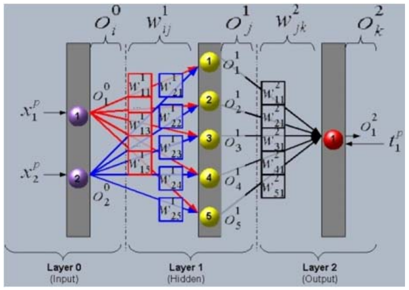
Machine Learning

10-701, Fall 2015

Perceptron and Artificial Neural Networks

Eric Xing



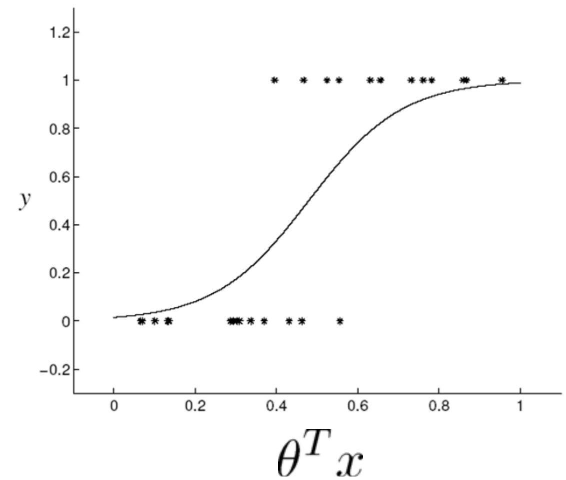
Lecture 7, October 1, 2015

Reading: Chap. 5 CB

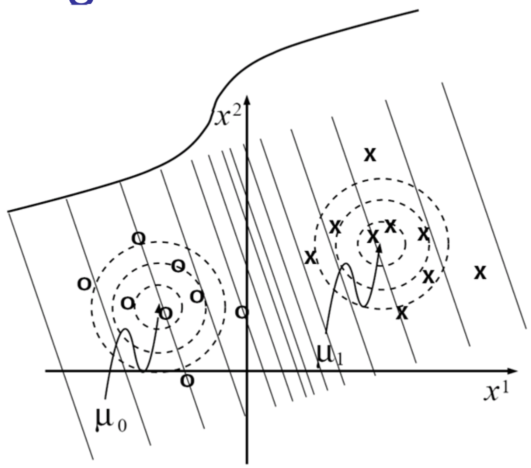
Recall Logistic Regression
(sigmoid classifier, MaxEnt classifier, …)

- The prediction rule:

$$p(y=1|x_n) = \frac{1}{1+\exp\left\{-\sum_{i=1}^{M}\theta_i x_i - \theta_0\right\}} = \frac{1}{1+e^{-\theta^T x}}$$



- In this case, learning p(y|x) amounts to learning ...?
  - Algorithm: gradient ascent



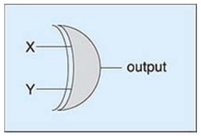
- What is the limitation?

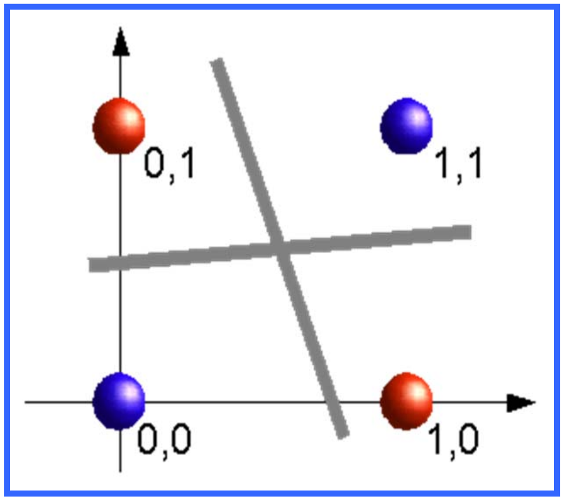
Learning highly non-linear functions

f: X → Y

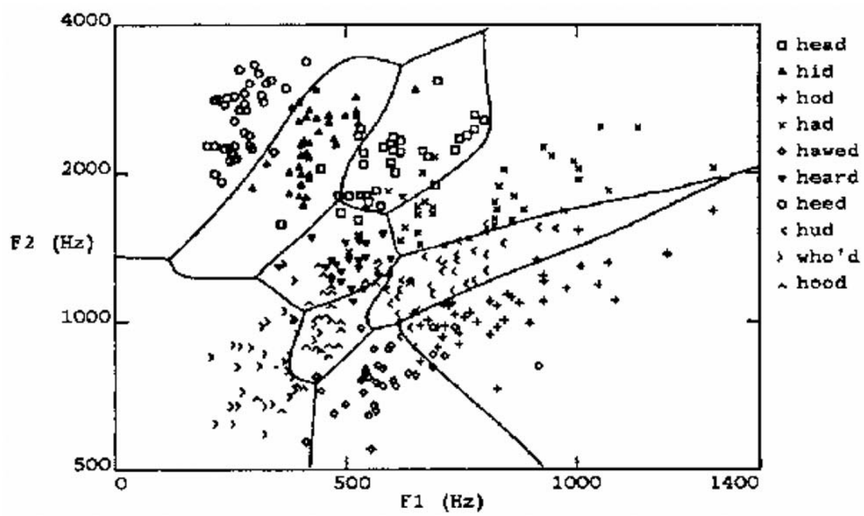
- f might be non-linear function
- X (vector of) continuous and/or discrete vars
- Y (vector of) continuous and/or discrete vars

The XOR gate

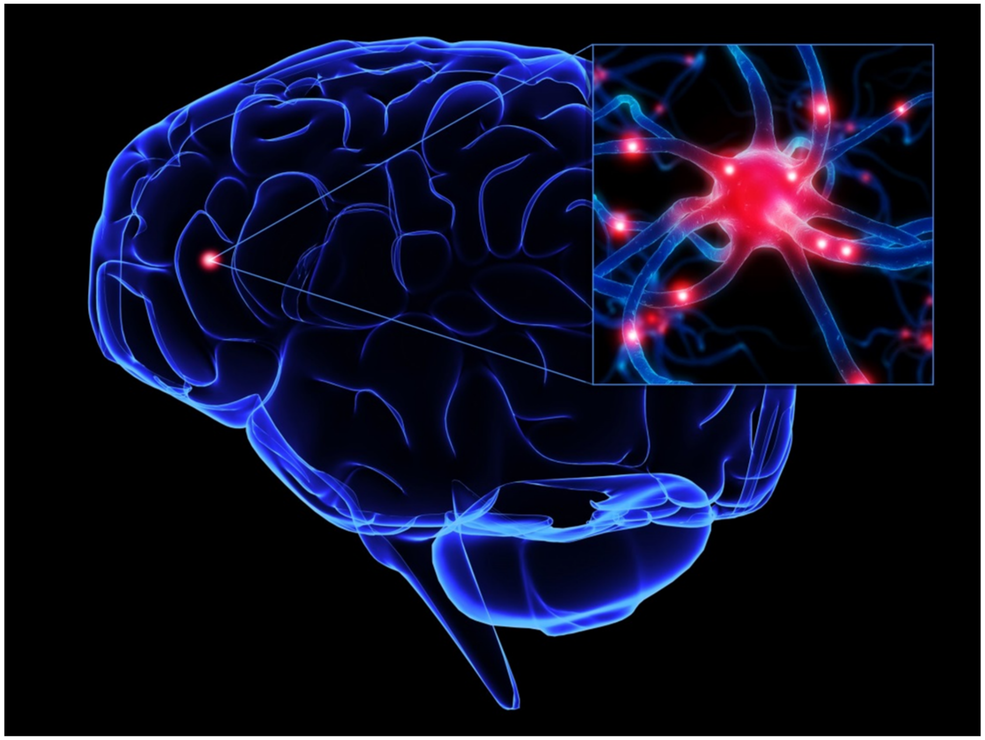




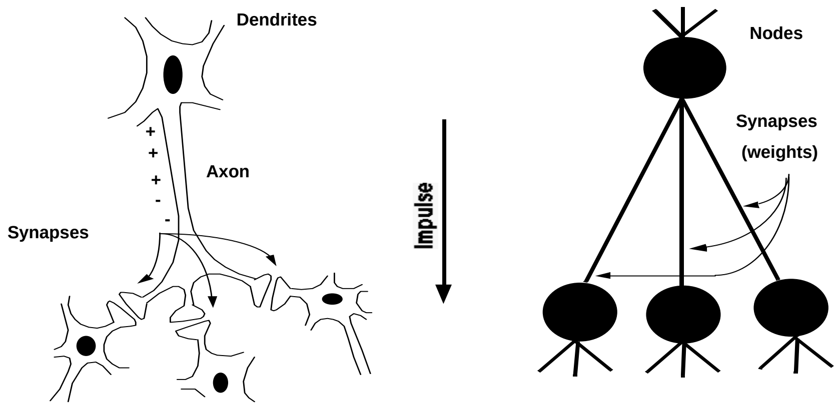
Speech recognition



Our brain is very good at this …

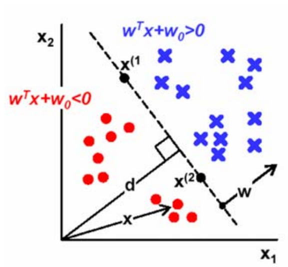


How a neuron works



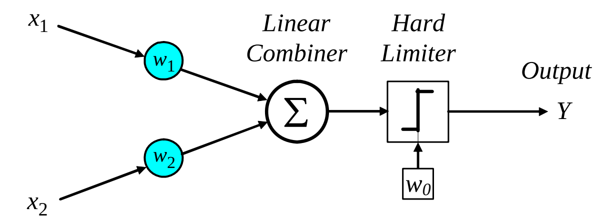
- Activation function:

$$X = \sum_{i=1}^{M} x_i w_i \qquad Y = \begin{cases} +1, & \text{if } X \geq \omega_0 \\ -1, & \text{if } X < \omega_0 \end{cases}$$



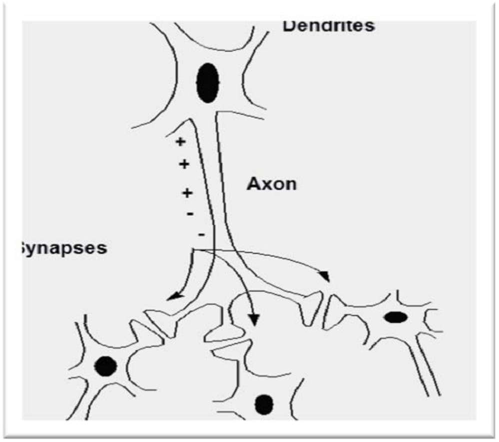
- An mathematical expression

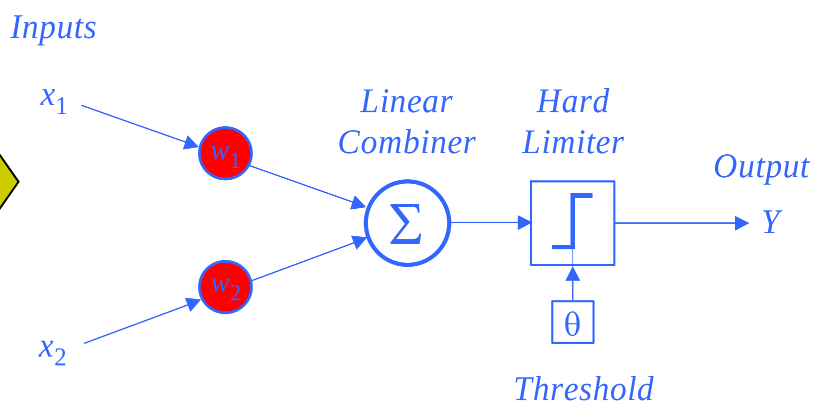
$$p(y=1 \mid x) = \frac{1}{1+\exp\left\{-\sum_{i=1}^{M} w_i x_i - \theta_0\right\}} = \frac{1}{1+e^{-w^T x}}$$



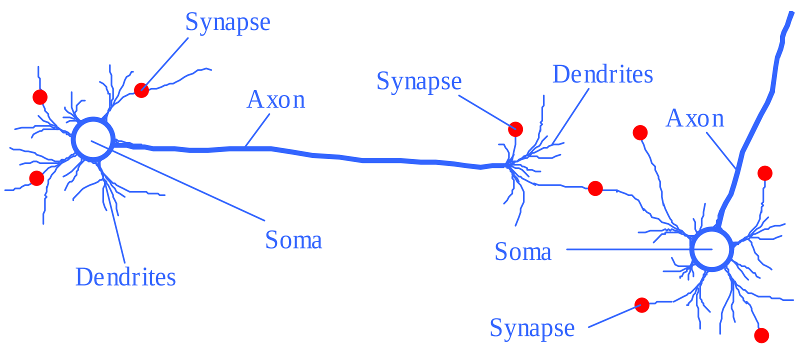
Perceptron and Neural Nets

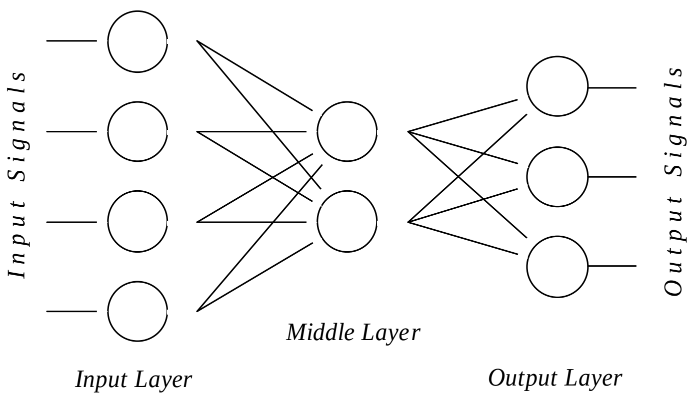
- From biological neuron to artificial neuron (perceptron)





