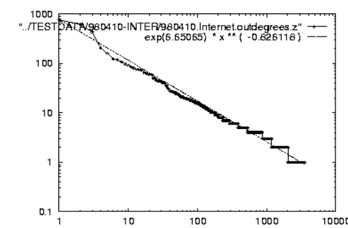
- Power-law-tail in- and out-degrees
- Power-law-tail scree plots
- **shrinking/constant** diameter
- Densification Power Law
- communities-within-communities

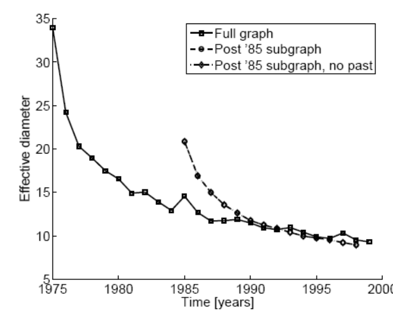
Q: how to achieve all of them?

A: Kronecker matrix product [Leskovec+05b]

Graph gen.: Problem dfn

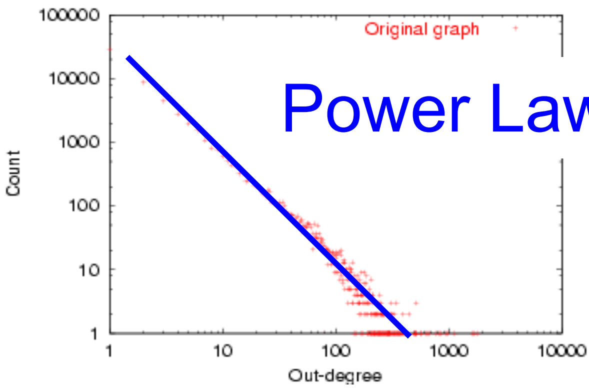
- Given a growing graph with count of nodes N_1, N_2, ...
- Generate a realistic sequence of graphs that will obey all the patterns
  - Static Patterns
    - S1 Power Law Degree Distribution
    - S2 Power Law eigenvalue and eigenvector distribution
    - Small Diameter
  - Dynamic Patterns
    - T2 Growth Power Law (2x nodes; 3x edges)
    - T1 Shrinking/Stabilizing Diameters



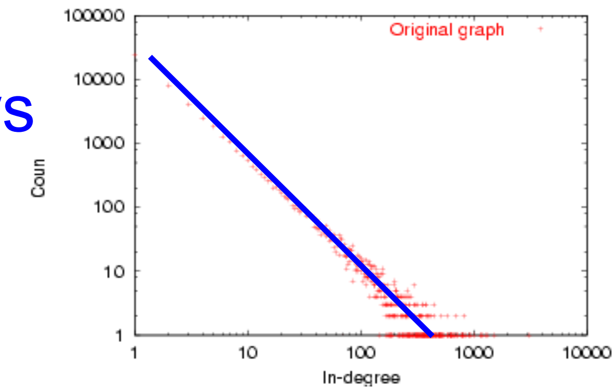


Graph Patterns

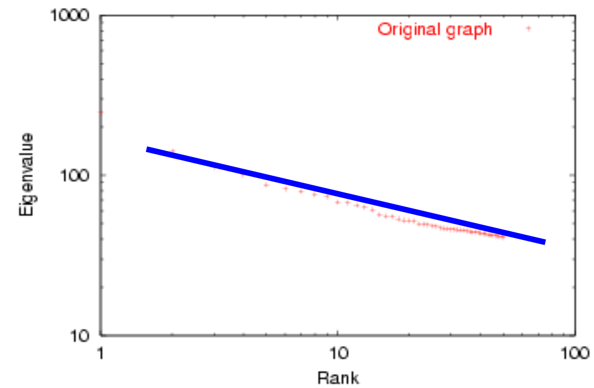
Power Laws



Count vs Indegree



Count vs Outdegree



Eigenvalue vs Rank

How to match all these properties (+ small diameters, etc)?

Hint: self-similarity

- A: RMAT/Kronecker generators
  - With self-similarity, we get all power-laws, automatically,
  - And small/shrinking diameter
  - And `no good cuts'

R-MAT: A Recursive Model for Graph Mining,
by D. Chakrabarti, Y. Zhan and C. Faloutsos,
SDM 2004, Orlando, Florida, USA

Realistic, Mathematically Tractable Graph Generation and Evolution, Using Kronecker Multiplication,
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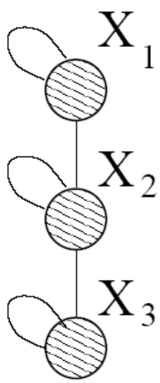
Kronecker Graphs

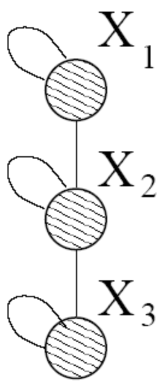


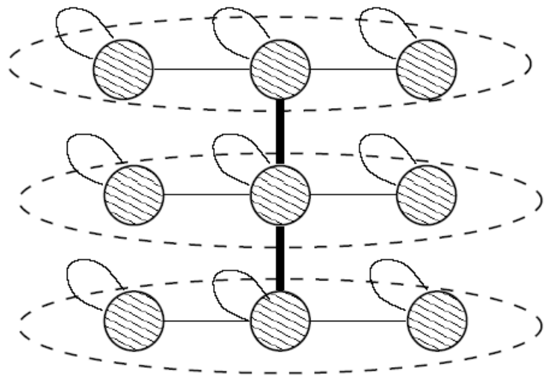
1	1	0
1	1	1
0	1	1

G_1

Adjacency matrix

Kronecker Graphs





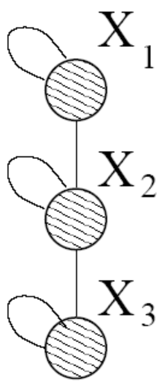
Intermediate stage

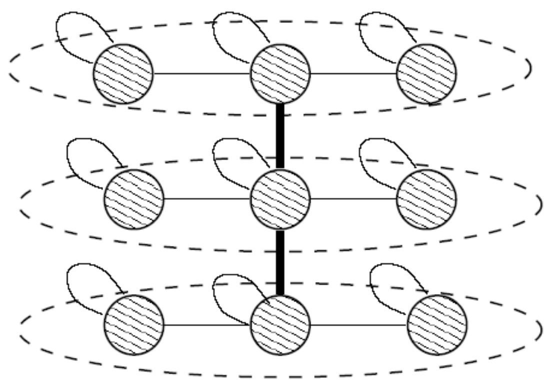
1	1	0
1	1	1
0	1	1

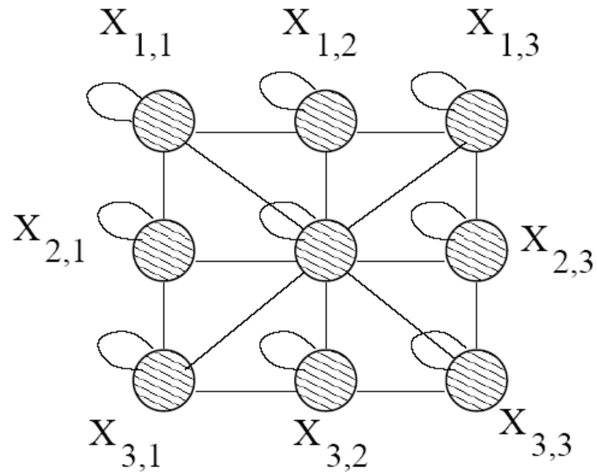
G_1

Adjacency matrix

Kronecker Graphs







Intermediate stage

1	1	0
1	1	1
0	1	1

G_1

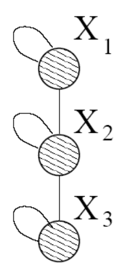
Adjacency matrix

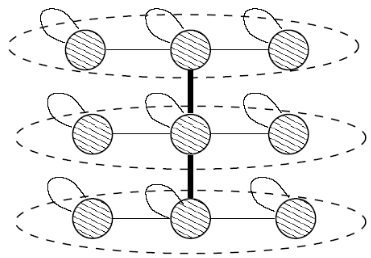
G_1	G_1	0
G_1	G_1	G_1
0	G_1	G_1

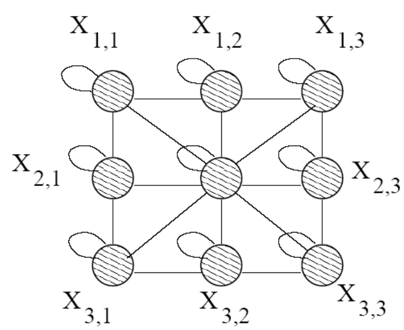
$G_2 = G_1 \otimes G_1$

Adjacency matrix

Kronecker product

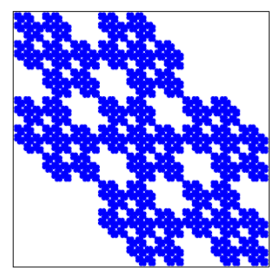






1	1	0
1	1	1
0	1	1

G_1	G_1	0
G_1	G_1	G_1
0	G_1	G_1



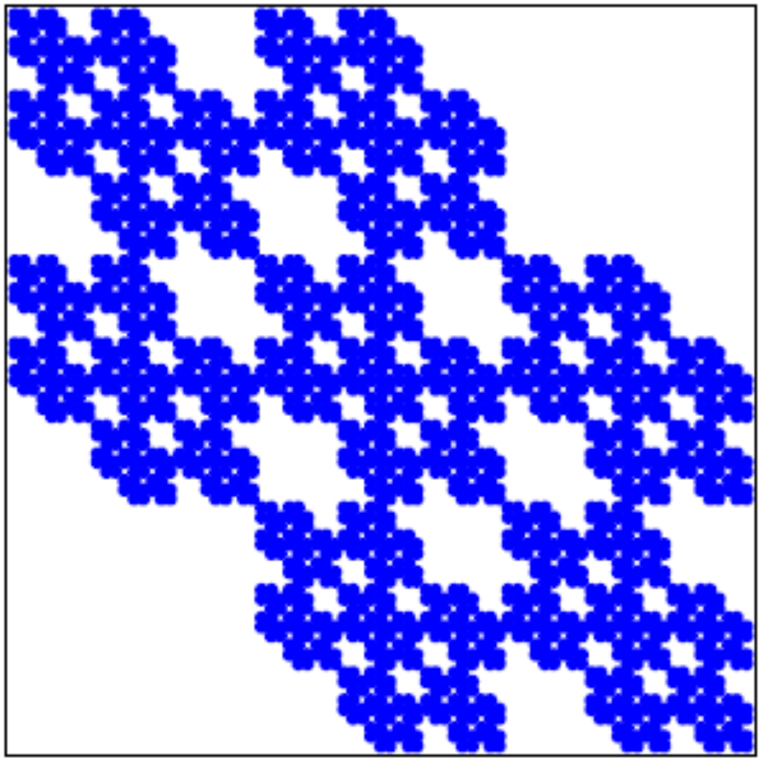
N

N*N

N**4

Kronecker Graphs

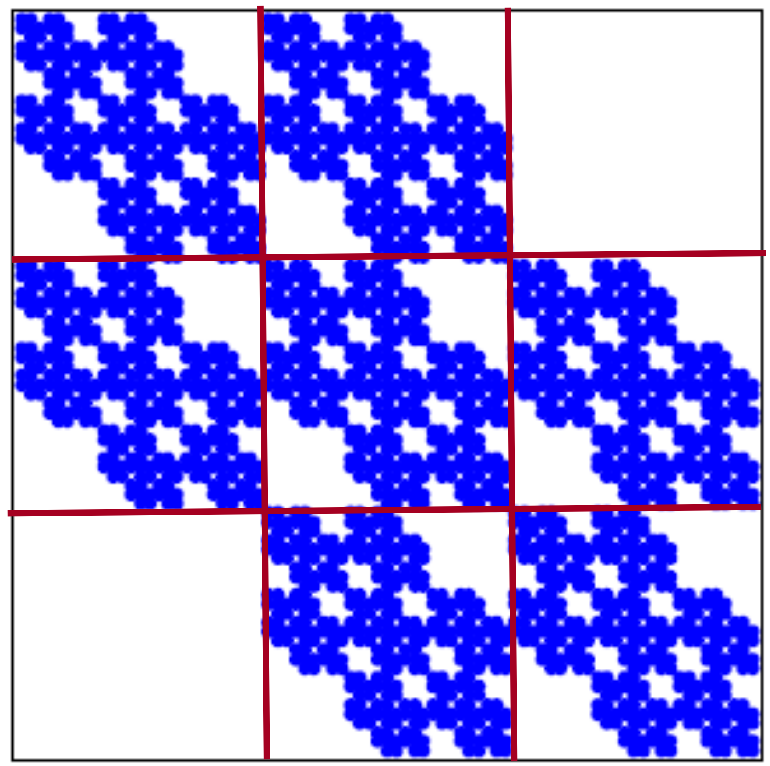
- Continuing multiplying with G_1 we obtain G_4 and so on …



