

Constructive Logic (15-317), Fall 2024

Assignment 4: Quantifiers and Arithmetic

Constructive Logic Staff
(Instructor: Karl Crary)

Due: Wednesday, September 25, 2024, 11:59 pm

This assignment is coding only, using Dcheck. Please submit a file named “hw.deriv” to “Homework 4.”

You can find documentation on Dcheck at cs.cmu.edu/~crary/dcheck/dcheck.pdf and a sample file at cs.cmu.edu/~crary/dcheck/example.deriv. (Be aware that the sample file uses several logics that we have not seen yet in class.)

1 Quantifiers

Using Dcheck, give derivations of the following judgements, if they are derivable. For the ones that are not derivable, simply put:

`deriv <name> = omitted`

Use system “AR” (even though you are not using arithmetic in this section), and name your derivations `task1`, `task2`, etc. Note: to avoid revealing the answer, the autograder will not provide feedback on any problem for which you answer `omitted`.

Distributivity

Task 1 (6 points). $(\exists x:\tau. A(x) \vee B(x)) \supset (\exists x:\tau. A(x)) \vee (\exists x:\tau. B(x))$ true

Task 2 (6 points). $(\exists x:\tau. A(x)) \vee (\exists x:\tau. B(x)) \supset (\exists x:\tau. A(x) \vee B(x))$ true

DeMorgan

These judgements are true in classical logic, but may or may not be true constructively.

Task 3 (4 points). $\neg(\forall x:\tau. A(x)) \supset \exists x:\tau. \neg A(x)$ true

Task 4 (4 points). $\neg(\exists x:\tau. A(x)) \supset \forall x:\tau. \neg A(x)$ true

Forall/Exists

Task 5 (3 points). $(\forall x:\tau. A(x)) \supset (\exists x:\tau. A(x))$ true

Task 6 (5 points). $(\forall x:\tau. A(x)) \wedge (\exists x:\tau. \top) \supset (\exists x:\tau. A(x))$ true

2 Arithmetic

Task 7 (12 points). $\forall x:\text{nat}.\forall y:\text{nat}.\forall z:\text{nat}.x = y \supset y = z \supset x = z$ true

Use system “AR” and name your derivation `task7`. (Caution: Many students find this proof challenging, so we recommend you start early.)

3 Context Mastery

Task 8 (4 points). Using Dcheck, give a derivation of the following judgement using contexts (*i.e.*, using the NDC system):

$$\vdash \neg A \vee \neg B \supset \neg(A \wedge B) \text{ true}$$

Name your derivation `task8`. **Instant feedback is turned off for this task, so be extra careful.** (Keep in mind that hypotheses are numbered from **right-to-left**, with the rightmost numbered **zero**.)

4 Verifications and Uses Mastery

Task 9 (4 points). Using Dcheck, give a derivation of the following judgement:

$$\neg P \vee \neg Q \supset \neg(P \wedge Q) \uparrow$$

Name your derivation `task9`. (Remember that Dcheck takes the propositions P and Q to be atomic.) **Instant feedback is turned off for this task, so be extra careful.**