- From biological neuron network to artificial neuron networks

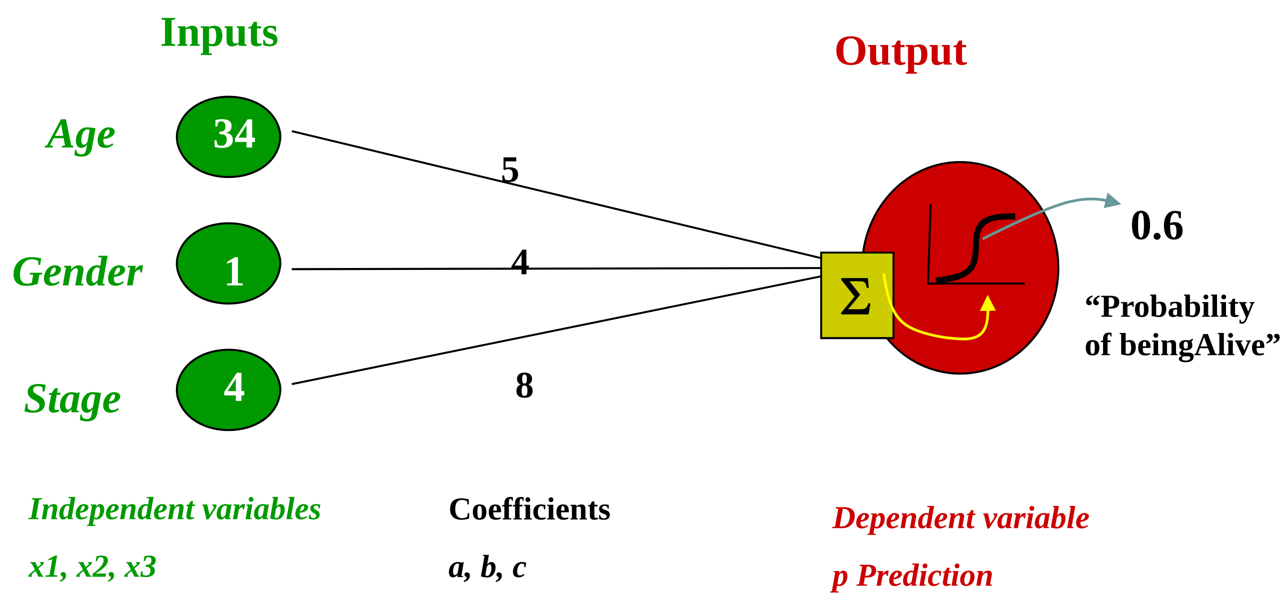




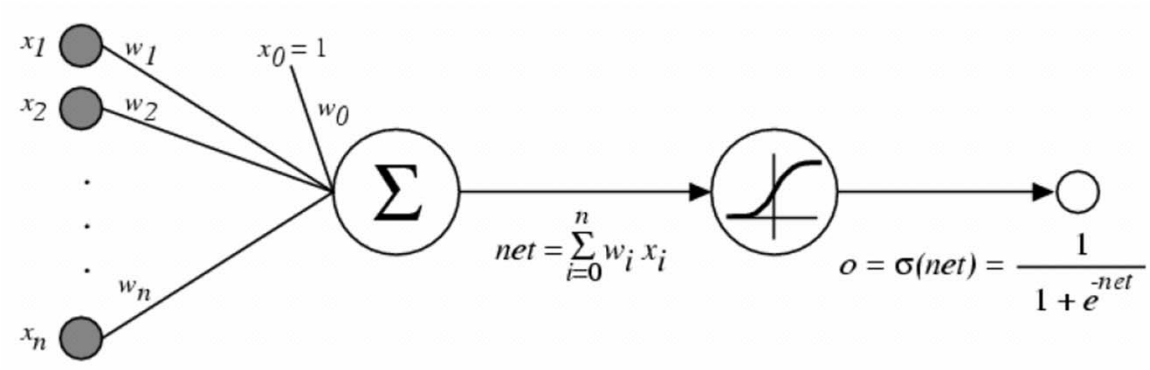
Jargon Pseudo-Correspondence

- Independent variable = input variable
- Dependent variable = output variable
- Coefficients = “weights”
- Estimates = “targets”

Logistic Regression Model (the sigmoid unit)



A perceptron learning algorithm



- Recall the nice property of sigmoid function $\frac{d\sigma}{dt} = \sigma(1-\sigma)$
- Consider regression problem f:X→Y , for scalar Y: $y = f(x) + \epsilon$
- We used to maximize the conditional data likelihood

$$\vec{w} = \arg\max_{\vec{w}} \ln \prod_i P(y_i | x_i; \vec{w})$$

- Here …

$$\vec{w} = \arg\min_{\vec{w}} \sum_i \frac{1}{2}(y_i - \hat{f}(x_i; \vec{w}))^2$$

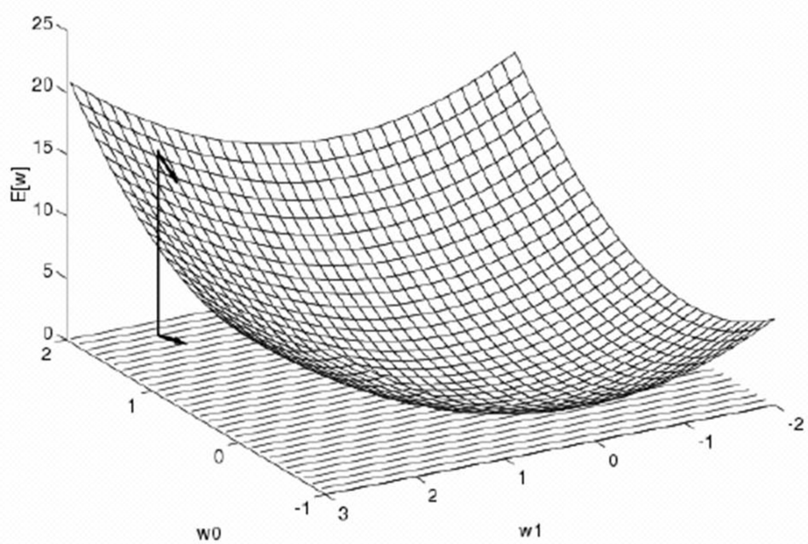
Gradient Descent

x_d = input

t_d = target output

o_d =observed unit output

w_i =weight i



Gradient

$$\nabla E[\vec{w}] \equiv \left[\frac{\partial E}{\partial w_0}, \frac{\partial E}{\partial w_1}, \cdots \frac{\partial E}{\partial w_n}\right]$$

Training rule:

$$\Delta \vec{w} = -\eta \nabla E[\vec{w}]$$

i.e.,

$$\Delta w_i = -\eta \frac{\partial E}{\partial w_i}$$

$$\frac{\partial E[\vec{w}]}{\partial w_j} = \frac{\partial}{\partial w_i} \frac{1}{2} \sum (t_d - o_d)^2$$

$$=$$

The perceptron learning rules

x_d = input

t_d = target output

o_d =observed unit output

w_i =weight i

$$\frac{\partial E_D[\vec{w}])}{\partial w_j} = \frac{\partial}{\partial w_i}\frac{1}{2}\sum_d (t_d - o_d)^2$$
$$= \frac{1}{2}\sum_d 2(t_d - o_d)\frac{\partial}{\partial w_i}(t_d - o_d)$$
$$= \sum_d (t_d - o_d)\big(-\frac{\partial o_d}{\partial w_i}\big)$$
$$= -\sum_d (t_d - o_d)\frac{\partial o_d}{\partial net_i}\frac{\partial net_d}{\partial w_i}$$
$$= -\sum_d (t_d - o_d)o_d(1 - o_d)x_d^i$$

Batch mode:

Do until converge:

1. compute gradient $\nabla E_D[w]$
2. $\vec{w} = \vec{w} - \eta \nabla E_D[\vec{w}]$

Incremental mode:

Do until converge:

- For each training example *d* in *D*
  1. compute gradient $\nabla E_d[w]$
  2. $\vec{w} = \vec{w} - \eta \nabla E_d[\vec{w}]$

  where

  $$\nabla E_d[\vec{w}] = -(t_d - o_d)o_d(1 - o_d)\vec{x}_d$$

MLE vs MAP

- Maximum conditional likelihood estimate

$$\vec{w} = \arg\max_{\vec{w}} \ln \prod_i P(y_i | x_i; \vec{w})$$

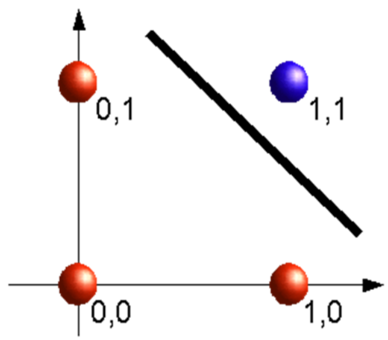
$$\boxed{\vec{w} \leftarrow \vec{w} + \eta \sum_d (t_d - o_d) o_d (1 - o_d) \vec{x}_d}$$

- Maximum a posteriori estimate

$$\vec{w} = \arg\max_{\vec{w}} \ln p(\vec{w}) \prod_i P(y_i | x_i; \vec{w})$$

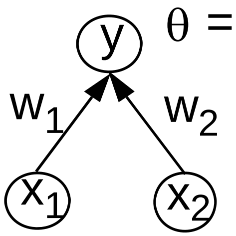
$$\boxed{\vec{w} \leftarrow \vec{w} + \eta \Big(\sum_d (t_d - o_d) o_d (1 - o_d) \vec{x}_d - \lambda \vec{w} \Big)}$$

What decision surface does a perceptron define?



NAND

x	y	Z (color)
0	0	**1**
0	1	**1**
1	0	**1**
1	1	**0**



$\theta = 0.5$

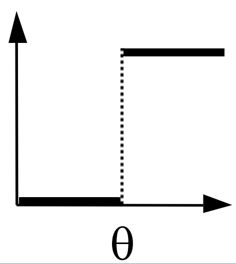
$f(x_1w_1 + x_2w_2) = y$

$f(0w_1 + 0w_2) = 1$

$f(0w_1 + 1w_2) = 1$

$f(1w_1 + 0w_2) = 1$

$f(1w_1 + 1w_2) = 0$

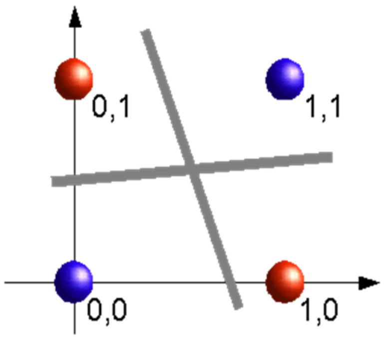


$$f(a) = \begin{cases} 1, \text{ for } a > \theta \\ 0, \text{ for } a \leq \theta \end{cases}$$

some possible values for w_1 and w_2

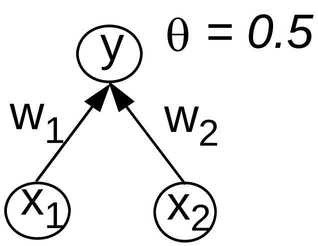
w_1	w_2
0.20	0.35
0.20	0.40
0.25	0.30
0.40	0.20

What decision surface does a perceptron define?



NAND

x	y	Z (color)
0	0	**0**
0	1	**1**
1	0	**1**
1	1	**0**



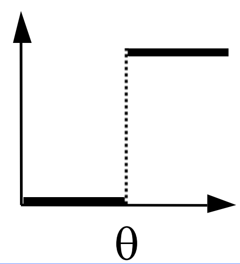
$f(x_1 w_1 + x_2 w_2) = y$

$f(0w_1 + 0w_2) = 0$

$f(0w_1 + 1w_2) = 1$

$f(1w_1 + 0w_2) = 1$

$f(1w_1 + 1w_2) = 0$

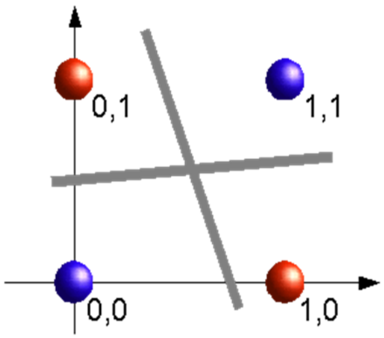


$$f(a) = \begin{cases} 1, & \text{for } a > \theta \\ 0, & \text{for } a \leq \theta \end{cases}$$

some possible values for w_1 and w_2

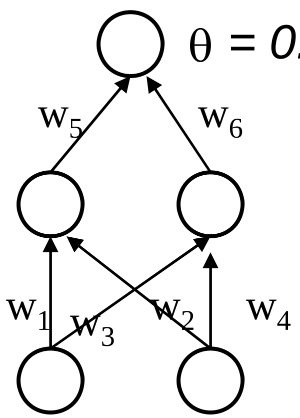
w_1	w_2

What decision surface does a perceptron define?