G_4 adjacency matrix

Kronecker Graphs

- Continuing multiplying with G_1 we obtain G_4 and so on …



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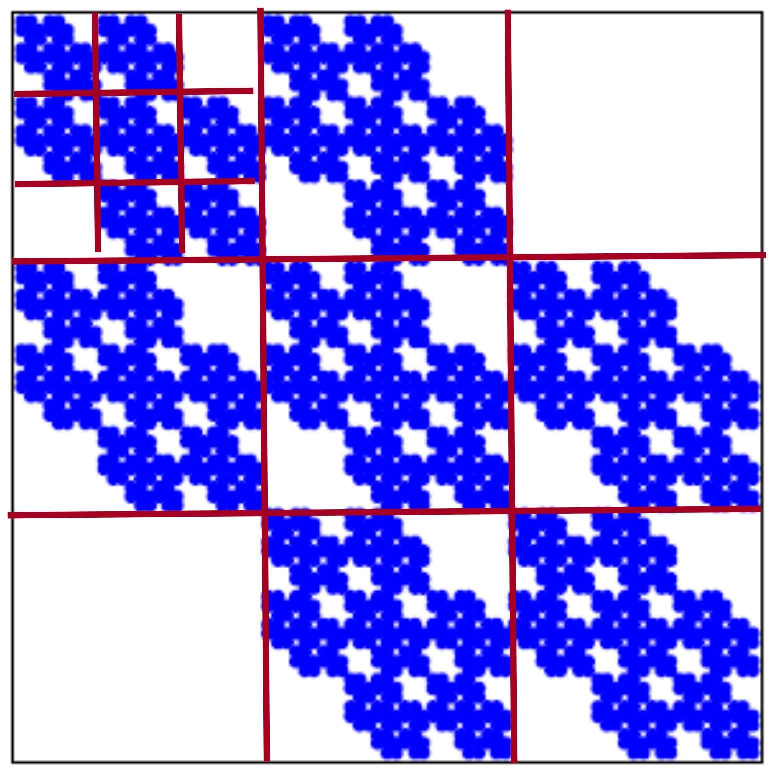

G_4 adjacency matrix

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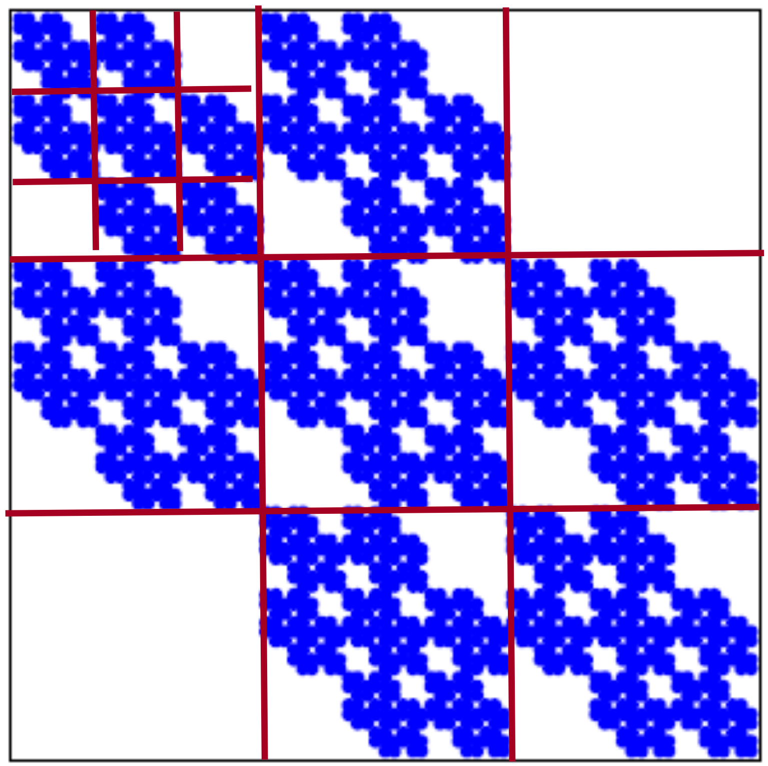
Kronecker Graphs

- Continuing multiplying with G_1 we obtain G_4 and so on …

Holes within holes; Communities within communities



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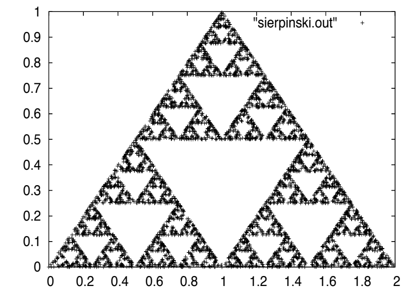


Properties:

- We can PROVE that
  - Degree distribution is multinomial ~ power law

new
  - Diameter: constant
  - Eigenvalue distribution: multinomial
  - First eigenvector: multinomial

Problem Definition

- Given a growing graph with nodes N_1, N_2, ...
- Generate a realistic sequence of graphs that will obey all the patterns
  - Static Patterns
    - ✓ Power Law Degree Distribution
    - ✓ Power Law eigenvalue and eigenvector distribution
    - ✓ Small Diameter
  - Dynamic Patterns
    - ✓ Growth Power Law
    - ✓ Shrinking/Stabilizing Diameters
- First generator for which we can **prove** all these properties

Impact: Graph500

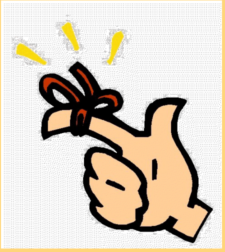
- Based on RMAT (= 2x2 Kronecker)
- Standard for graph benchmarks
- http://www.graph500.org/
- Competitions 2x year, with all major entities: LLNL, Argonne, ITC-U. Tokyo, Riken, ORNL, Sandia, PSC, …

To iterate is human, to recurse is devine

R-MAT: A Recursive Model for Graph Mining, by D. Chakrabarti, Y. Zhan and C. Faloutsos, SDM 2004, Orlando, Florida, USA

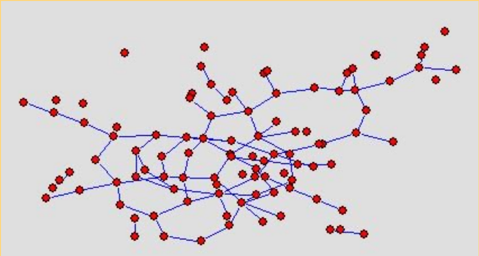
Conclusions - Generators

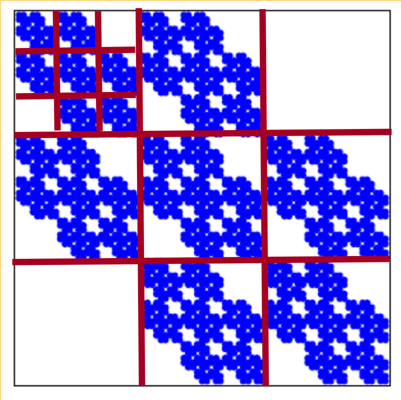
- Erdos-Renyi: phase transition
- Preferential attachment (Barabasi)
  - Power-law-tail in degree distribution
- Variations
- Recursion – Kronecker graphs
  - Numerous power-laws, + small diameters

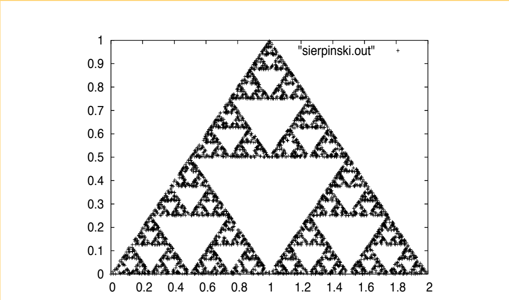


Answer:

- Q: How to generate realistic graphs?
- A: self-similarity – ‘Kronecker’ graphs







Resources

Generators:

- Kronecker (christos@cs.cmu.edu)
- BRITE http://www.cs.bu.edu/brite/
- INET: http://topology.eecs.umich.edu/inet

Other resources

Visualization - graph algo's:

- Graphviz: http://www.graphviz.org/
- pajek: http://vlado.fmf.uni-lj.si/pub/networks/pajek/

Kevin Bacon web site:
http://www.cs.virginia.edu/oracle/

References

- [Aiello+, '00] William Aiello, Fan R. K. Chung, Linyuan Lu: *A random graph model for massive graphs*. STOC 2000: 171-180
- [Albert+] Reka Albert, Hawoong Jeong, and Albert-Laszlo Barabasi: *Diameter of the World Wide Web*, Nature 401 130-131 (1999)
- [Barabasi, '03] Albert-Laszlo Barabasi *Linked: How Everything Is Connected to Everything Else and What It Means* (Plume, 2003)

References, cont'd

- [Barabasi+, '99] Albert-Laszlo Barabasi and Reka Albert. *Emergence of scaling in random networks*. Science, 286:509--512, 1999
- [Broder+, '00] Andrei Broder, Ravi Kumar, Farzin Maghoul, Prabhakar Raghavan, Sridhar Rajagopalan, Raymie Stata, Andrew Tomkins, and Janet Wiener. *Graph structure in the web*, WWW, 2000

References, cont'd

- [Chakrabarti+, '04] *RMAT: A recursive graph generator*, D. Chakrabarti, Y. Zhan, C. Faloutsos, SIAM-DM 2004
- [Dill+, '01] Stephen Dill, Ravi Kumar, Kevin S. McCurley, Sridhar Rajagopalan, D. Sivakumar, Andrew Tomkins: *Self-similarity in the Web*. VLDB 2001: 69-78

References, cont'd

- [Fabrikant+, '02] A. Fabrikant, E. Koutsoupias, and C.H. Papadimitriou. *Heuristically Optimized Trade-offs: A New Paradigm for Power Laws in the Internet*. ICALP, Malaga, Spain, July 2002
- [FFF, 99] M. Faloutsos, P. Faloutsos, and C. Faloutsos, "*On power-law relationships of the Internet topology*," in SIGCOMM, 1999.

References, cont'd

- [Jovanovic+, '01] M. Jovanovic, F.S. Annexstein, and K.A. Berman. *Modeling Peer-to-Peer Network Topologies through "Small-World" Models and Power Laws*. In TELFOR, Belgrade, Yugoslavia, November, 2001
- [Kumar+ '99] Ravi Kumar, Prabhakar Raghavan, Sridhar Rajagopalan, Andrew Tomkins: *Extracting Large-Scale Knowledge Bases from the Web*. VLDB 1999: 639-650

References, cont'd

- [Leskovec+05b] Jure Leskovec, Deepayan Chakrabarti, Jon Kleinberg, Christos Faloutsos *Realistic, Mathematically Tractable Graph Generation and Evolution, Using Kronecker Multiplication* (ECML/PKDD 2005), Porto, Portugal, 2005.

References, cont'd

- [Leskovec+07] Jure Leskovec and Christos Faloutsos, Scalable Modeling of Real Graphs using Kronecker Multiplication, ICML 2007.

