

15-312 Foundations of Programming Languages

Midterm Examination

February 26, 2004

Name: _____

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- This is a closed-book exam; only one double-sided sheet of notes is permitted.
- Write your answer legibly in the space provided.
- There are 12 pages in this exam, including 3 worksheets.
- It consists of 3 questions worth a total of 100 points.
- Read through the entire exam before answering any problems.
- You have 85 minutes for this exam.

Problem 1	Problem 2	Problem 3	Total
15	45	40	100

1. Substitution and Alpha-Conversion (15 pts)

1. (8 pts) Suppose f has the type $\forall\alpha.\forall\beta.\beta \rightarrow \alpha$. What is the type of $f[\beta \rightarrow \gamma]$?

2. (7 pts) Complete the typing derivation below:

$$x:\text{int} \vdash (\lambda x:\text{bool}.x) :$$

2. Type Safety (45 pts)

Suppose we add support for lists to MinML by adding the following syntax, typing rules, and operational semantics:

Syntax

$$\begin{aligned} \tau &::= \dots \mid \tau \text{ list} \\ e &::= \dots \mid \text{nil} \mid e_1 :: e_2 \mid \text{isnil } e \mid \text{hd } e \mid \text{tl } e \\ v &::= \dots \mid \text{nil} \mid v_1 :: v_2 \end{aligned}$$

Typing Rules

$$\frac{}{\Gamma \vdash \text{nil} : \tau \text{ list}} \quad \frac{\Gamma \vdash e_1 : \tau \quad \Gamma \vdash e_2 : \tau \text{ list}}{\Gamma \vdash (e_1 :: e_2) : \tau \text{ list}}$$

$$\frac{\Gamma \vdash e : \tau \text{ list}}{\Gamma \vdash \text{isnil } e : \text{bool}} \quad \frac{\Gamma \vdash e : \tau \text{ list}}{\Gamma \vdash \text{hd } e : \tau} \quad \frac{\Gamma \vdash e : \tau \text{ list}}{\Gamma \vdash \text{tl } e : \tau \text{ list}}$$

Operational Semantics

$$\frac{e_1 \mapsto e'_1}{e_1 :: e_2 \mapsto e'_1 :: e_2} \quad \frac{e \mapsto e'}{v :: e \mapsto v :: e'}$$

$$\frac{e \mapsto e'}{\text{hd } e \mapsto \text{hd } e'} \quad \frac{}{\text{hd}(v_1 :: v_2) \mapsto v_1} \quad \frac{e \mapsto e'}{\text{tl } e \mapsto \text{tl } e'} \quad \frac{}{\text{tl}(v_1 :: v_2) \mapsto v_2}$$

- (7 pts) The operational semantics is incomplete. Give appropriate evaluation rules for `isnil`.

2. (4 pts) State the progress lemma.

3. (15 pts) Give a counterexample to the progress lemma.

4. (4 pts) State the preservation lemma.

5. (15 pts) Show the cases for the proof of the preservation lemma pertaining to τ_1 and isnil . If you use any of the main lemmas (*i.e.*, inversion, canonical forms, or substitution) you must state the lemma, but you need not prove it.

(Extra space.)

3. Continuations and Polymorphism (40 pts)

In class we developed an abstract machine called the “C-machine” for MinML. Below you will extend the abstract machine to PolyMinML. That is, you will extend the machine to support the constructs for polymorphism. For your convenience, the relevant syntax is:

$$\begin{aligned} e &::= \dots \mid \Lambda\alpha.e \mid e[\tau] \\ v &::= \dots \mid \Lambda\alpha.e \end{aligned}$$

(*Hint*: I recommend thinking about both part 1 and 2 before answering either.)

1. (7 pts) Extend the syntax of stack frames with any additional frames necessary to support polymorphism.

$$f ::= \dots \mid$$

2. (10 pts) Give any additional operational rules necessary to support polymorphism.

3. (11 pts) Now consider PolyMinML plus first-class continuations. Give a term P with the type $\forall\alpha.\forall\beta.((\alpha \rightarrow \beta) \rightarrow \alpha) \rightarrow \alpha$. The start of the term is given for you below. Do **not** solve the problem trivially by using an infinite loop or failure. (*Hint*: You will need a `letcc` and a `throw`.)

$$P = \Lambda\alpha.\Lambda\beta.\lambda f:((\alpha \rightarrow \beta) \rightarrow \alpha).$$

4. (12 pts) Using your extended C-machine, evaluate the following program with the empty stack:

$$P[\text{int}][\text{bool}](\lambda g:(\text{int} \rightarrow \text{bool}). \text{if } g \text{ 3 then 5 else 7})$$

(*Suggestion*: The evaluation is somewhat long; we recommend that you make definitions, as appropriate, to cut down on writing. If your solution to part 3 is incomplete, perform the evaluation as far as possible given what you have.)

(Extra space.)

Worksheet

Worksheet

Worksheet