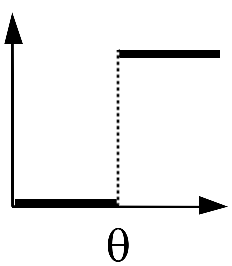


NAND

x	y	Z (color)
0	0	**0**
0	1	**1**
1	0	**1**
1	1	**0**



θ = *0.5 for all units*



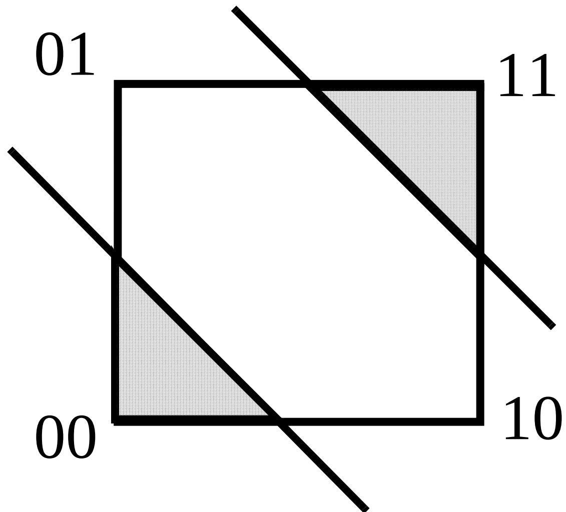
$$f(a) = \begin{cases} 1, \text{ for } a > \theta \\ 0, \text{ for } a \leq \theta \end{cases}$$

a possible set of values for $(w_1, w_2, w_3, w_4, w_5, w_6)$:
$(0.6,-0.6,-0.7,0.8,1,1)$