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References, cont'd

- [Pennock+, '02] David M. Pennock, Gary William Flake, Steve Lawrence, Eric J. Glover, C. Lee Giles: *Winners don't take all: Characterizing the competition for links on the web* Proc. Natl. Acad. Sci. USA 99(8): 5207-5211 (2002)

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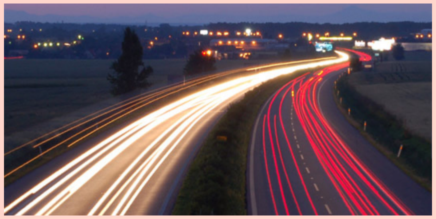
References, cont'd

- [Watts+ Strogatz, '98] D. J. Watts and S. H. Strogatz *Collective dynamics of 'small-world' networks*, Nature, 393:440-442 (1998)
- [Watts, '03] Duncan J. Watts *Six Degrees: The Science of a Connected Age* W.W. Norton & Company; (February 2003)

Graph mining: tools



Main outline

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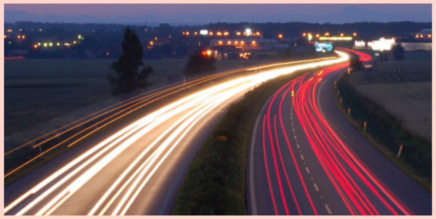
- Indexing
- Mining
  - Graphs – patterns
  - Graphs – generators and tools
  - Association rules
  - …

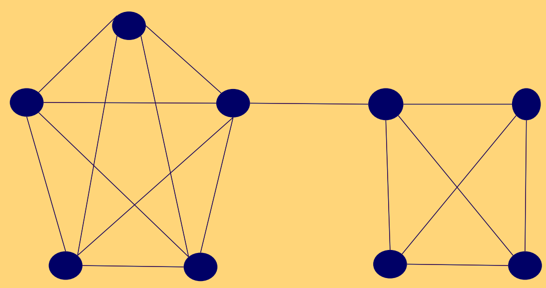


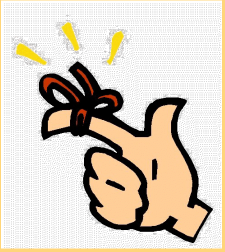
Detailed outline

- Graphs – generators
- Graphs – tools
  - Community detection / graph partitioning
    - Algo's
    - Observation: 'no good cuts'
  - (Node proximity – personalized RWR)
  - Influence/virus propagation & immunization
  - 'Belief Propagation' & fraud detection
  - Anomaly detection

Problem

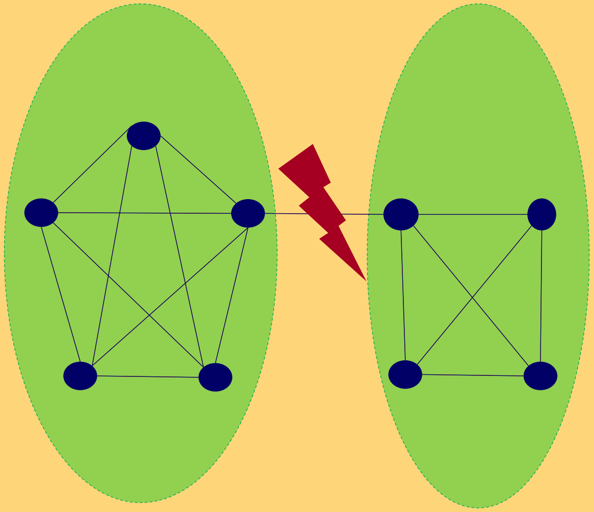
- Given a graph, and *k*
- Break it into *k* (disjoint) communities





Short answer

- METIS [Karypis, Kumar]



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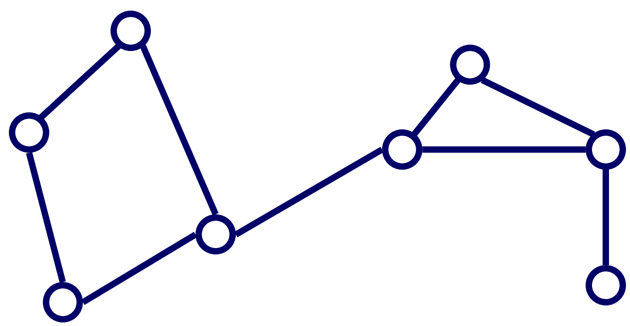

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Problem

- Given a graph, and k
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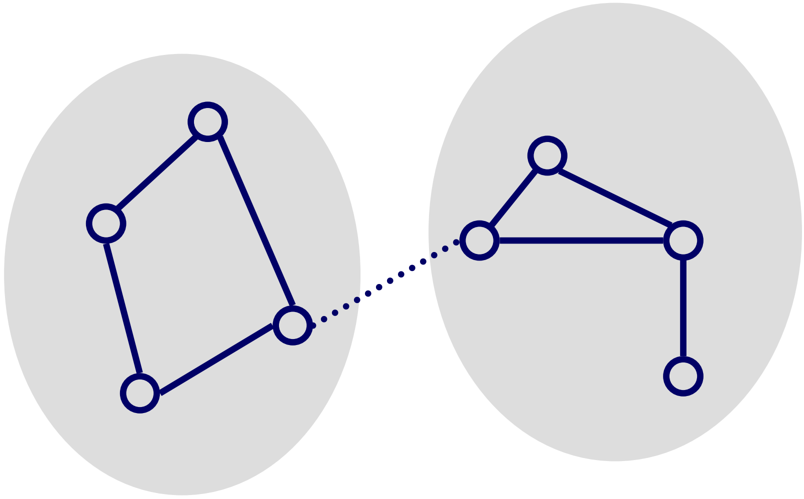

$k = 2$

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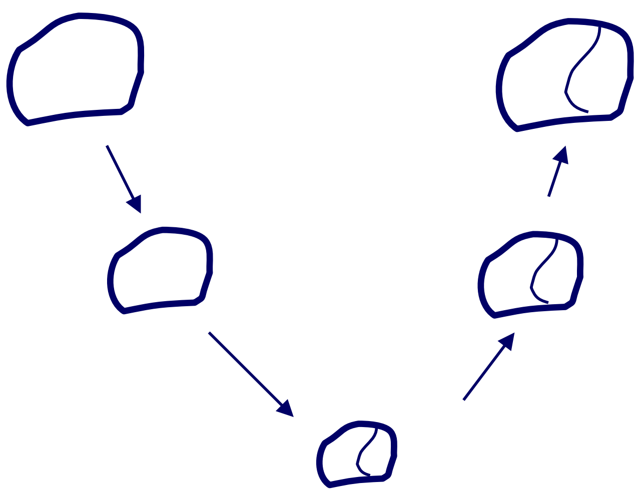
Solution #1: METIS

- Arguably, the best algorithm
- Open source, at
  - http://www.cs.umn.edu/~metis
- and *many* related papers, at same url
- Main idea:
  - coarsen the graph;
  - partition;
  - un-coarsen



Solution #1: METIS

- G. Karypis and V. Kumar. *METIS 4.0: Unstructured graph partitioning and sparse matrix ordering system*. TR, Dept. of CS, Univ. of Minnesota, 1998.
- <and many extensions>

Solution #2

(problem: hard clustering, *k* pieces)

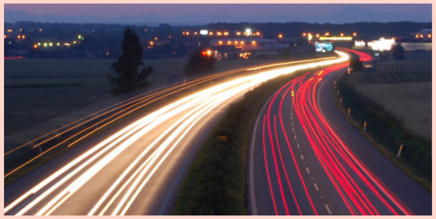
Spectral partitioning:

- Consider the 2nd smallest eigenvector of the (normalized) Laplacian

Solutions #3, …

Many more ideas:

- Clustering on the $\mathbf{A}^2$ (square of adjacency matrix) [Zhou, Woodruff, PODS'04]
- Minimum cut / maximum flow [Flake+, KDD'00]
- …



Detailed outline

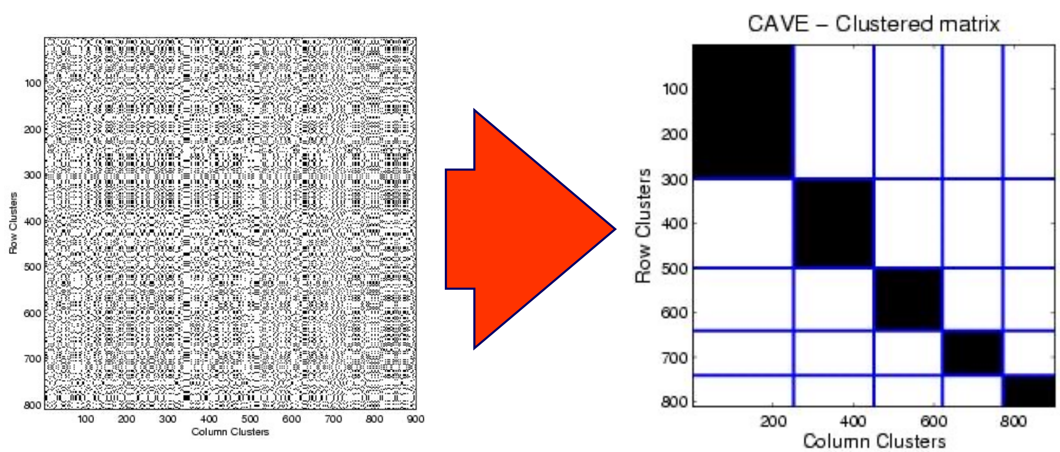
- Motivation
- Hard clustering – k pieces
- Hard co-clustering – (k,l) pieces
- Hard clustering – optimal # pieces
- Soft clustering – matrix decompositions
- Observations

Problem definition

- Given a bi-partite graph, and *k*, *l*
- Divide it into *k* row groups and *l* row groups
- (Also applicable to uni-partite graph)

Co-clustering

- Given data matrix and the number of row and column groups *k* and *l*
- Simultaneously
  - Cluster rows into *k* disjoint groups
  - Cluster columns into *l* disjoint groups



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details

Co-clustering

- Let X and Y be discrete random variables
  - X and Y take values in $\{1, 2, \ldots, m\}$ and $\{1, 2, \ldots, n\}$
  - $p(X, Y)$ denotes the joint probability distribution—if not known, it is often estimated based on co-occurrence data
  - Application areas: text mining, market-basket analysis, analysis of browsing behavior, etc.
- Key Obstacles in Clustering Contingency Tables
  - High Dimensionality, Sparsity, Noise
  - Need for robust and scalable algorithms

Reference:

1. Dhillon et al. Information-Theoretic Co-clustering, KDD'03

$$
\begin{array}{cc}
 & n \\
m & \begin{bmatrix} .05 & .05 & .05 & 0 & 0 & 0 \\ .05 & .05 & .05 & 0 & 0 & 0 \\ 0 & 0 & 0 & .05 & .05 & .05 \\ 0 & 0 & 0 & .05 & .05 & .05 \\ .04 & .04 & 0 & .04 & .04 & .04 \\ .04 & .04 & .04 & 0 & .04 & .04 \end{bmatrix}
\end{array}
$$

eg, terms x documents

$$
\begin{array}{cc}
 & k \\
m & \begin{bmatrix} .5 & 0 & 0 \\ .5 & 0 & 0 \\ 0 & .5 & 0 \\ 0 & .5 & 0 \\ 0 & 0 & .5 \\ 0 & 0 & .5 \end{bmatrix}
\end{array}
\begin{array}{cc}
 & l \\
k & \begin{bmatrix} .3 & 0 \\ 0 & .3 \\ .2 & .2 \end{bmatrix}
\end{array}
\begin{array}{cc}
 & n \\
l & \begin{bmatrix} .36 & .36 & .28 & 0 & 0 & 0 \\ 0 & 0 & 0 & .28 & .36 & .36 \end{bmatrix}
\end{array}
=
\begin{bmatrix} .054 & .054 & .042 & 0 & 0 & 0 \\ .054 & .054 & .042 & 0 & 0 & 0 \\ 0 & 0 & 0 & .042 & .054 & .054 \\ 0 & 0 & 0 & .042 & .054 & .054 \\ .036 & .036 & 028 & .028 & .036 & .036 \\ .036 & .036 & .028 & .028 & .036 & .036 \end{bmatrix}
$$

med. doc

cs doc

$$\begin{bmatrix} .05 & .05 & .05 & 0 & 0 & 0 \\ .05 & .05 & .05 & 0 & 0 & 0 \\ 0 & 0 & 0 & .05 & .05 & .05 \\ 0 & 0 & 0 & .05 & .05 & .05 \\ .04 & .04 & 0 & .04 & .04 & .04 \\ .04 & .04 & .04 & 0 & .04 & .04 \end{bmatrix}$$

med. terms

cs terms

common terms

term group x doc. group

$$\begin{bmatrix} .5 & 0 & 0 \\ .5 & 0 & 0 \\ 0 & .5 & 0 \\ 0 & .5 & 0 \\ 0 & 0 & .5 \\ 0 & 0 & .5 \end{bmatrix} \begin{bmatrix} .3 & 0 \\ 0 & .3 \\ .2 & .2 \end{bmatrix} \begin{bmatrix} .36 & .36 & .28 & 0 & 0 & 0 \\ 0 & 0 & 0 & .28 & .36 & .36 \end{bmatrix} = \begin{bmatrix} .054 & .054 & .042 & 0 & 0 & 0 \\ .054 & .054 & .042 & 0 & 0 & 0 \\ 0 & 0 & 0 & .042 & .054 & .054 \\ 0 & 0 & 0 & .042 & .054 & .054 \\ .036 & .036 & 028 & .028 & .036 & .036 \\ .036 & .036 & .028 & .028 & .036 & .036 \end{bmatrix}$$

doc x doc group

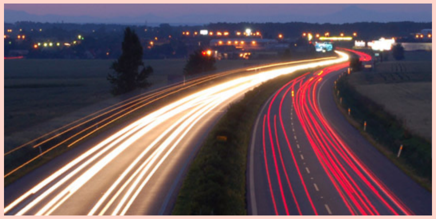
term x term-group

Co-clustering

Observations

- uses KL divergence, instead of L2
- the middle matrix is **not** diagonal
  - Like in the Tucker tensor decomposition
- s/w at:

www.cs.utexas.edu/users/dml/Software/cocluster.html



Detailed outline

- Motivation
- Hard clustering – k pieces
- Hard co-clustering – (k,l) pieces
- Hard clustering – optimal # pieces
- Soft clustering – matrix decompositions
- Observations

Problem with Information Theoretic Co-clustering

- Number of row and column groups must be specified

Desiderata:

