Constructive Logic (15-317), Fall 2024
Assignment 2: Harmony

Constructive Logic Staff
(Instructor: Karl Crary)

Due: Wednesday, September 11, 2024, 11:59 pm

This assignment will have a written portion and a coding portion. You will submit both portions through Gradescope, to the assignments labelled "Homework 2 (written)" and "Homework 2 (code)." Please submit a file named "`hw.pdf`" to the former, and a file named "`hw.deriv`" to the latter.

We recommend that you typeset your written solutions. Most students use LaTeX, but other software is acceptable. (Please put each task on its own page to speed up grading.) If you choose not to typeset your solutions, be aware that you are answerable for your handwriting. Any that the grader has difficulty reading (in the sole judgement of the grader), will be marked wrong.

For the coding portion you will use Dcheck. You can find documentation on Dcheck at `cs.cmu.edu/~crary/dcheck/dcheck.pdf` and a sample file at `cs.cmu.edu/~crary/dcheck/example.deriv`. (Be aware that the sample file uses several logics that we have not seen yet in class.)

1 More natural deduction

Using Dcheck, give derivations of each of following judgements (naming them `task1`, `task2`, `task3`, and `task4`):

Task 1 (3 points).

$$A \wedge \neg A \supset B \ \mathsf{true}$$

Task 2 (3 points).

$$(A \supset B) \supset (A \supset B \supset C) \supset (A \supset C) \ \mathsf{true}$$

Task 3 (3 points).

$$\neg(A \vee B) \supset (\neg A \wedge \neg B) \ \mathsf{true}$$

Task 4 (3 points).

$$(\neg A \wedge \neg B) \supset \neg(A \vee B) \ \mathsf{true}$$

2 Contexts

Task 5 (4 points). The following proof uses floating hypotheses. In Dcheck, write the exact same proof using context notation. **Do not simplify the proof, and do not drop or reorder any hypotheses.** Name your derivation `task5`.

$$
\dfrac{\dfrac{\dfrac{\dfrac{[(A \supset A) \wedge (A \supset A)\ \mathsf{true}]_u}{A \supset A\ \mathsf{true}}\,\wedge E2 \qquad \dfrac{\dfrac{[(A \supset A) \wedge (A \supset A)\ \mathsf{true}]_u}{A \supset A\ \mathsf{true}}\,\wedge E1 \qquad [A\ \mathsf{true}]_w}{A\ \mathsf{true}}\,\supset E}{A\ \mathsf{true}}\,\supset E}{(A \supset A)\ \mathsf{true}}\,\supset I^w}{((A \supset A) \wedge (A \supset A)) \supset (A \supset A)\ \mathsf{true}}\,\supset I^u
$$

3 Harmony

Solve the following problems in your written submission.

Task 6 (10 points). Consider a new connective[1] $\odot$ with the following elimination rule:

$$
\dfrac{A \odot B\ \mathsf{true} \qquad \begin{array}{c}[A\ \mathsf{true}]_u \quad [B\ \mathsf{true}]_v \\ \vdots \\ C\ \mathsf{true}\end{array}}{C\ \mathsf{true}}\,\odot E^{u,v}
$$

(Normally we take the perspective that introduction rules come first to define a connective, but this time we'll go in the opposite direction.)

a. Come up with a correct set of (zero or more) introduction rules for this connective.

b. Show that the connective is locally sound for your choice of introduction rules.

c. Show that the connective is locally complete for your choice of introduction rules.

Task 7 (10 points). Consider a new connective[2] $\ltimes$ with the following introduction and elimination rules:

$$
\dfrac{A\ \mathsf{true} \qquad \begin{array}{c}[A\ \mathsf{true}]_u \\ \vdots \\ B\ \mathsf{true}\end{array}}{A \ltimes B\ \mathsf{true}}\,\ltimes I^u \qquad \dfrac{A \ltimes B\ \mathsf{true}}{B\ \mathsf{true}}\,\ltimes E
$$

a. Is this connective locally sound? If so, provide the local reduction. If not, repair the connective by giving replacement elimination rule(s). (The repaired connective should be both locally sound and locally complete, but you do not need to prove it.)

b. Is this connective locally complete? If so, provide the local expansion. If not, repair the connective by giving replacement elimination rule(s). (The repaired connective should be both locally sound and locally complete, but you do not need to prove it.)

[1]in Latex: `\odot`

[2]in Latex: `\ltimes`