Non Linear Separation

Meningitis
No cough
Headache

Flu
Cough
Headache

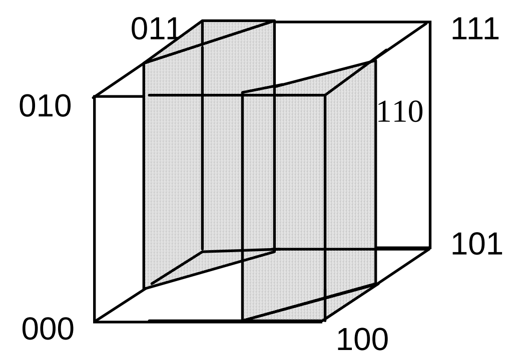


No treatment

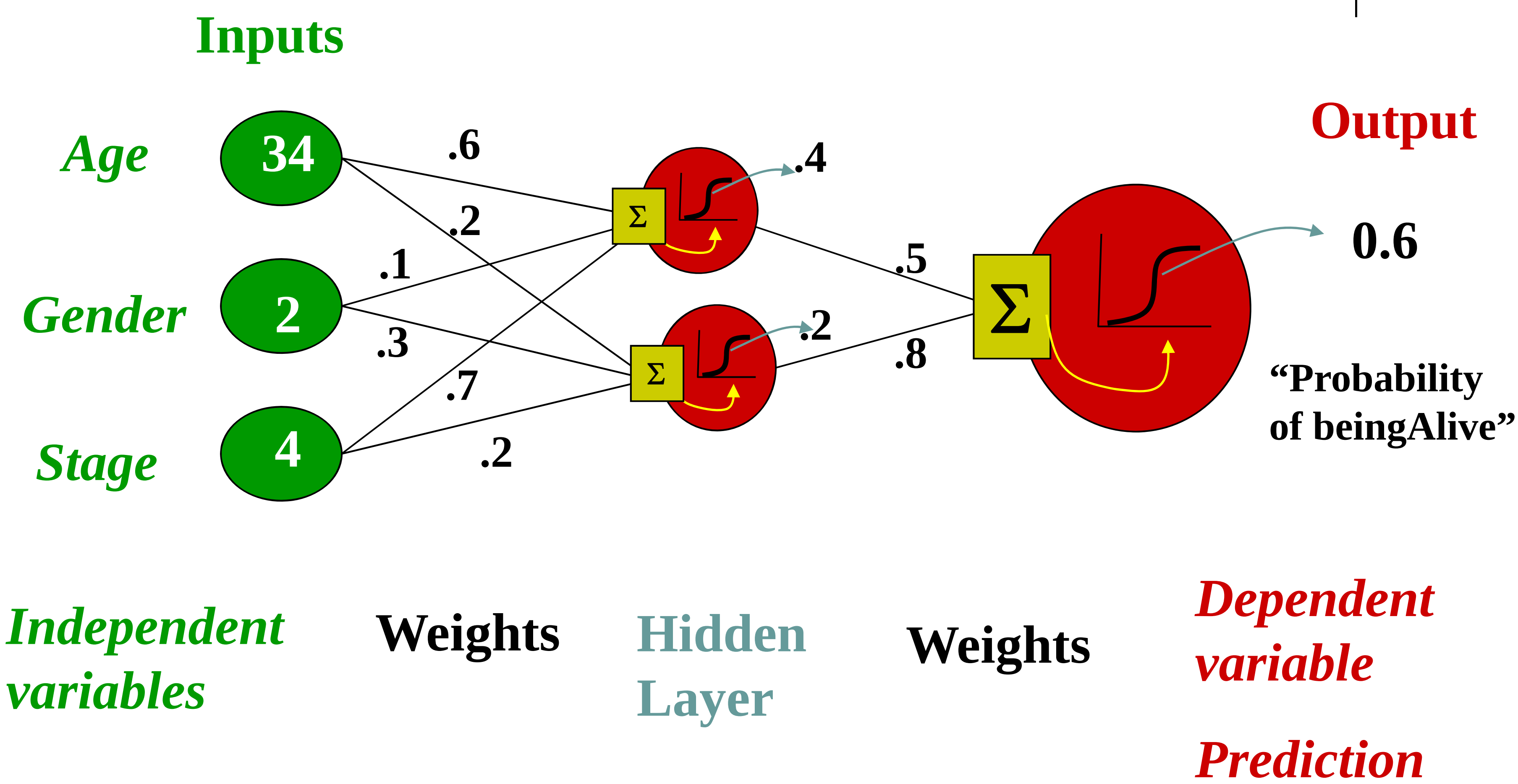
Treatment

No disease
No cough
No headache

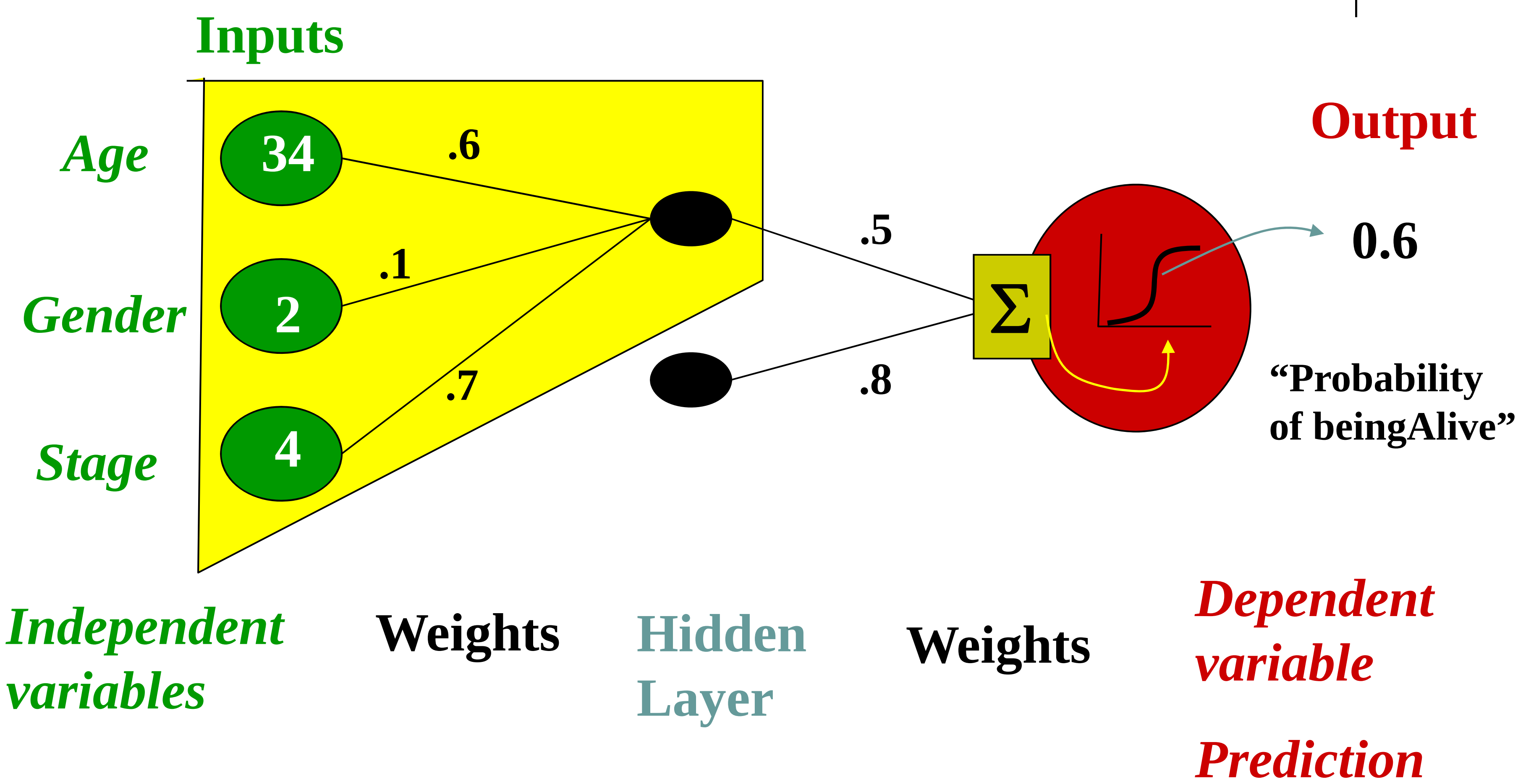
Pneumonia
Cough
No headache

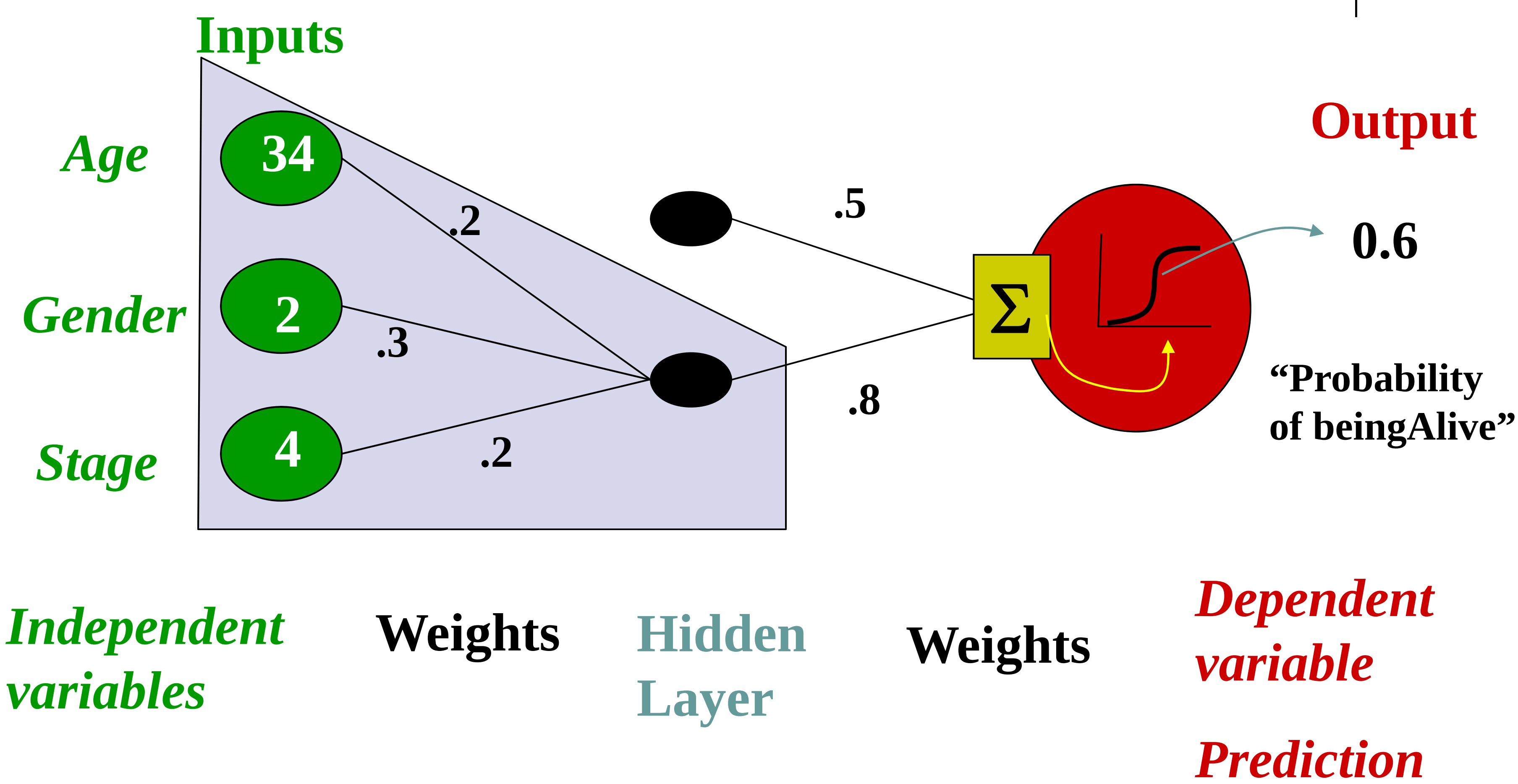


Neural Network Model

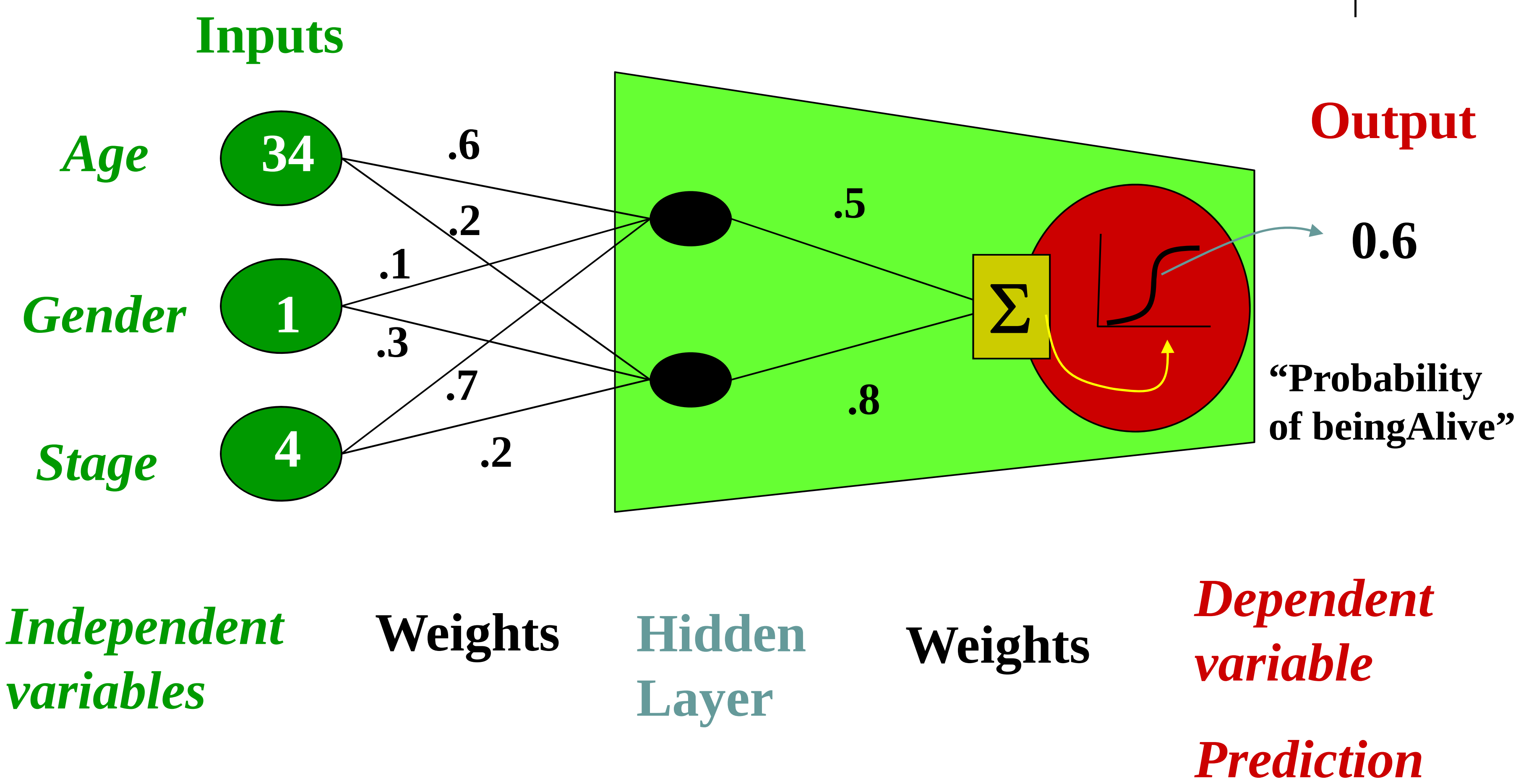


“Combined logistic models”

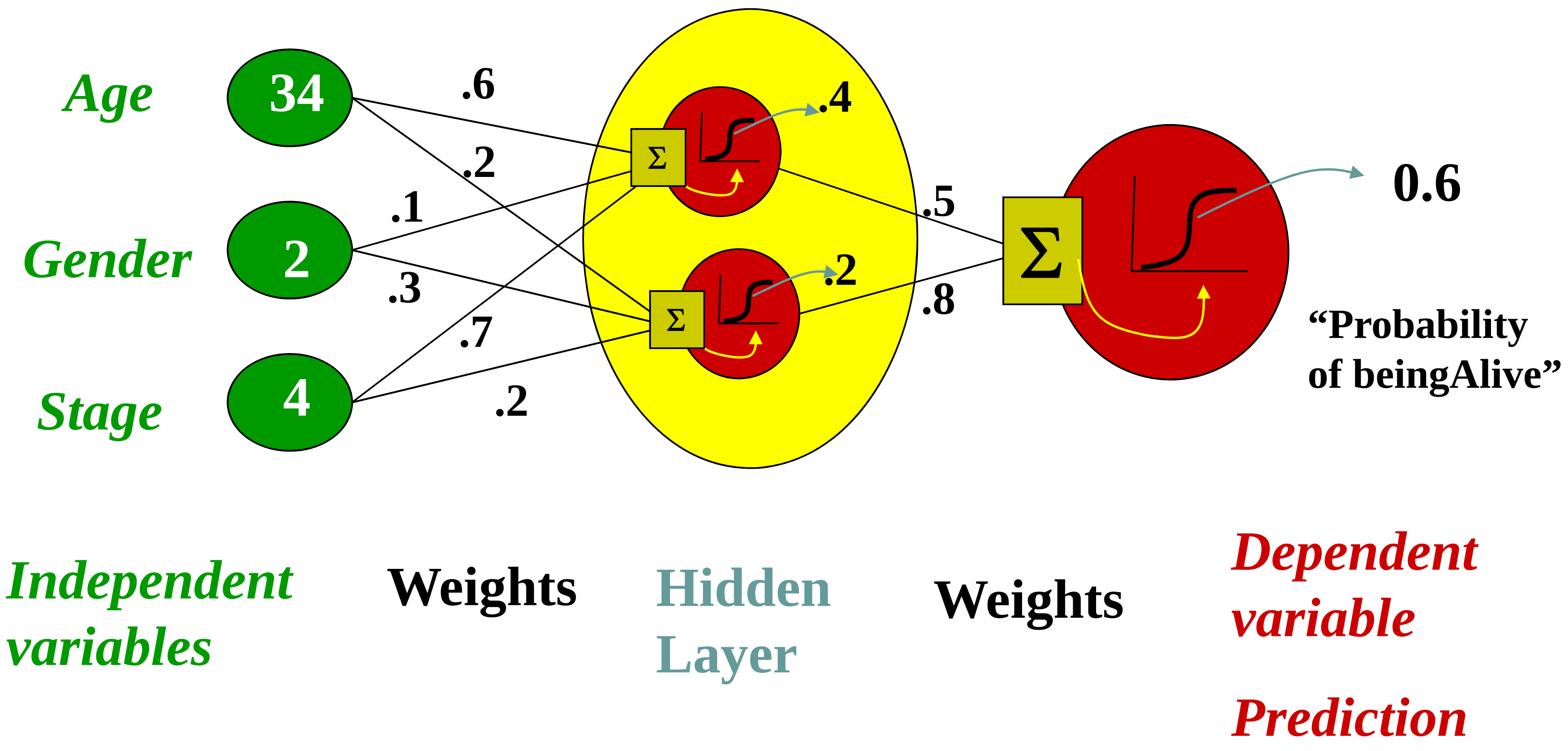




Inputs
Age
34
Gender
2
Stage
4
.2
.3
.2
.5
.8
Σ
Output
0.6
"Probability
of beingAlive"
Independent
variables
Weights
Hidden
Layer
Weights
Dependent
variable
Prediction

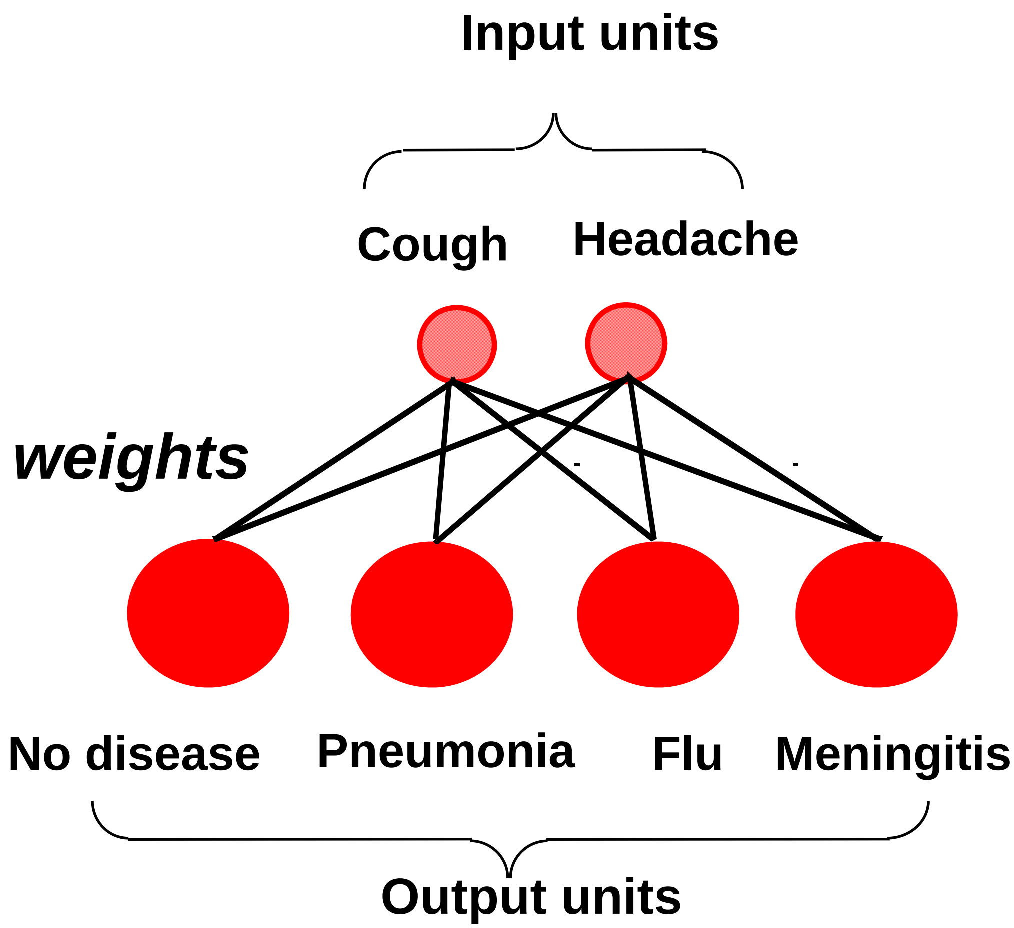

Inputs
Age
34
Gender
1
Stage
4
.6
.2
.1
.3
.7
.2
.5
.8
Σ
Output
0.6
"Probability
of beingAlive"
Independent
variables
Weights
Hidden
Layer
Weights
Dependent
variable
Prediction

Not really, no target for hidden units...