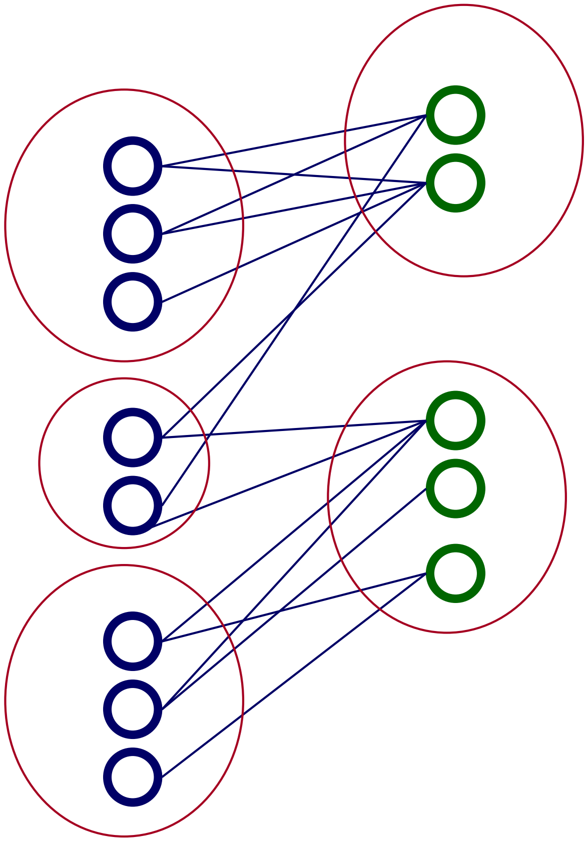
✓ Simultaneously discover row and column groups

✗ Fully Automatic: No "magic numbers"

✓ Scalable to large graphs

Graph partitioning

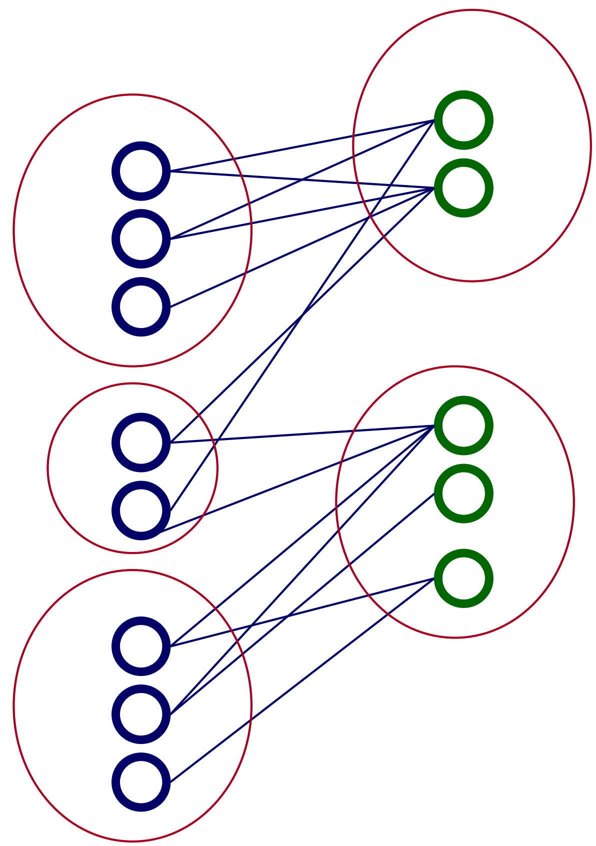
- Documents x terms
- Customers x products
- Users x web-sites



Graph partitioning

- Documents x terms
- Customers x products
- Users x web-sites

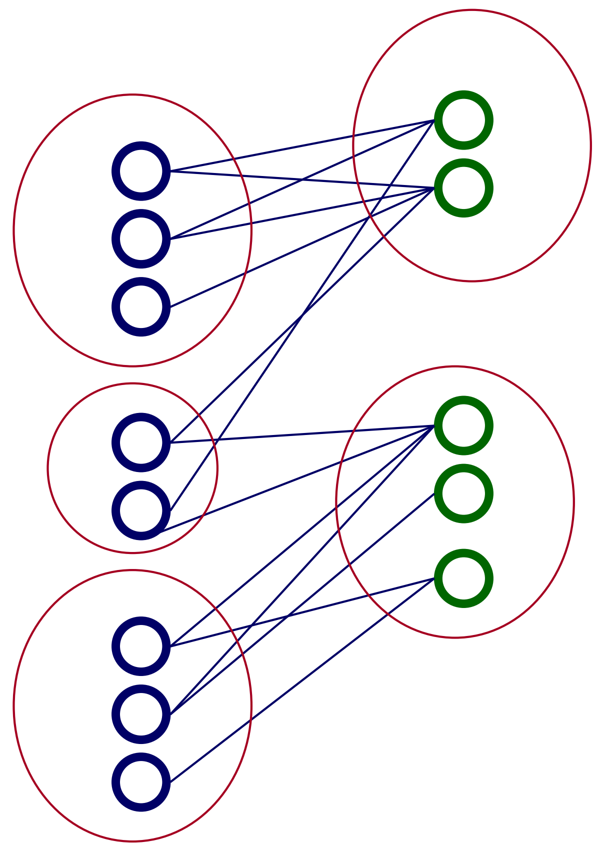
- Q: HOW MANY PIECES?



Graph partitioning

- Documents x terms
- Customers x products
- Users x web-sites

- Q: HOW MANY PIECES?
- A: MDL/ compression



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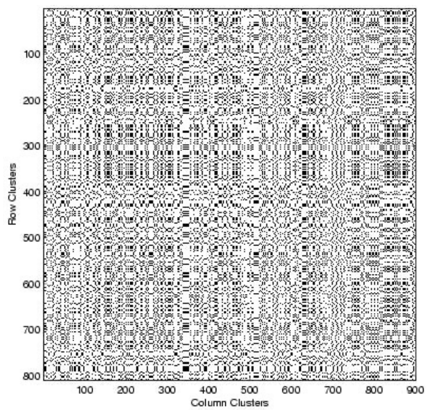
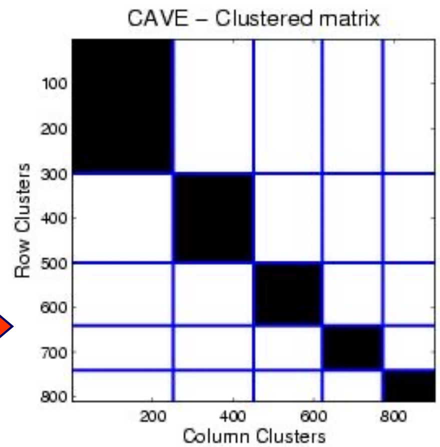
Cross-association





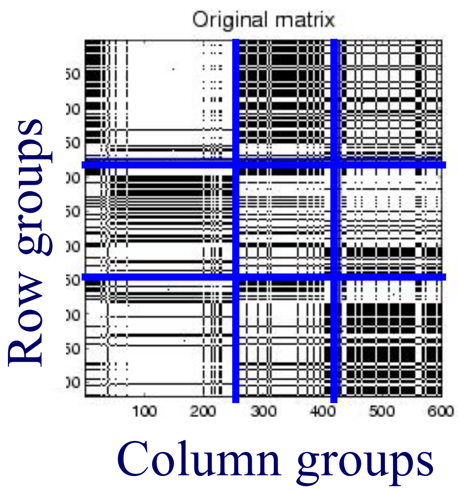
Desiderata:

- ✓ Simultaneously discover row and column groups
- ✓ Fully Automatic: No “magic numbers”
- ✓ Scalable to large matrices

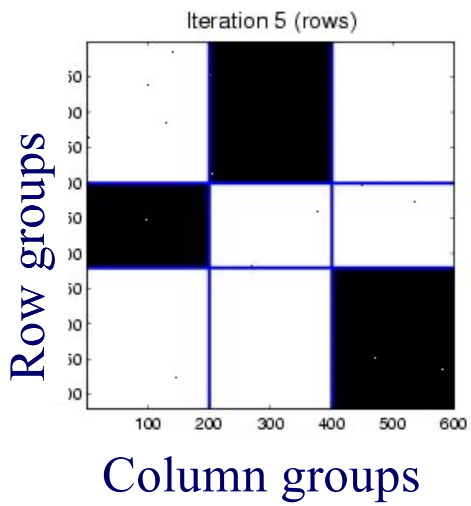
Reference:

1. Chakrabarti et al. Fully Automatic Cross-Associations, KDD’04

What makes a cross-association "good"?

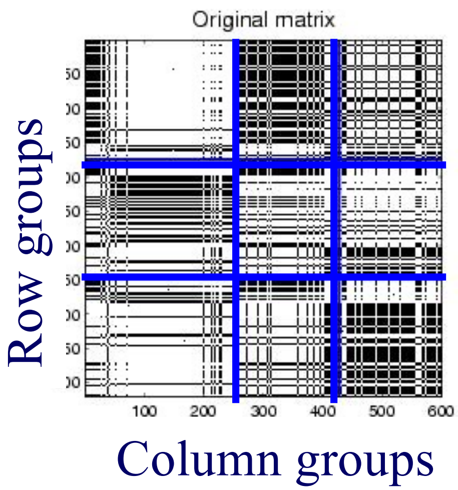


versus

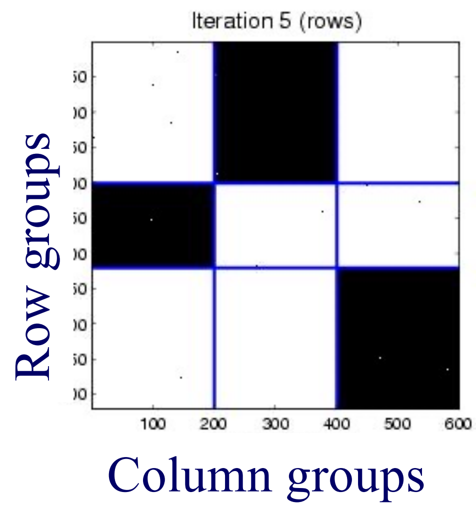


Why is this better?

What makes a cross-association "good"?



versus



Why is this better?

simpler; easier to describe

easier to compress!

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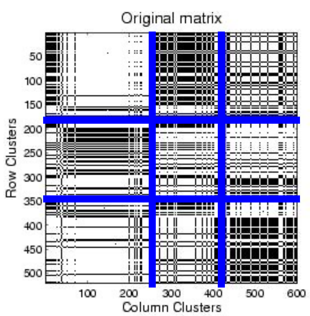
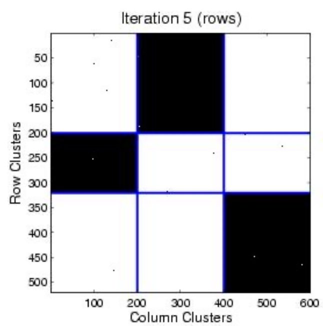
What makes a cross-association "good"?





Problem definition: given an encoding scheme
- decide on the # of col. and row groups *k* and *l*
- and reorder rows and columns,
- to achieve best compression