Recall perceptrons

$$\vec{w} \leftarrow \vec{w} + \eta \sum_d (t_d - o_d) o_d (1 - o_d) \vec{x}_d$$



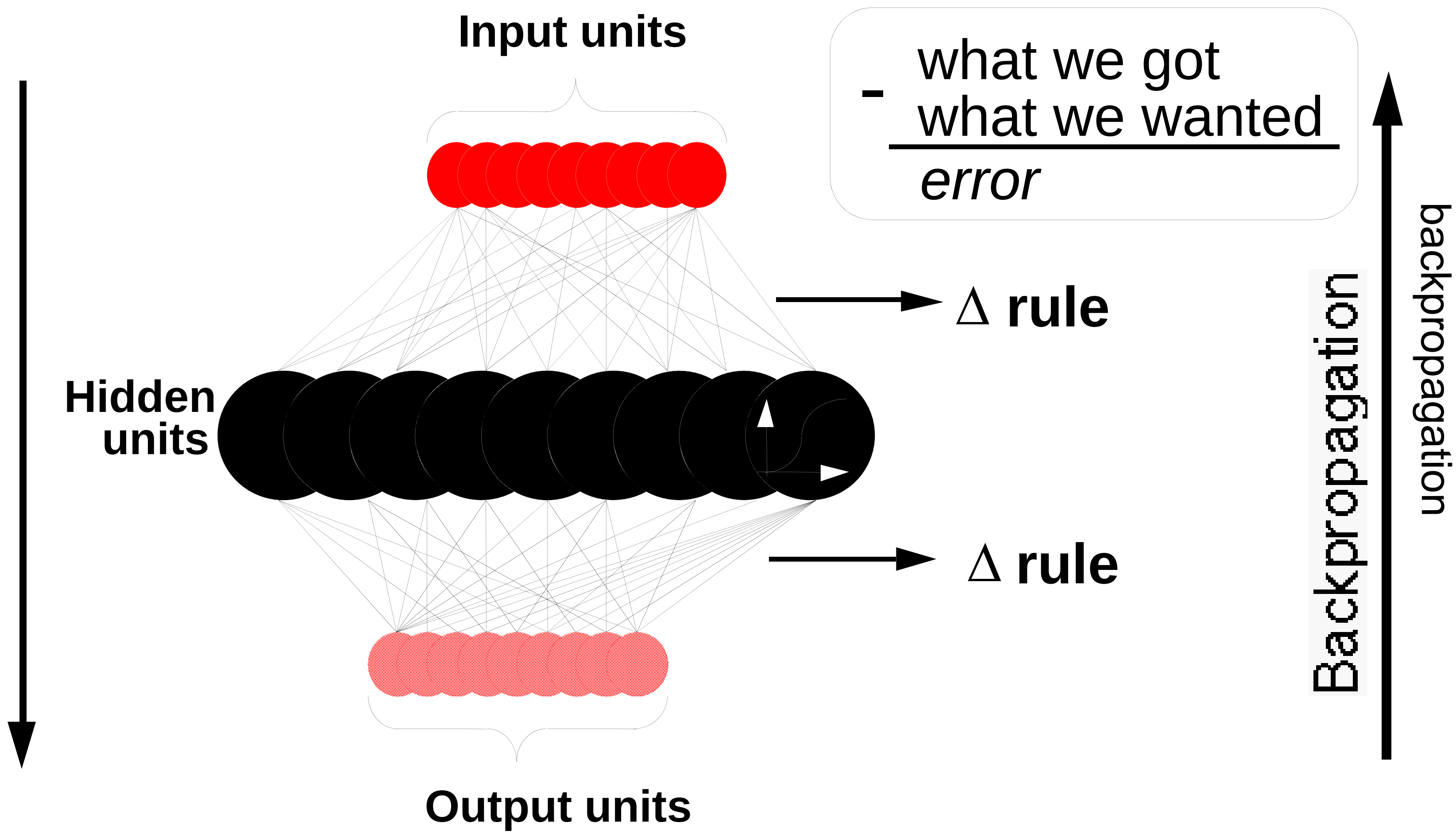
Δ **rule**

change weights to decrease the error

- what we got
what we wanted

error

Hidden Units and Backpropagation



Backpropagation Algorithm

x_d = input

t_d = target output

o_d =observed unit output

w_i =weight i

$$\vec{w} \leftarrow \vec{w} + \eta \sum_d (t_d - o_d) o_d (1 - o_d) \vec{x}_d$$

- Initialize all weights to small random numbers

  Until convergence, Do

1. Input the training example to the network and compute the network outputs

1. For each output unit k

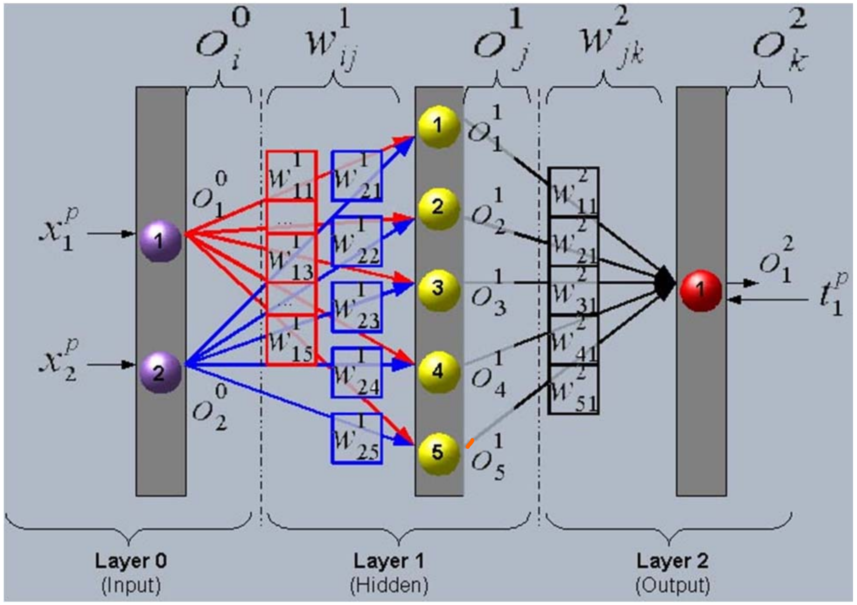
$$\delta_k \leftarrow o_k^2(1 - o_k^2)(t - o_k^2)$$

2. For each hidden unit h

$$\delta_h \leftarrow o_h^1(1 - o_h^1) \sum_{k \in outputs} w_{h,k} \delta_k$$

3. Undate each network weight $w_{i,j}$

$$w_{i,j} \leftarrow w_{i,j} + \Delta w_{i,j} \text{ where } \Delta w_{i,j} = \eta \delta_j x^j$$



More on Backpropatation

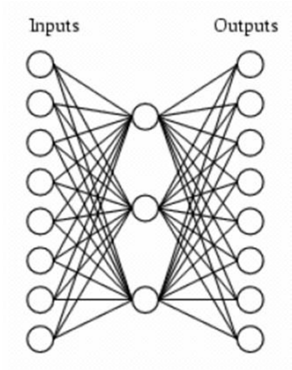
- It is doing gradient descent over entire network weight vector
- Easily generalized to arbitrary directed graphs
- Will find a local, not necessarily global error minimum
  - In practice, often works well (can run multiple times)
- Often include weight *momentum* α

$$\Delta w_{i,j}(t) = \eta \delta_j x_{i,j} + \alpha \Delta w_{i,j}(t-1)$$

- Minimizes error over *training* examples
  - Will it generalize well to subsequent testing examples?
- Training can take thousands of iterations, → very slow!
- Using network after training is very fast

Learning Hidden Layer Representation

- A network:



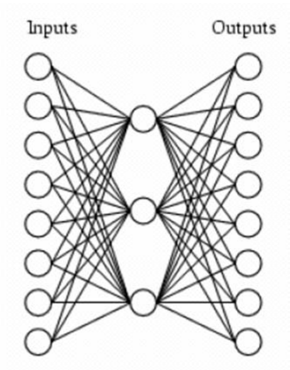
- A target function:

Input		Output
10000000	$\rightarrow$	10000000
01000000	$\rightarrow$	01000000
00100000	$\rightarrow$	00100000
00010000	$\rightarrow$	00010000
00001000	$\rightarrow$	00001000
00000100	$\rightarrow$	00000100
00000010	$\rightarrow$	00000010
00000001	$\rightarrow$	00000001

- Can this be learned?

Learning Hidden Layer Representation

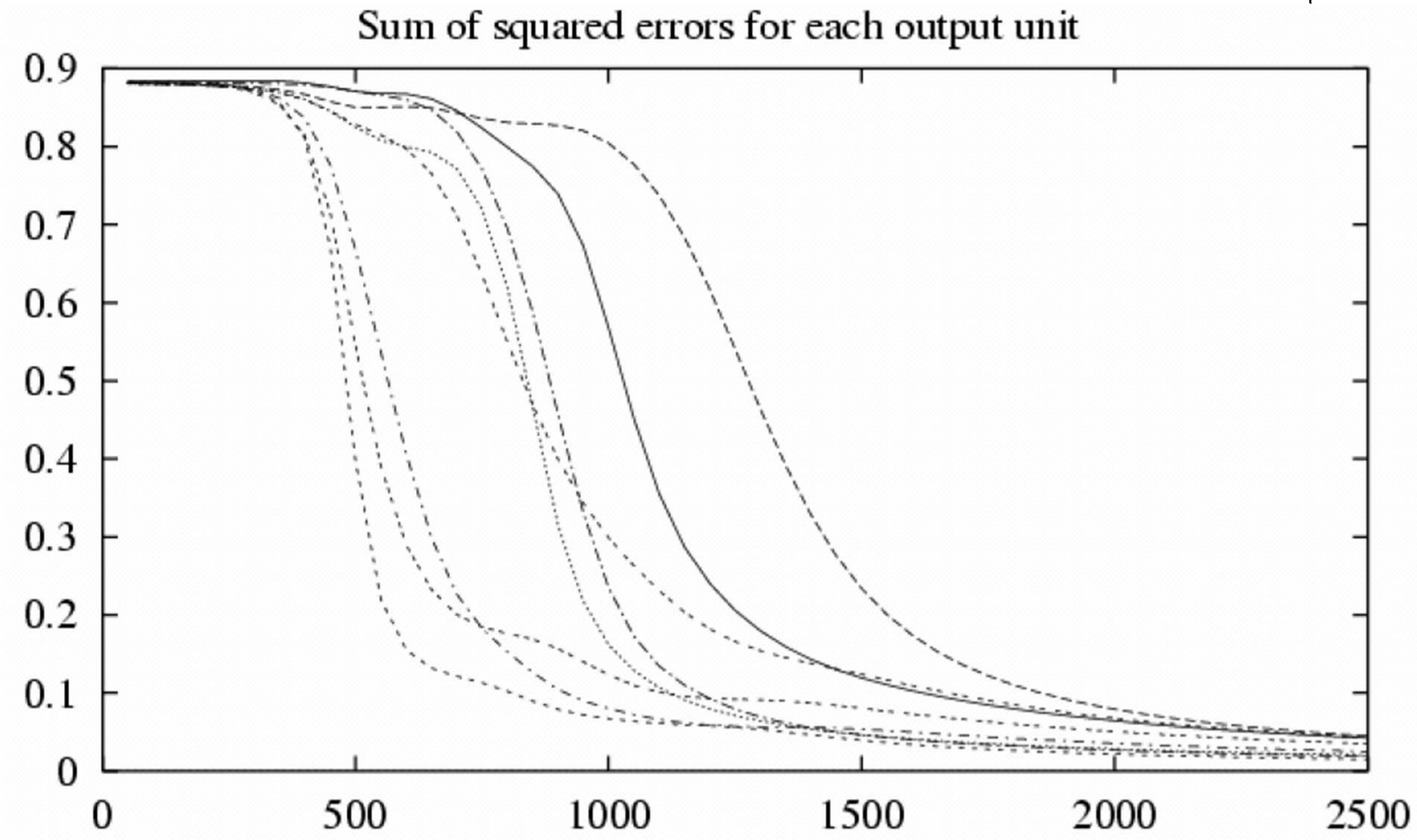
- A network:



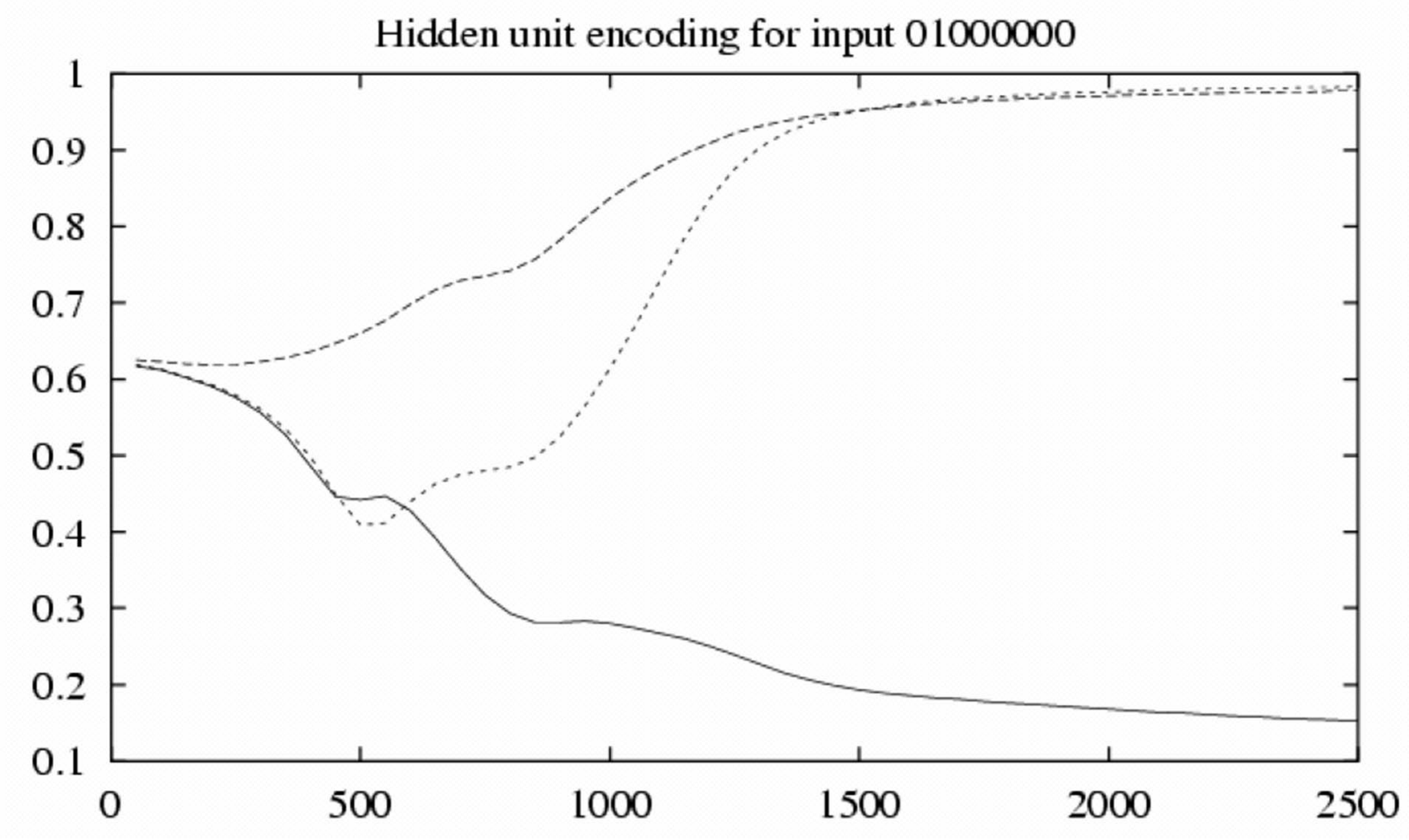
- Learned hidden layer representation:

Input		Hidden Values				Output
10000000	→	.89	.04	.08	→	10000000
01000000	→	.01	.11	.88	→	01000000
00100000	→	.01	.97	.27	→	00100000
00010000	→	.99	.97	.71	→	00010000
00001000	→	.03	.05	.02	→	00001000
00000100	→	.22	.99	.99	→	00000100
00000010	→	.80	.01	.98	→	00000010
00000001	→	.60	.94	.01	→	00000001

Training


Sum of squared errors for each output unit
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
500
1000
1500
2000
2500

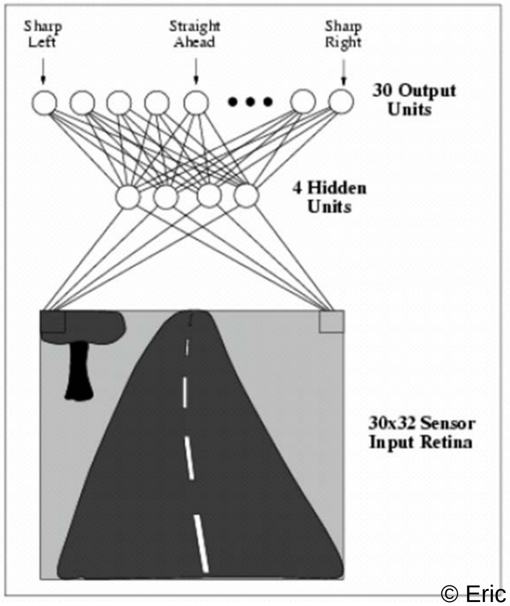
Training

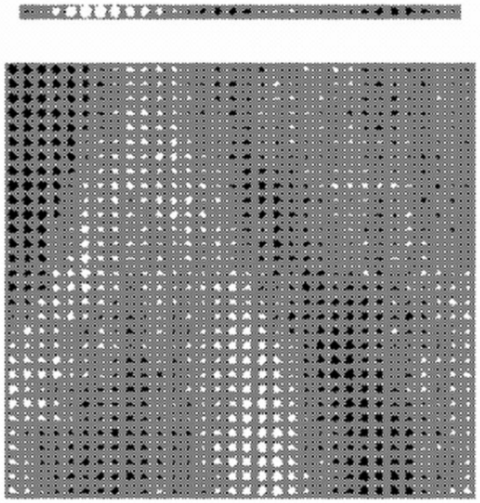

Hidden unit encoding for input 01000000
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
500
1000
1500
2000
2500

The "Driver" Network

ALVINN

[Pomerleau 1993]

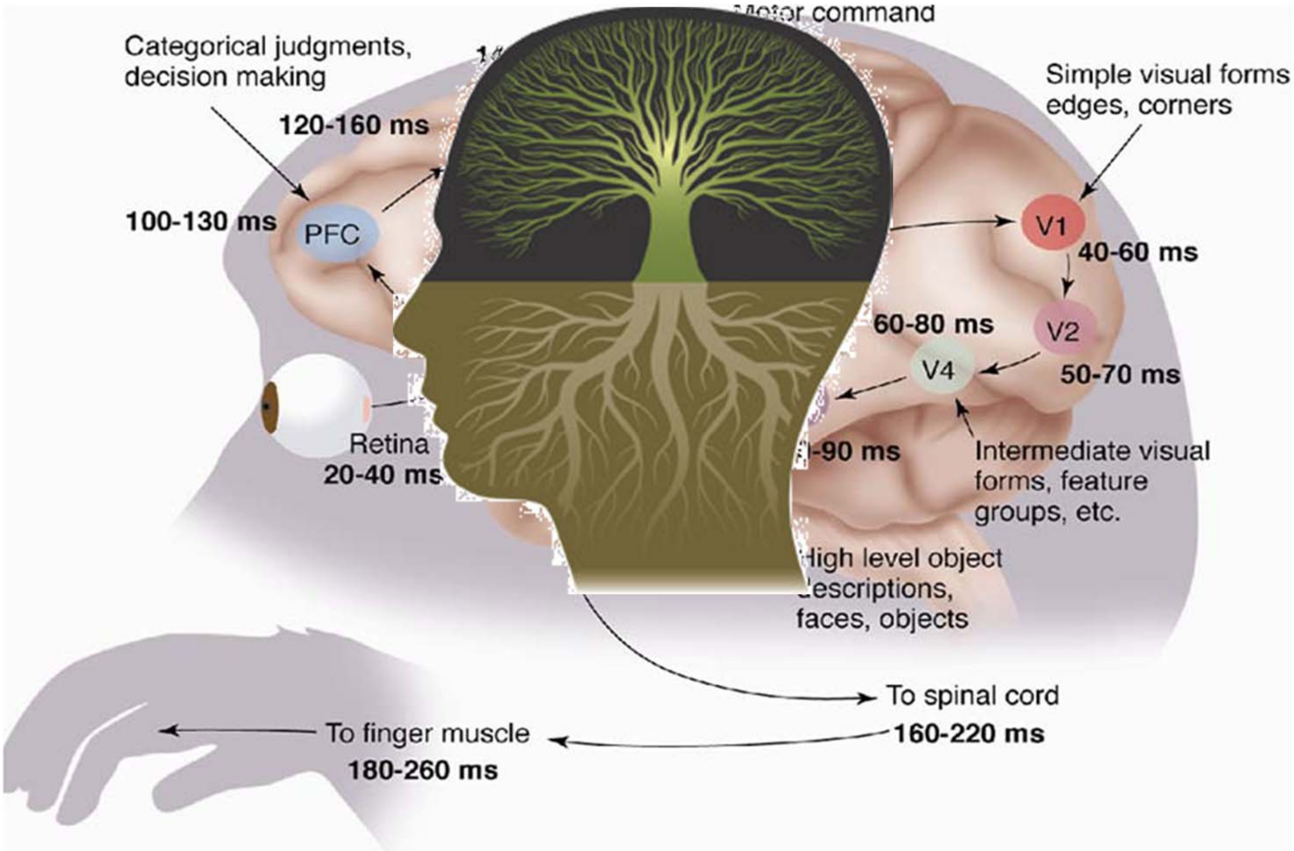




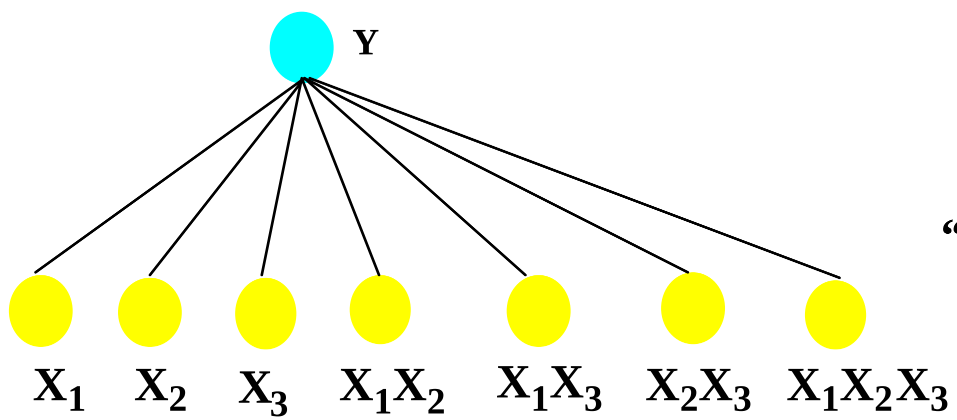
Artificial neural networks – what you should know

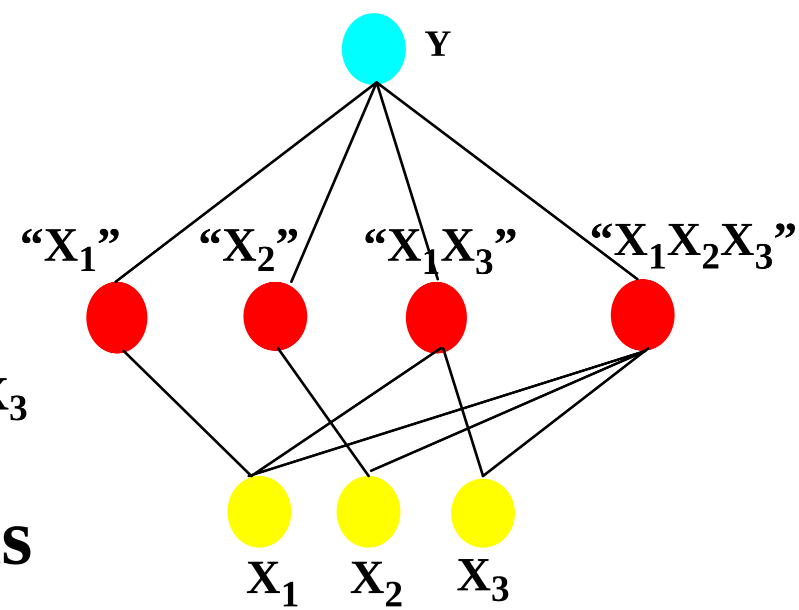
- Highly expressive non-linear functions
- Highly parallel network of logistic function units
- Minimizing sum of squared training errors
  - Gives MLE estimates of network weights if we assume zero mean Gaussian noise on output values
- Minimizing sum of sq errors plus weight squared (regularization)
  - MAP estimates assuming weight priors are zero mean Gaussian
- Gradient descent as training procedure
  - How to derive your own gradient descent procedure
- Discover useful representations at hidden units
- Local minima is greatest problem
- Overfitting, regularization, early stopping

Modern ANN topics: "Deep" Learning



Non-linear LR vs. ANN

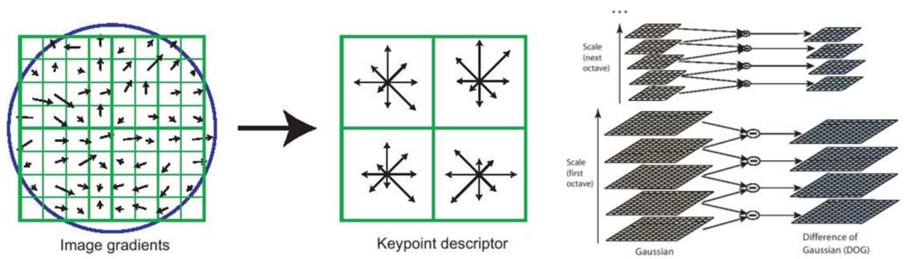




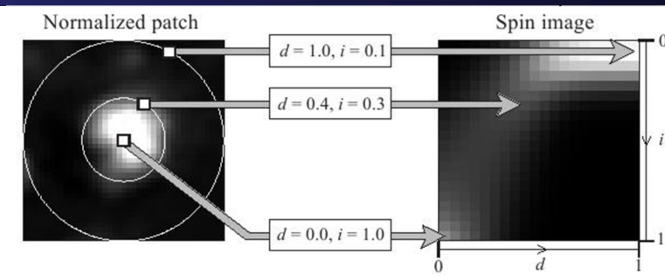
$(2^3\text{-}1)$ possible combinations

$$Y = a(X_1) + b(X_2) + c(X_3) + d(X_1X_2) + \ldots$$

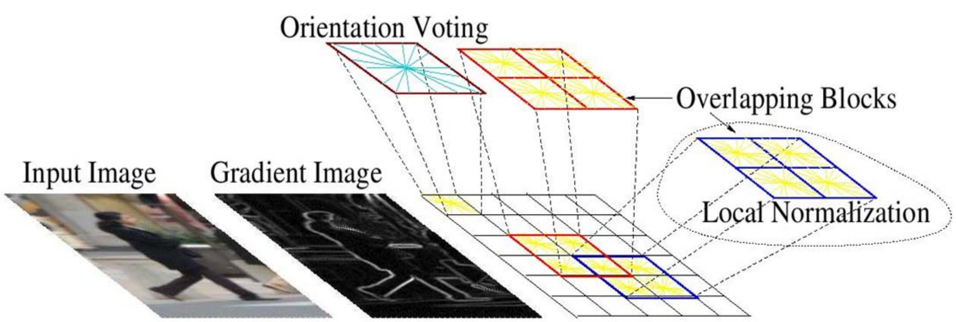
Computer vision features

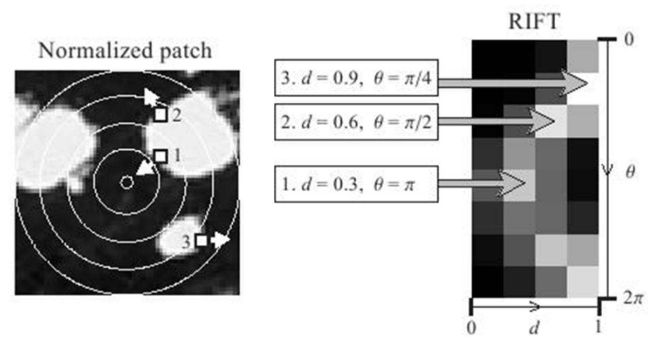


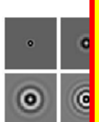
SIFT

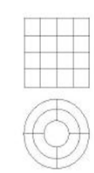


Spin image









(e)