details

Main Idea

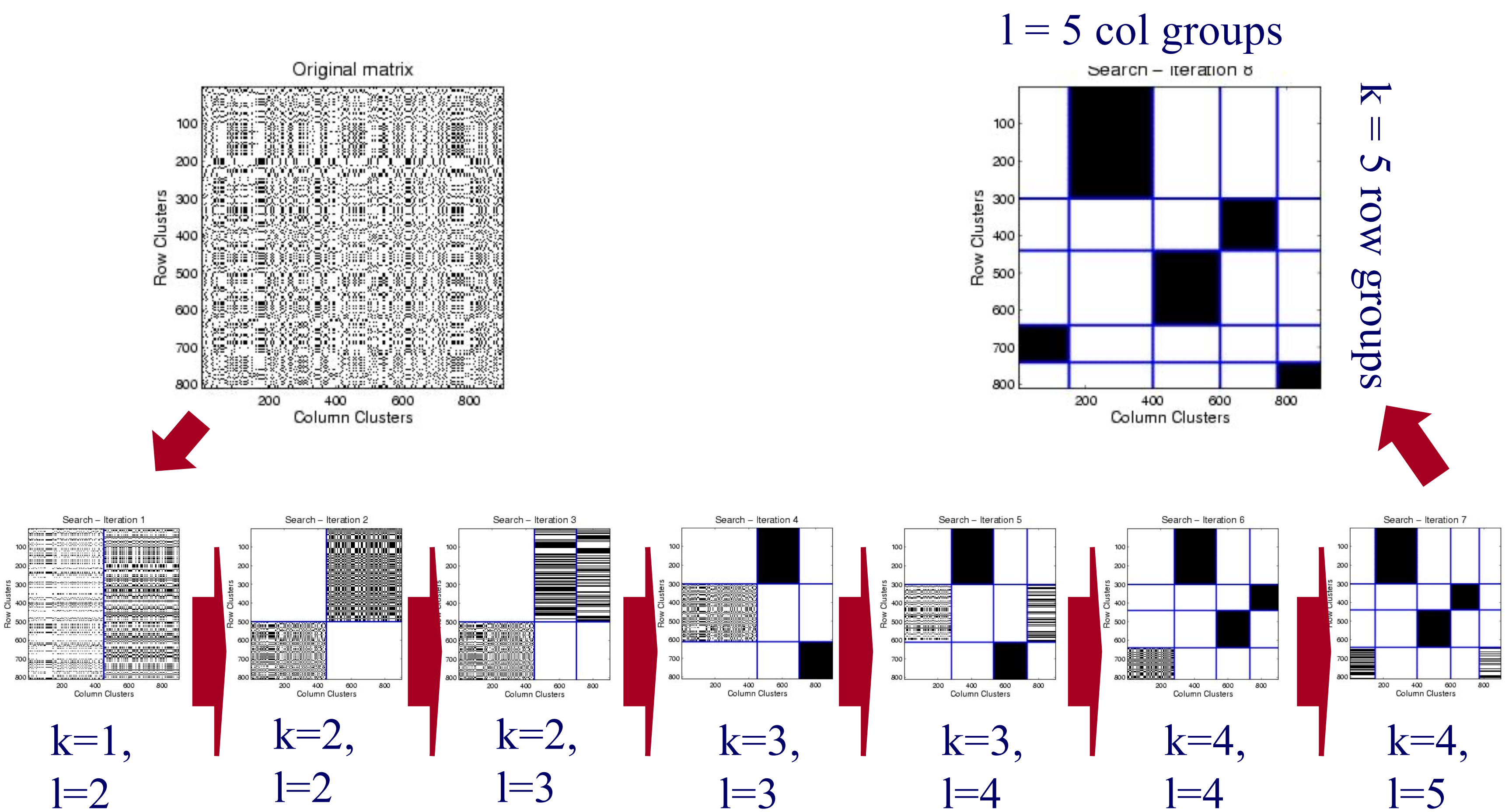
Good Compression → Better Clustering

Total Encoding Cost = $\underbrace{\Sigma_i \ \text{size}_i * H(x_i)}_{\text{Code Cost}}$ + $\underbrace{\text{Cost of describing cross-associations}}_{\text{Description Cost}}$

Minimize the total cost (# bits)

for lossless compression

Algorithm


l = 5 col groups
k = 5 row groups
Original matrix
Row Clusters
Column Clusters
Search – Iteration 8
Search – Iteration 1
Search – Iteration 2
Search – Iteration 3
Search – Iteration 4
Search – Iteration 5
Search – Iteration 6
Search – Iteration 7
k=1, l=2
k=2, l=2
k=2, l=3
k=3, l=3
k=3, l=4
k=4, l=4
k=4, l=5

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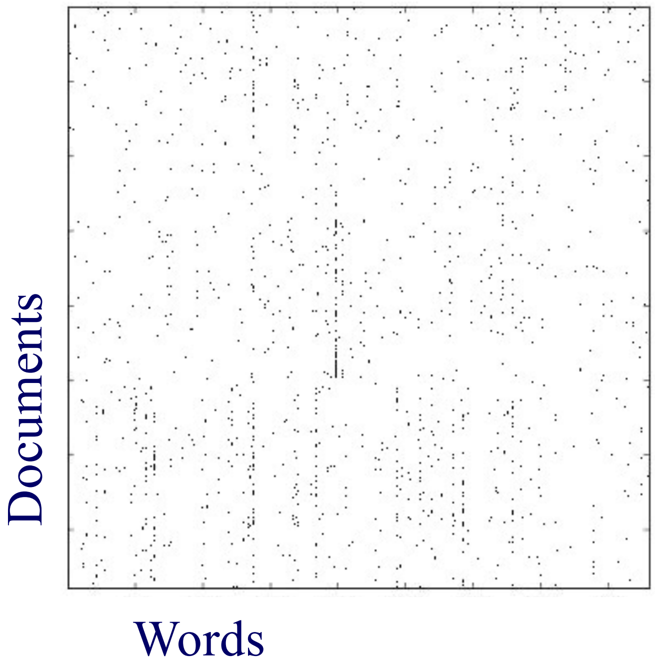
Experiments



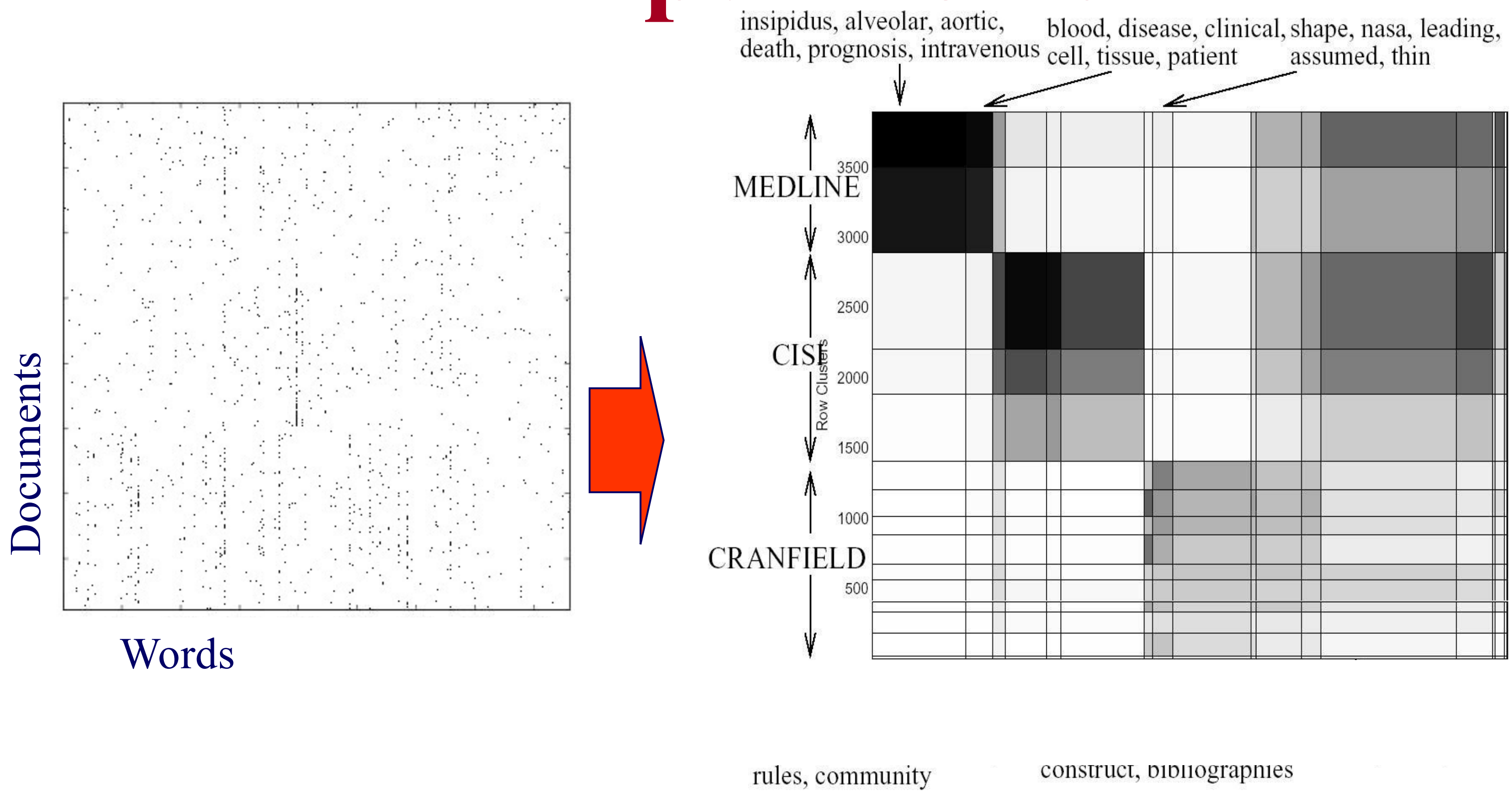
"CLASSIC"

- 3,893 documents
- 4,303 words
- 176,347 "dots"

Combination of 3 sources:

- MEDLINE (medical)
- CISI (info. retrieval)
- CRANFIELD (aerodynamics)

Experiments



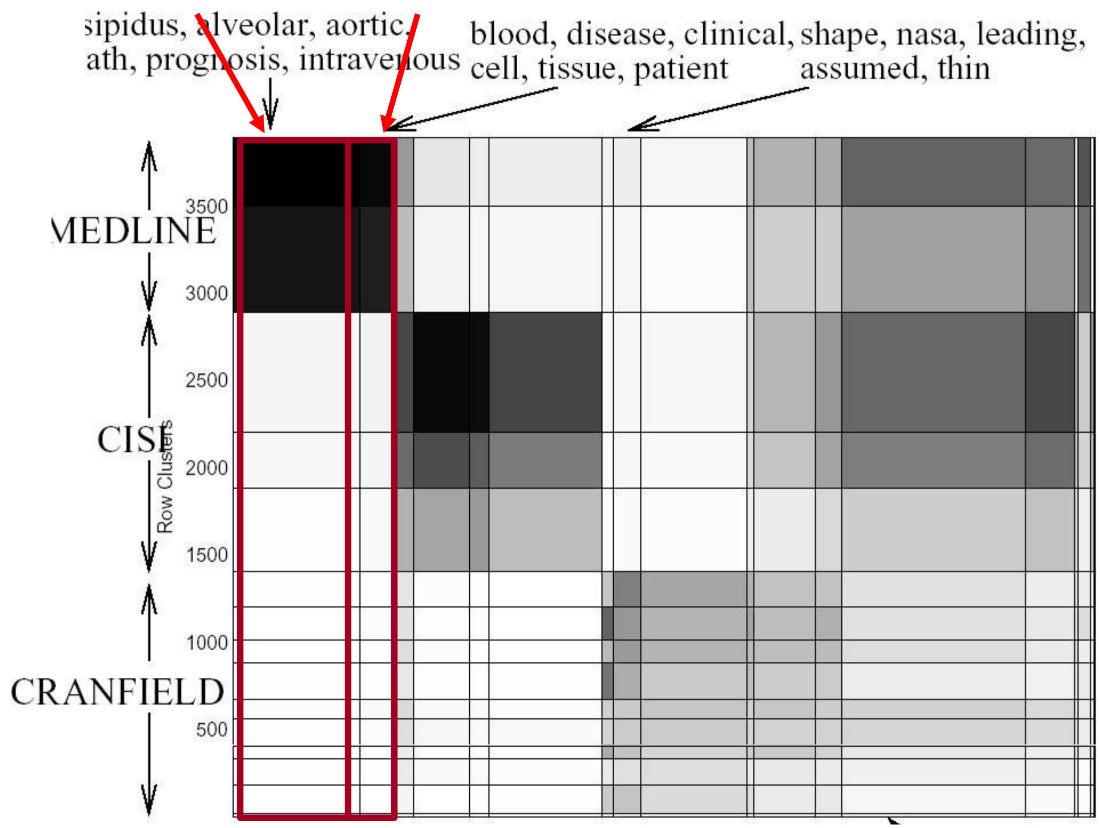
“CLASSIC” graph of documents & words:
k=15, l=19

Experiments

insipidus, alveolar, aortic, death, prognosis, intravenous

blood, disease, clinical, cell, tissue, patient

MEDLINE (medical)



"CLASSIC" graph of documents & words: k=15, l=19

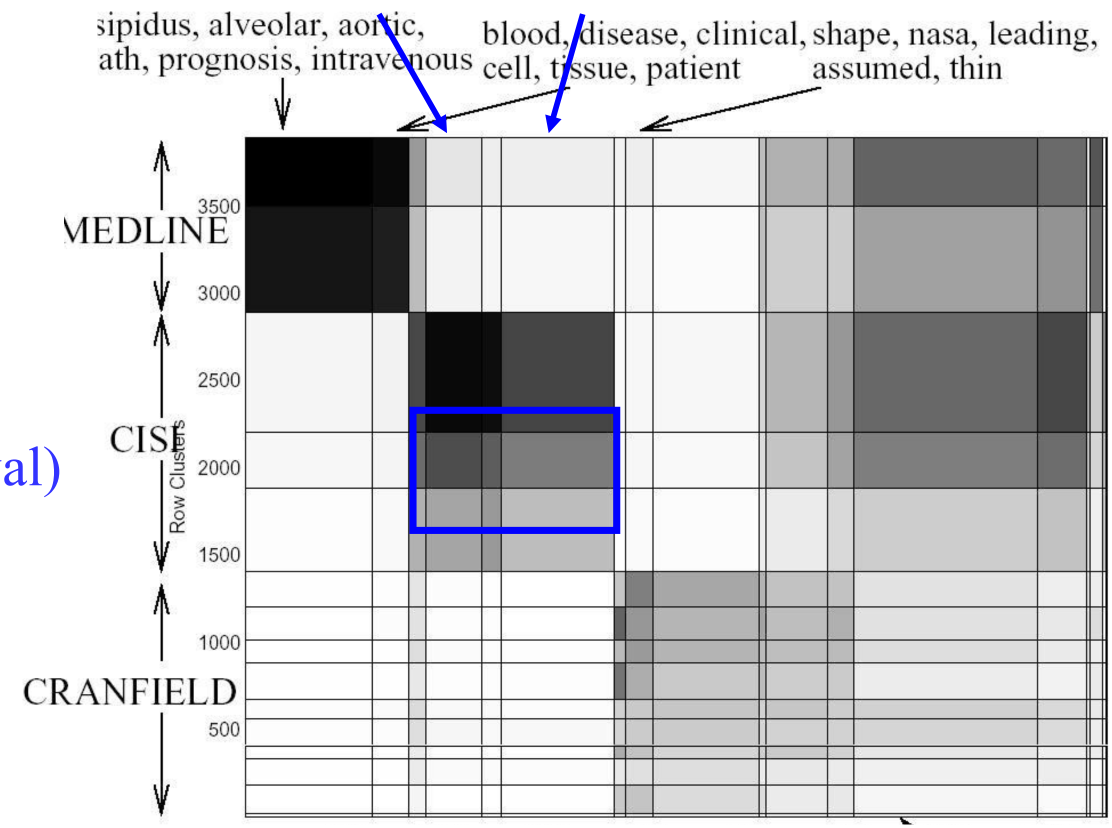
Experiments

providing, studying, records, development, students, rules

abstract, notation, works, construct, bibliographies

MEDLINE (medical)

CISI (Information Retrieval)



"CLASSIC" graph of documents & words: k=15, l=19

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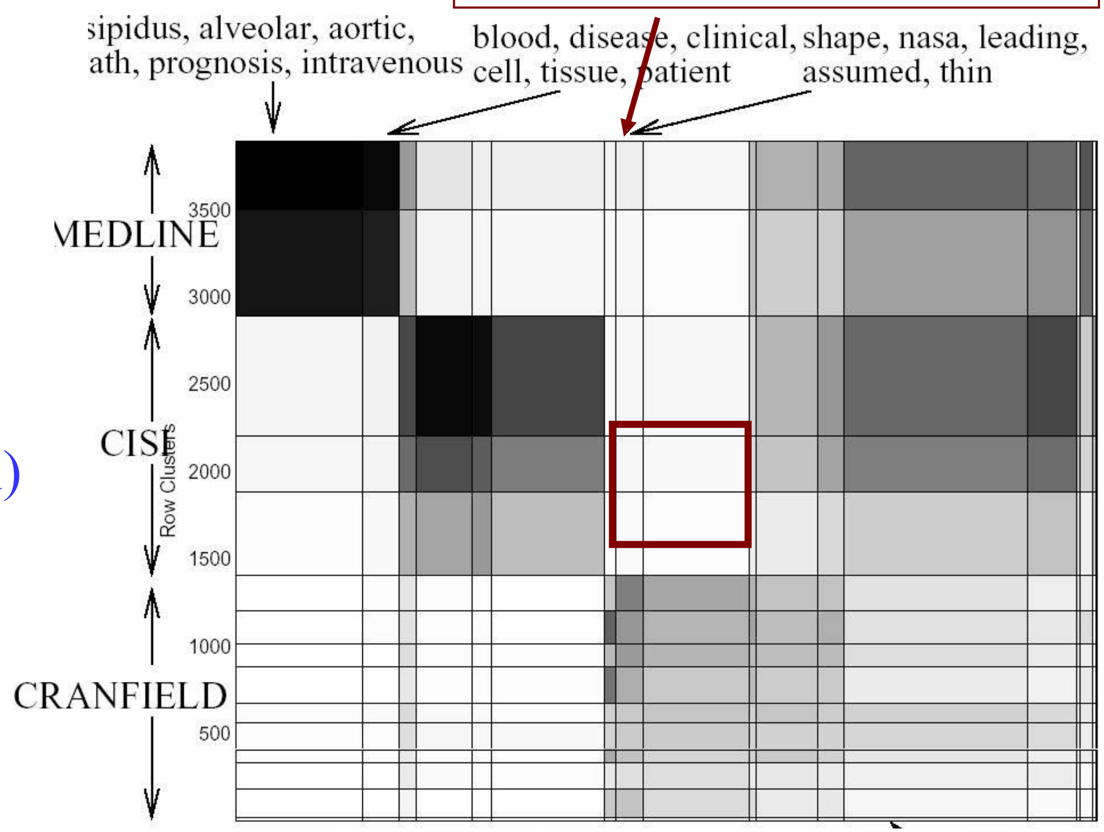
Experiments

shape, nasa, leading,
assumed, thin

MEDLINE
(medical)

CISI
(Information Retrieval)

CRANFIELD
(aerodynamics)



“CLASSIC” graph of documents & words:
k=15, l=19

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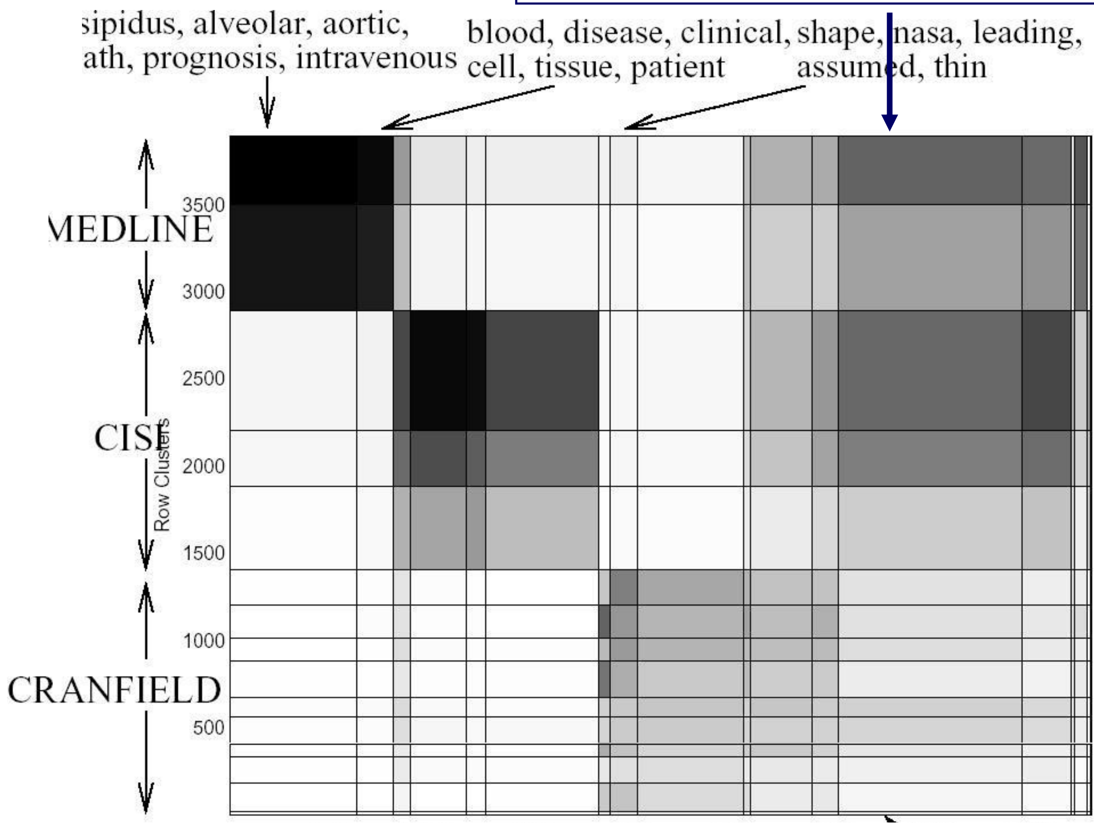
Experiments

paint, examination, fall, raise, leave, based

MEDLINE (medical)

CISI (Information Retrieval)

CRANFIELD (aerodynamics)



“CLASSIC” graph of documents & words: k=15, l=19

Algorithm

Code for cross-associations (matlab):

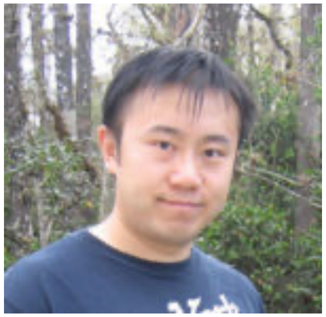
www.cs.cmu.edu/~deepay/mywww/software/CrossAssociations-01-27-2005.tgz

Variations and extensions:

- 'Autopart' [Chakrabarti, PKDD'04]
- www.cs.cmu.edu/~deepay

Algorithm

- Hadoop implementation [ICDM'08]



Spiros Papadimitriou, Jimeng Sun: DisCo: Distributed Co-clustering with Map-Reduce: A Case Study towards Petabyte-Scale End-to-End Mining. ICDM 2008: 512-521

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Detailed outline

- Motivation
- Hard clustering – *k* pieces
- Hard co-clustering – (*k,l*) pieces
- Hard clustering – optimal # pieces
- (Soft clustering – matrix decompositions
  - PCA, ICA, non-negative matrix factorization, …)
- Observations

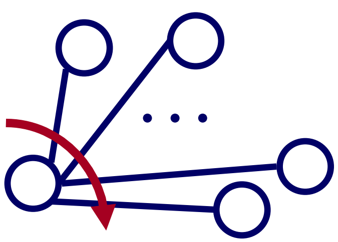
Detailed outline

- Motivation
- Hard clustering – *k* pieces
- Hard co-clustering – (*k*,*l*) pieces
- Hard clustering – optimal # pieces
- (Soft clustering)
- Observations

Observation #1

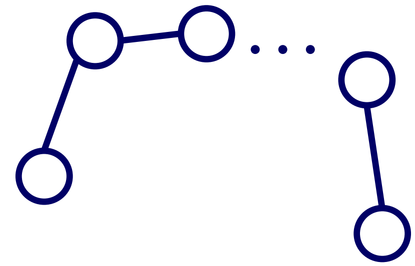
- Skewed degree distributions – there are nodes with huge degree (>O(10^4), in facebook/linkedIn popularity contests!)
- TRAP: '*find all pairs of nodes, within 2 steps from each other*'

Gaussian trap



1M

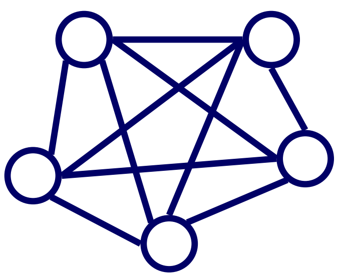
Observation #2

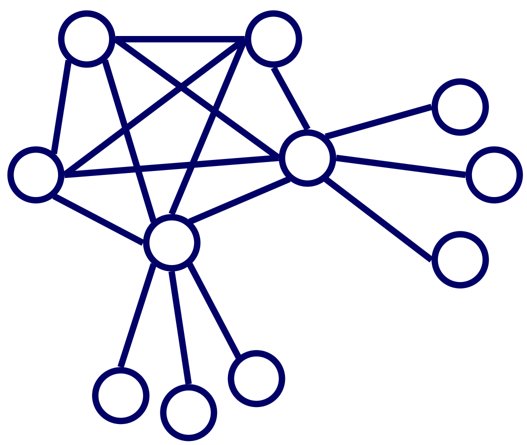


- TRAP: *shortest-path between two nodes*
- (cheat: look for 2, at most 3-step paths)
- Why:
  - If they are close (within 2-3 steps): solved
  - If not, after ~6 steps, you'll have ~ the whole graph, and the path won't be very meaningful, anyway.