Drawbacks of feature engineering

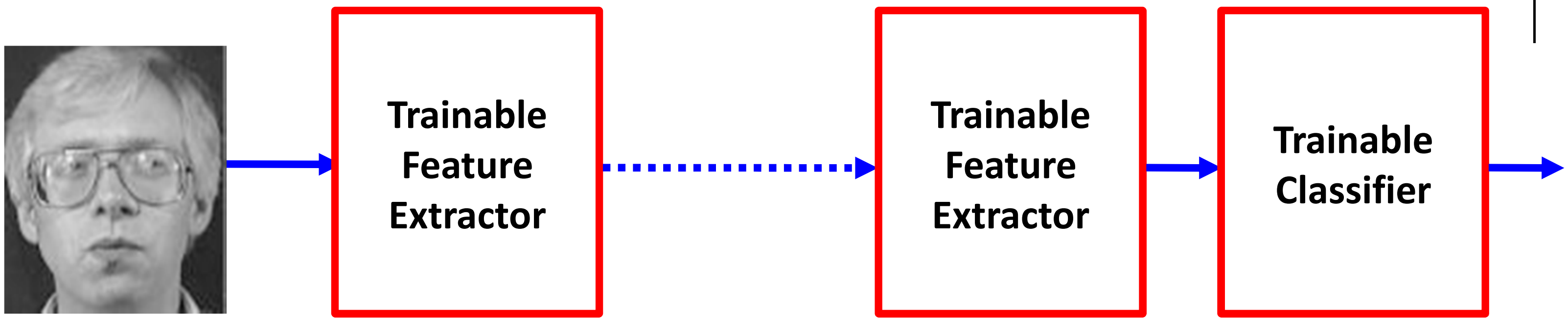
1. **Needs expert knowledge**
2. **Time consuming hand-tuning**

Textons

GLOH

e and Ng

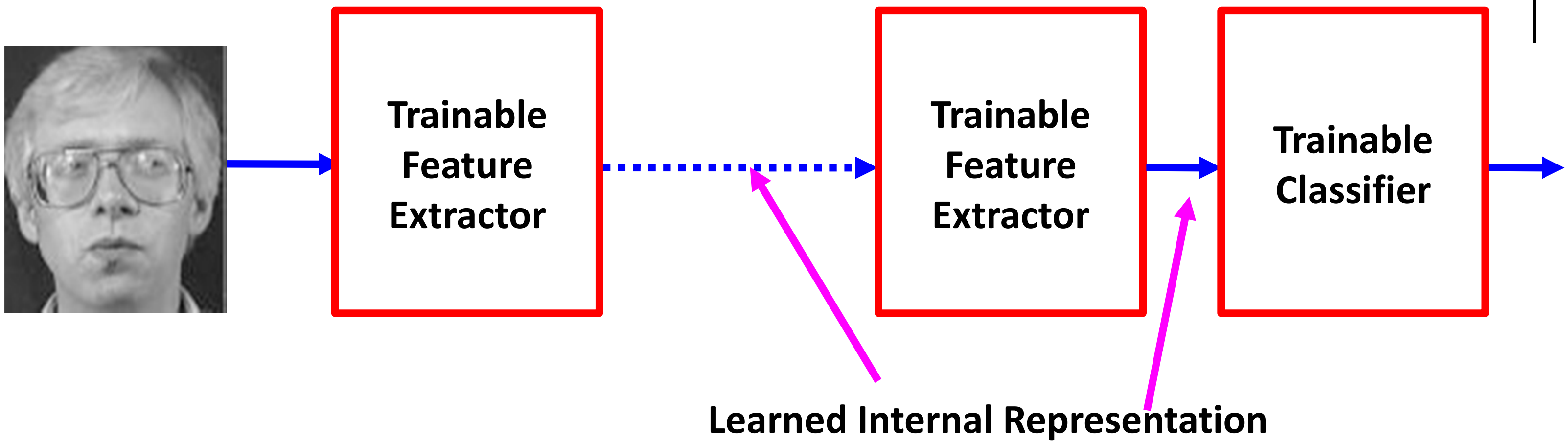
Using ANN to hierarchical representation



Good Representations are hierarchical

- In Language: hierarchy in syntax and semantics
  – Words->Parts of Speech->Sentences->Text
  – Objects,Actions,Attributes...-> Phrases -> Statements -> Stories
- In Vision: part-whole hierarchy
  – Pixels->Edges->Textons->Parts->Objects->Scenes

“Deep” learning: learning hierarchical representations



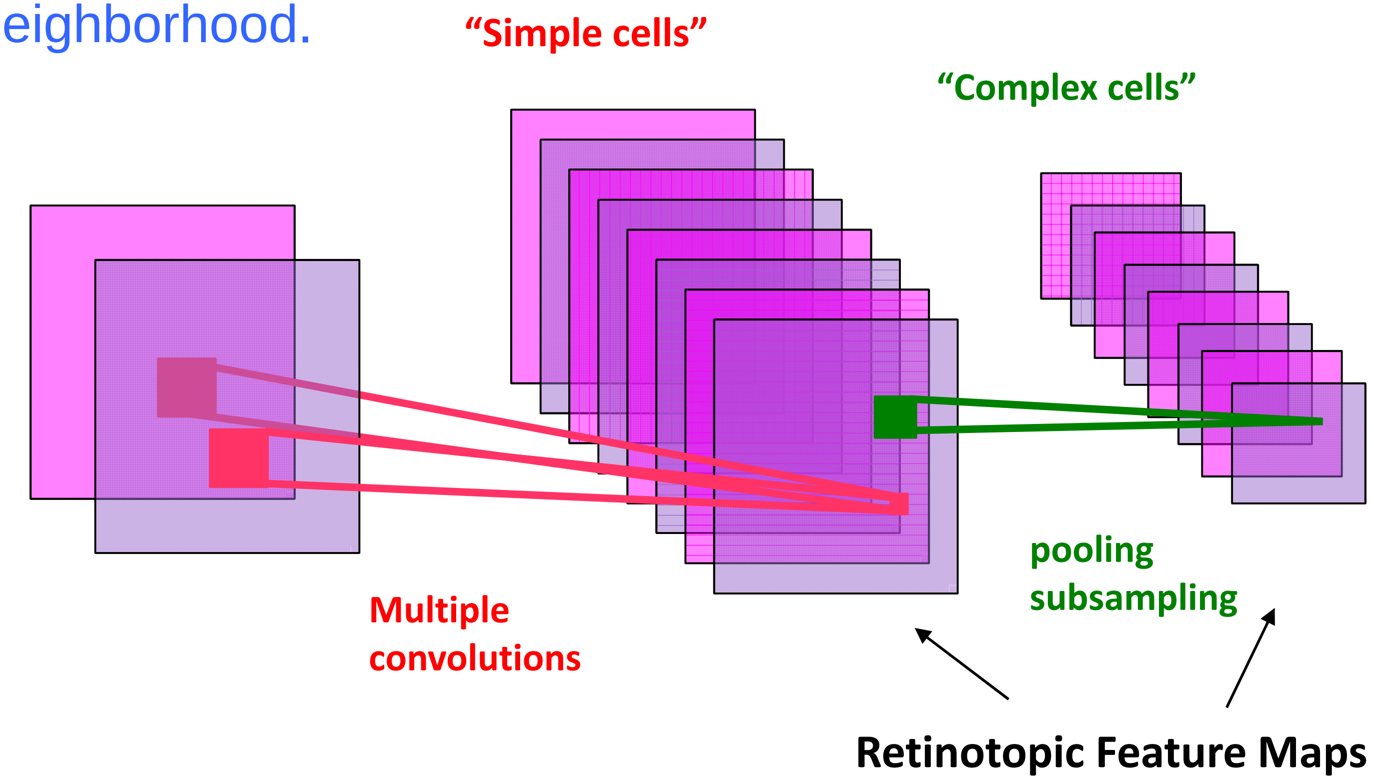
- Deep Learning: learning a hierarchy of internal representations
- From low-level features to mid-level invariant representations, to object identities
- Representations are increasingly invariant as we go up the layers
- using multiple stages gets around the specificity/invariance dilemma

"Deep" models

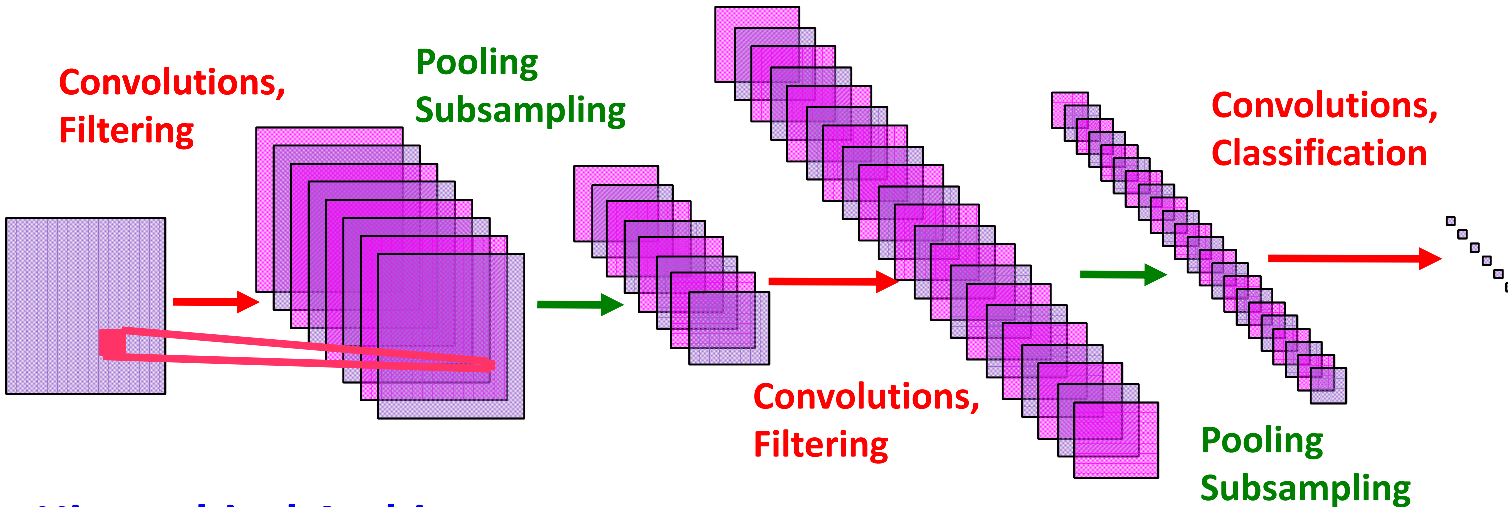
- Neural Networks: Feed-forward*
  - You have seen it
- Autoencoders (multilayer neural net with target output = input)
  - Probabilistic -- Directed: PCA, Sparse Coding
  - Probabilistic -- Undirected: MRFs and RBMs*
- Recursive Neural Networks*
- Convolutional Neural Nets

Filtering + NonLinearity + Pooling = 1 stage of a Convolutional Net

- [Hubel & Wiesel 1962]:
  - simple cells detect local features
  - complex cells "pool" the outputs of simple cells within a retinotopic neighborhood.