Observation #3

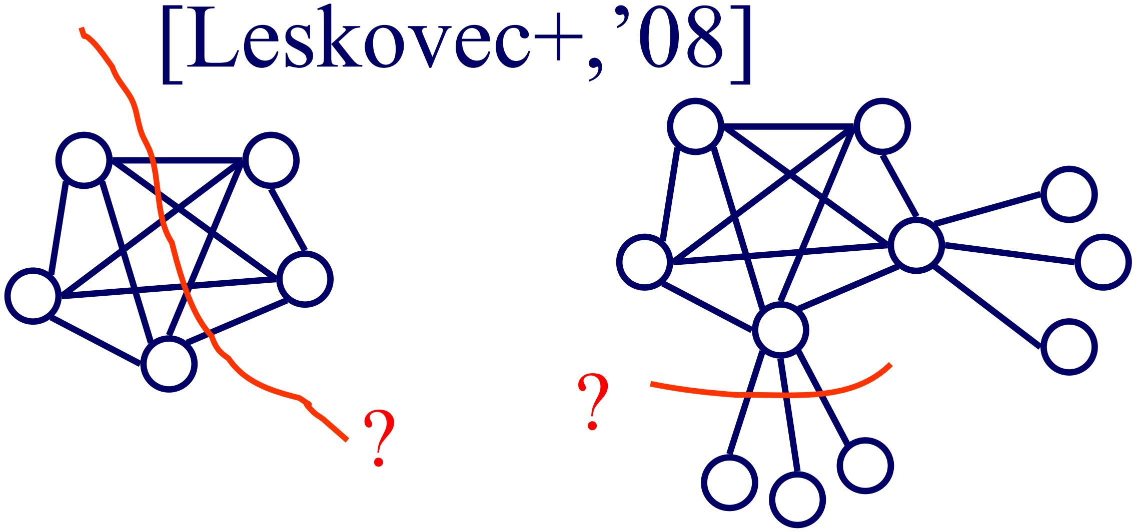
- Maybe there are no good cuts: ``jellyfish'' shape [Tauro+'01], [Siganos+,'06], strange behavior of cuts [Chakrabarti+'04], [Leskovec+,'08]



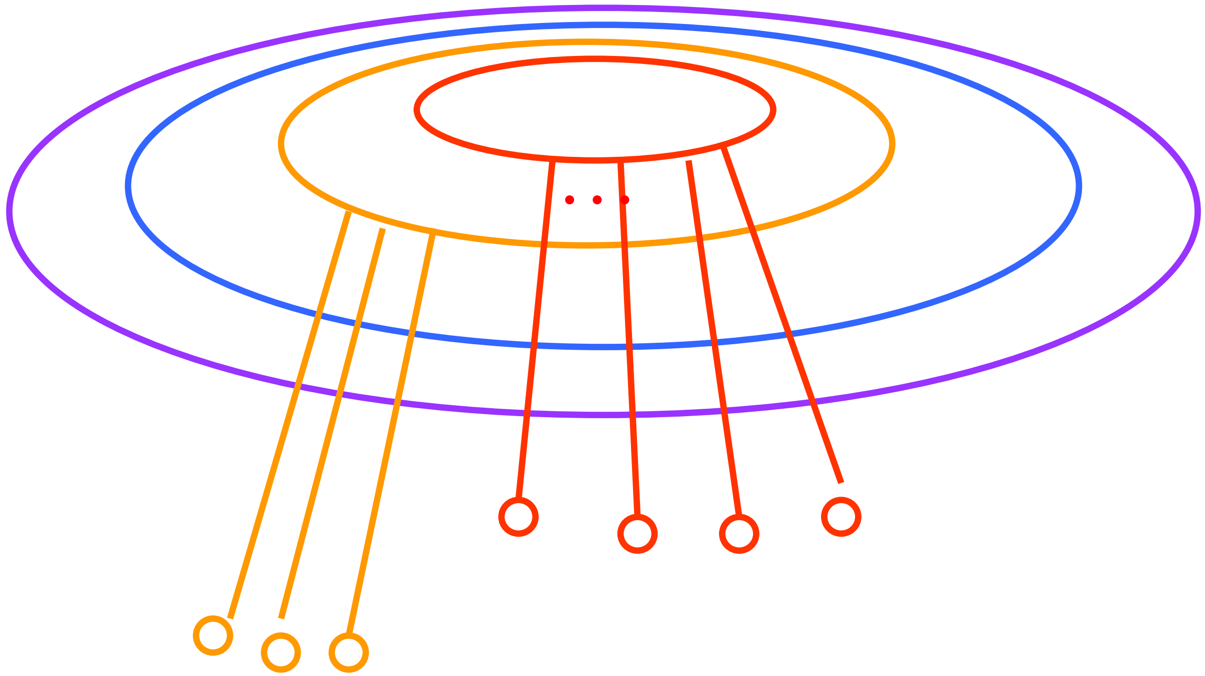


Observation #3

- Maybe there are no good cuts: ``jellyfish'' shape [Tauro+'01], [Siganos+,'06], strange behavior of cuts [Chakrabarti+,'04], [Leskovec+,'08]



Jellyfish model [Tauro+]



A Simple Conceptual Model for the Internet Topology, L. Tauro, C. Palmer, G. Siganos, M. Faloutsos, Global Internet, November 25-29, 2001

Jellyfish: A Conceptual Model for the AS Internet Topology G. Siganos, Sudhir L Tauro, M. Faloutsos, J. of Communications and Networks, Vol. 8, No. 3, pp 339-350, Sept. 2006.

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Strange behavior of min cuts

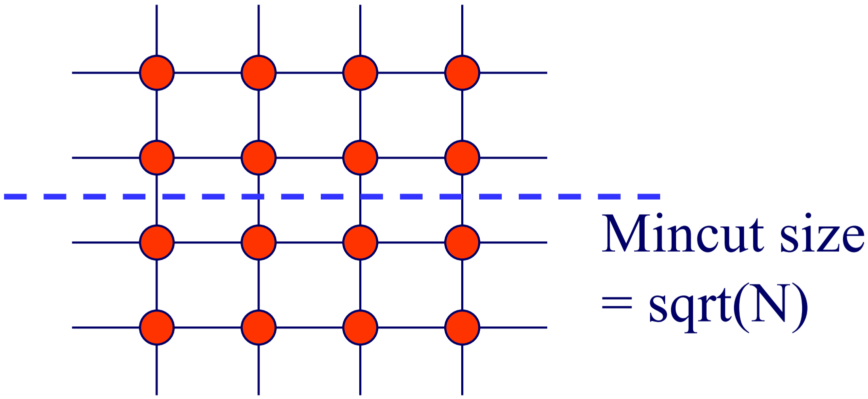
- ‘negative dimensionality’ (!)

NetMine: New Mining Tools for Large Graphs, by D. Chakrabarti, Y. Zhan, D. Blandford, C. Faloutsos and G. Blelloch, in the SDM 2004 Workshop on Link Analysis, Counter-terrorism and Privacy

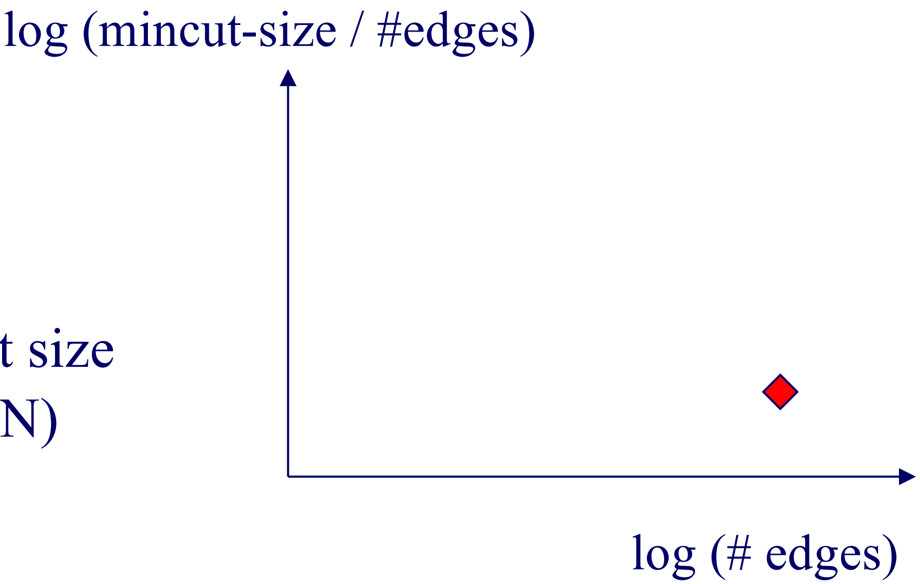
Statistical Properties of Community Structure in Large Social and Information Networks, J. Leskovec, K. Lang, A. Dasgupta, M. Mahoney. WWW 2008.

"Min-cut" plot

- Do min-cuts recursively.



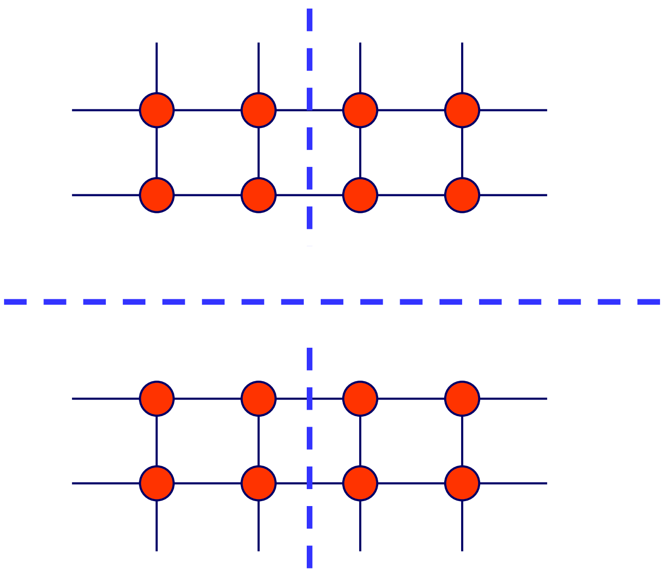
N nodes



"Min-cut" plot

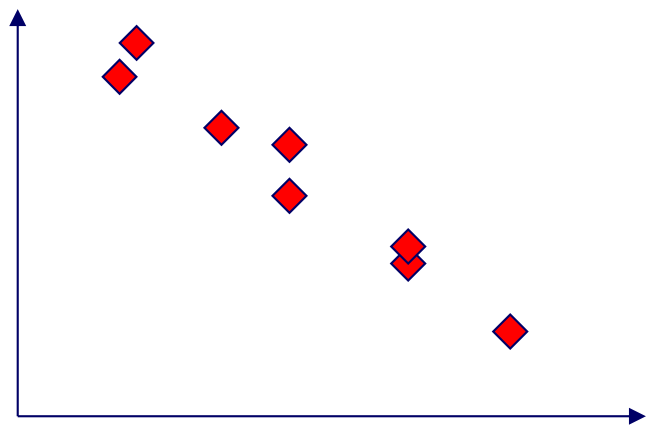
- Do min-cuts recursively.

New min-cut



N nodes

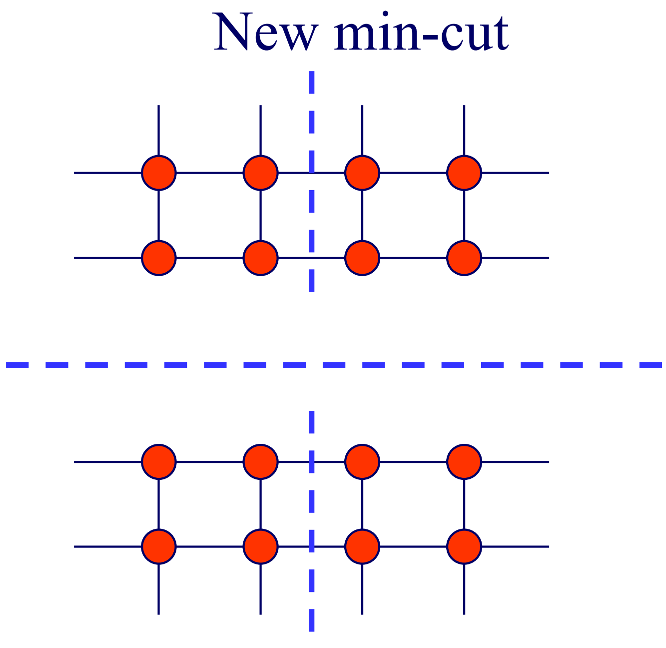
log (mincut-size / #edges)



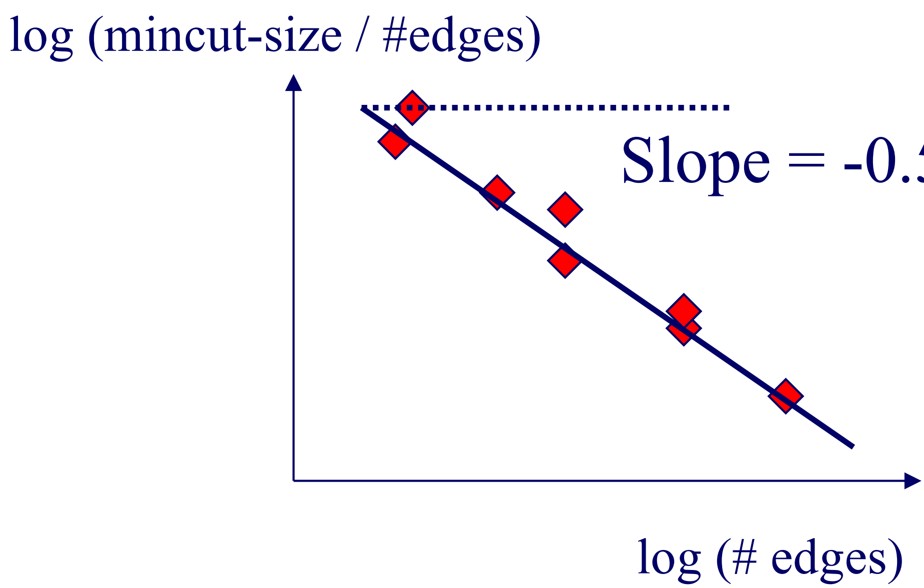
log (# edges)

“Min-cut” plot

- Do min-cuts recursively.



N nodes



For a d-dimensional grid, the slope is -1/d

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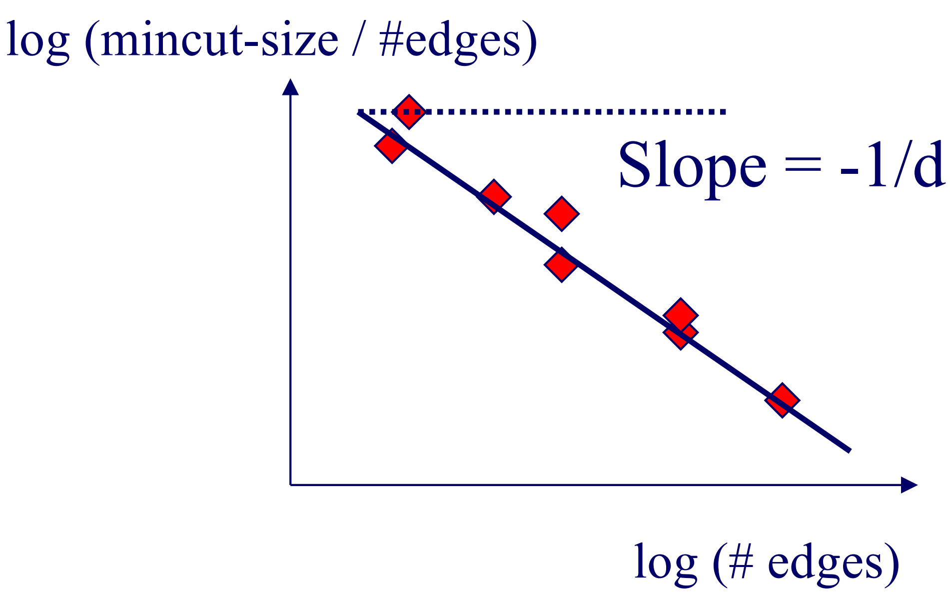
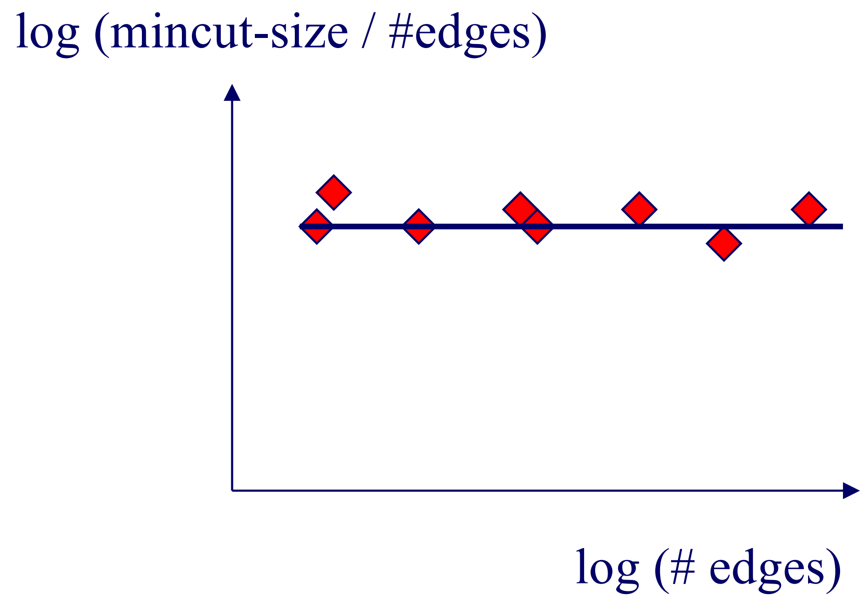
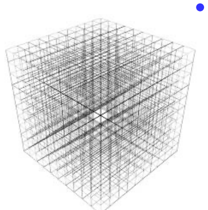
“Min-cut” plot





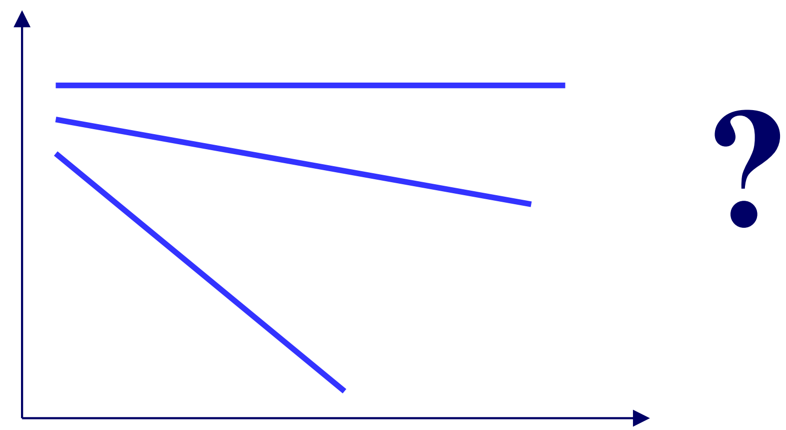
For a d-dimensional d, the slope is -1/d



For a random graph, the slope is 0

"Min-cut" plot

- What does it look like for a real-world graph?



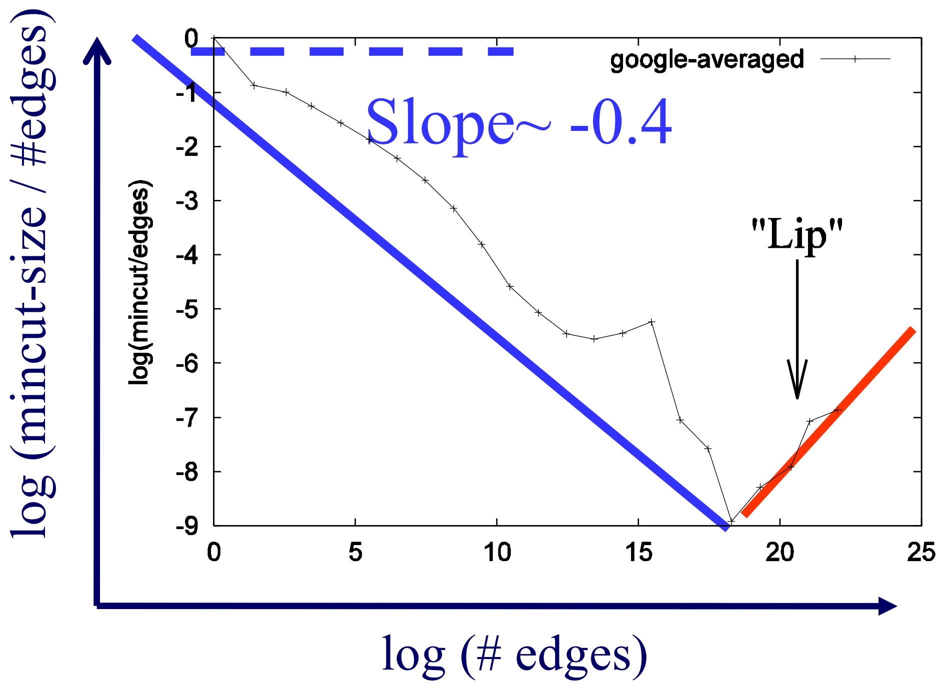
Experiments

- Datasets:
  - Google Web Graph: 916,428 nodes and 5,105,039 edges
  - Lucent Router Graph: Undirected graph of network routers from www.isi.edu/scan/mercator/maps.html; 112,969 nodes and 181,639 edges
  - User ➔ Website Clickstream Graph: 222,704 nodes and 952,580 edges

NetMine: New Mining Tools for Large Graphs, by D. Chakrabarti, Y. Zhan, D. Blandford, C. Faloutsos and G. Blelloch, in the SDM 2004 Workshop on Link Analysis, Counter-terrorism and Privacy

Experiments

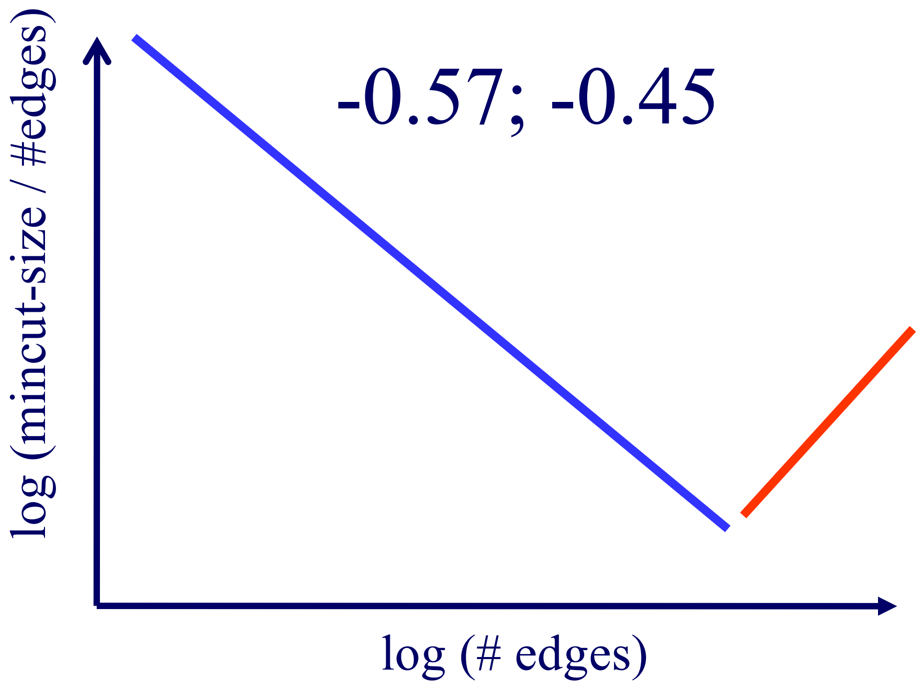
- Used the METIS algorithm [Karypis, Kumar, 1995]



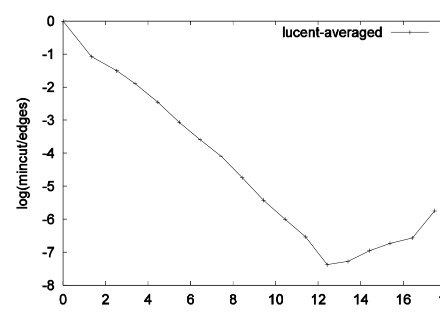
- Google Web graph
- Values along the y-axis are averaged
- We observe a "lip" for large edges
- Slope of -0.4, corresponds to a 2.5-dimensional grid!

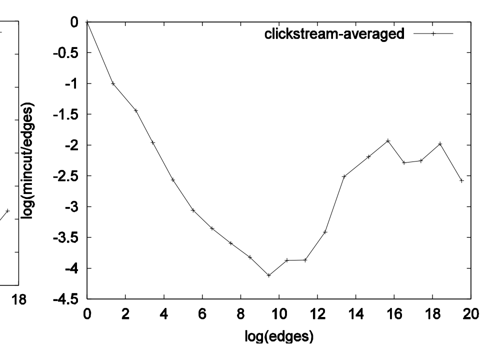
Experiments

- Used the METIS algorithm [Karypis, Kumar, 1995]



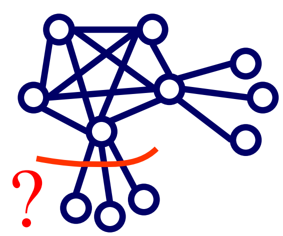
- Similarly, for
  - Lucent routers
  - clickstream



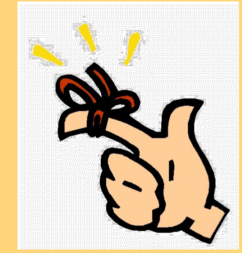


Conclusions – Practitioner's guide

- Hard clustering – *k* pieces **METIS**
- Hard co-clustering – (*k*,*l*) pieces **Co-clustering**
- Hard clustering – optimal # pieces **Cross-associations**
- Observations

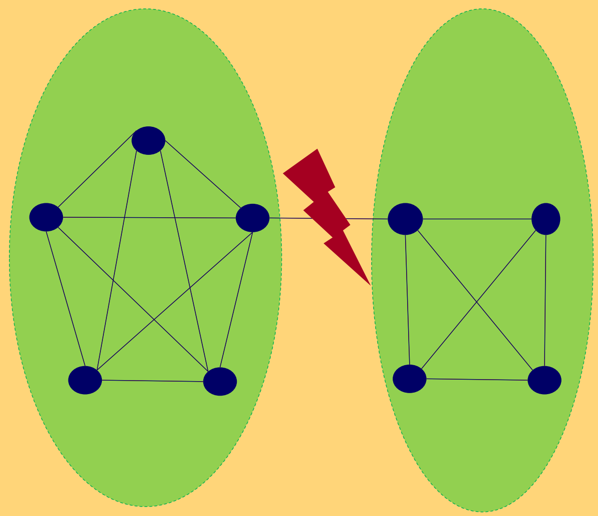


'jellyfish': Maybe, there are <u>no</u> good cuts



Short answer

- METIS [Karypis, Kumar]



But: maybe there are NO good cuts!