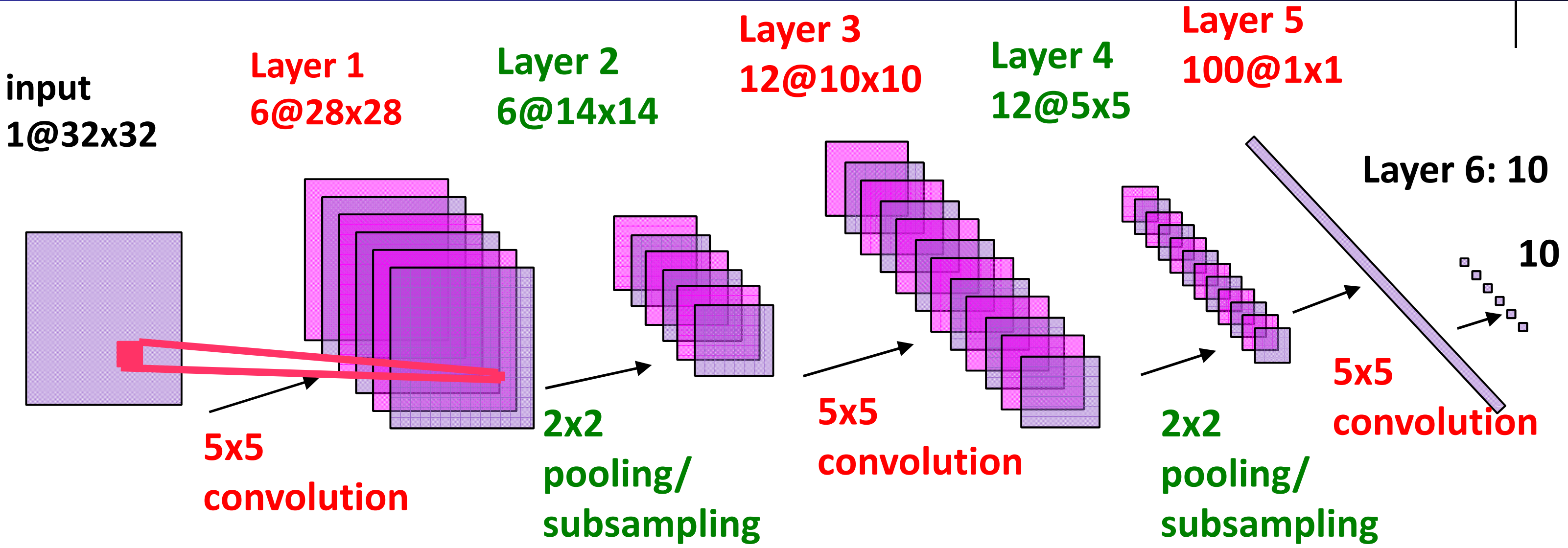


Convolutional Network: Multi-Stage Trainable Architecture



- **Hierarchical Architecture**
  - Representations are more global, more invariant, and more abstract as we go up the layers
- **Alternated Layers of Filtering and Spatial Pooling**
  - Filtering detects conjunctions of features
  - Pooling computes local disjunctions of features
- **Fully Trainable**
  - All the layers are trainable

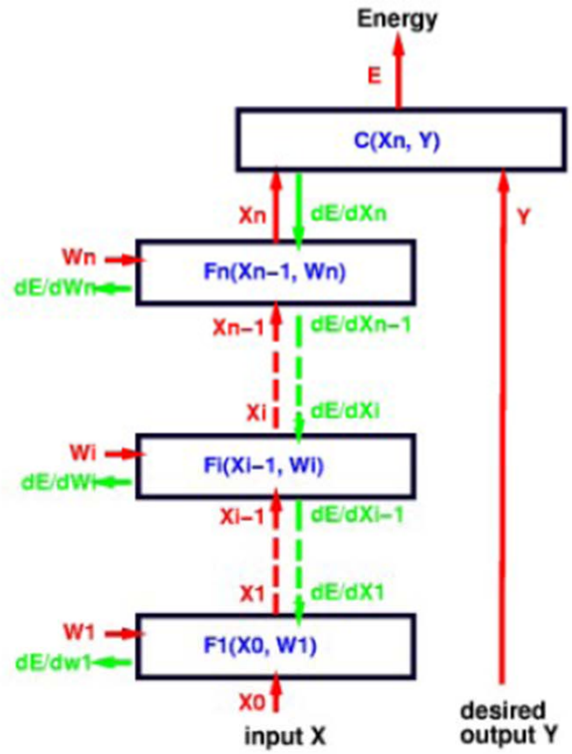
Convolutional Net Architecture for Hand-writing recognition



- Convolutional net for handwriting recognition (400,000 synapses)
  - Convolutional layers (simple cells): all units in a feature plane share the same weights
  - Pooling/subsampling layers (complex cells): for invariance to small distortions.
  - Supervised gradient-descent learning using back-propagation
  - The entire network is trained end-to-end. All the layers are trained simultaneously.
  - [LeCun et al. Proc IEEE, 1998]

How to train?

To compute all the derivatives, we use a backward sweep called the **back-propagation algorithm** that uses the recurrence equation for $\frac{\partial E}{\partial X_i}$

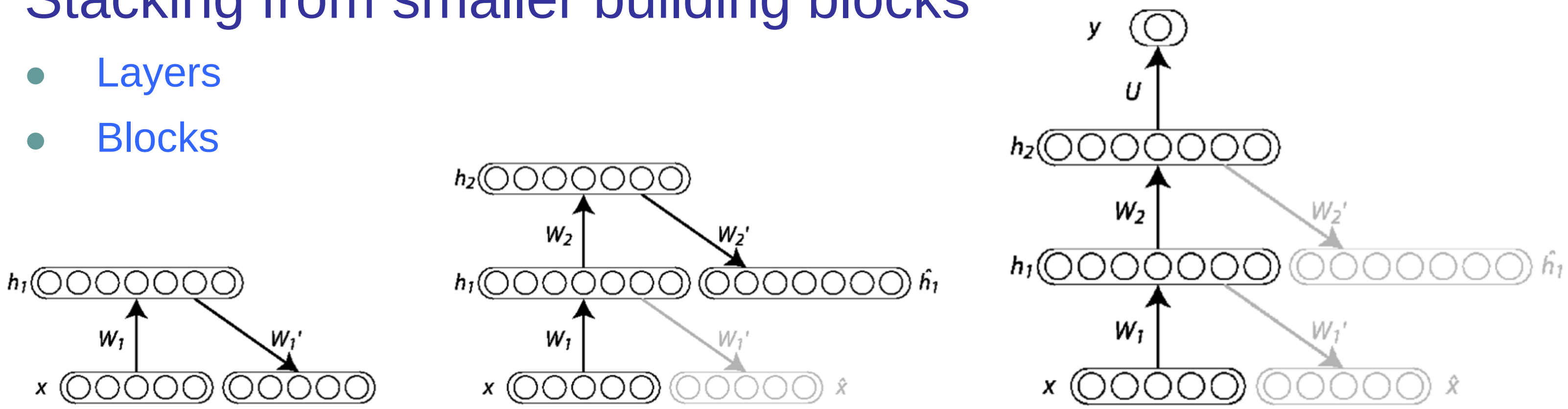


- $\frac{\partial E}{\partial X_n} = \frac{\partial C(X_n, Y)}{\partial X_n}$
- $\frac{\partial E}{\partial X_{n-1}} = \frac{\partial E}{\partial X_n} \frac{\partial F_n(X_{n-1}, W_n)}{\partial X_{n-1}}$
- $\frac{\partial E}{\partial W_n} = \frac{\partial E}{\partial X_n} \frac{\partial F_n(X_{n-1}, W_n)}{\partial W_n}$
- $\frac{\partial E}{\partial X_{n-2}} = \frac{\partial E}{\partial X_{n-1}} \frac{\partial F_{n-1}(X_{n-2}, W_{n-1})}{\partial X_{n-2}}$
- $\frac{\partial E}{\partial W_{n-1}} = \frac{\partial E}{\partial X_{n-1}} \frac{\partial F_{n-1}(X_{n-2}, W_{n-1})}{\partial W_{n-1}}$
- ....etc, until we reach the first module.
- we now have all the $\frac{\partial E}{\partial W_i}$ for $i \in [1, n]$.

But this is very slow !!!

Some new ideas to speed up

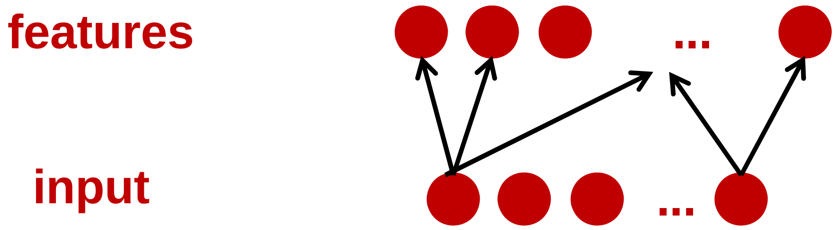
- Stacking from smaller building blocks
  - Layers
  - Blocks



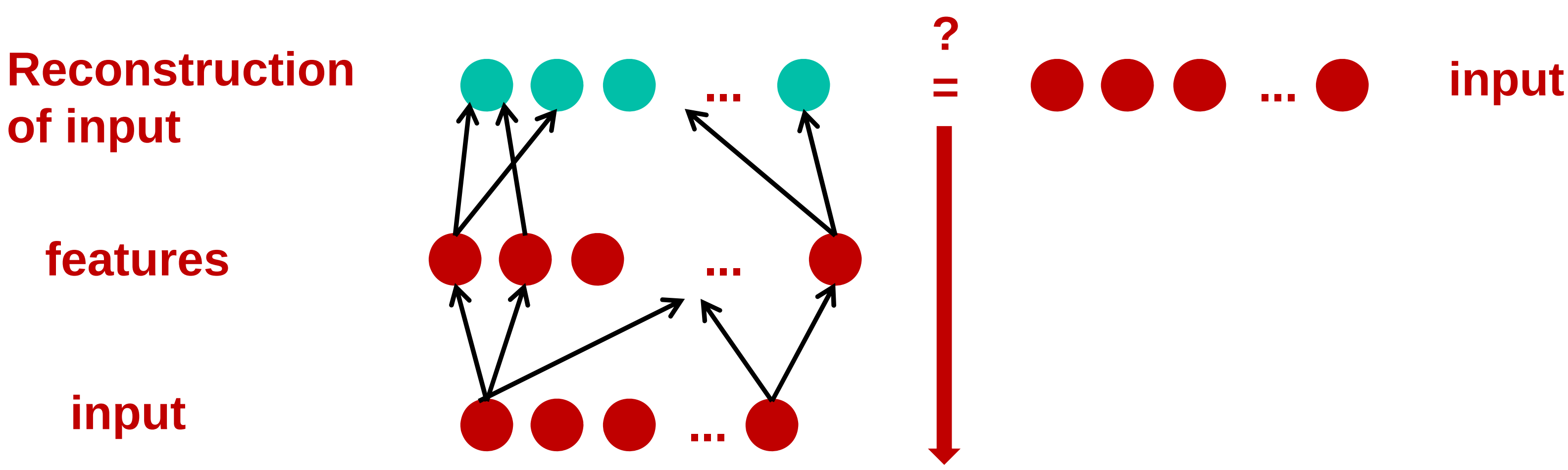
- Approximate Inference
  - Undirected connections for all layers (Markov net) [Related work: Salakhutdinov and Hinton, 2009]
  - Block Gibbs sampling or mean-field
  - Hierarchical probabilistic inference
- Layer-wise Unsupervised Learning

Layer-wise Unsupervised Pre-training

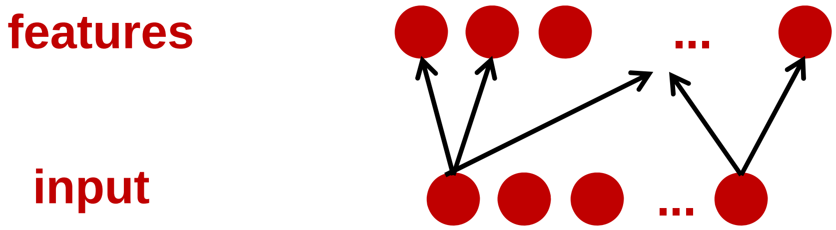
Layer-wise Unsupervised Pre-training



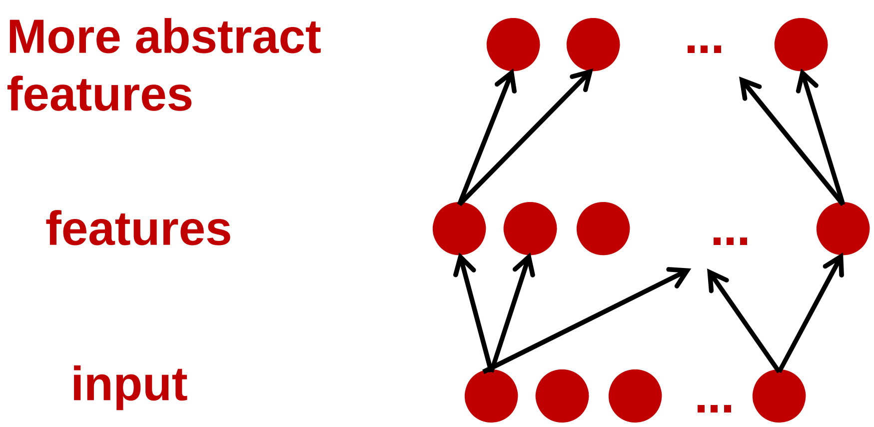
Layer-wise Unsupervised Pre-training



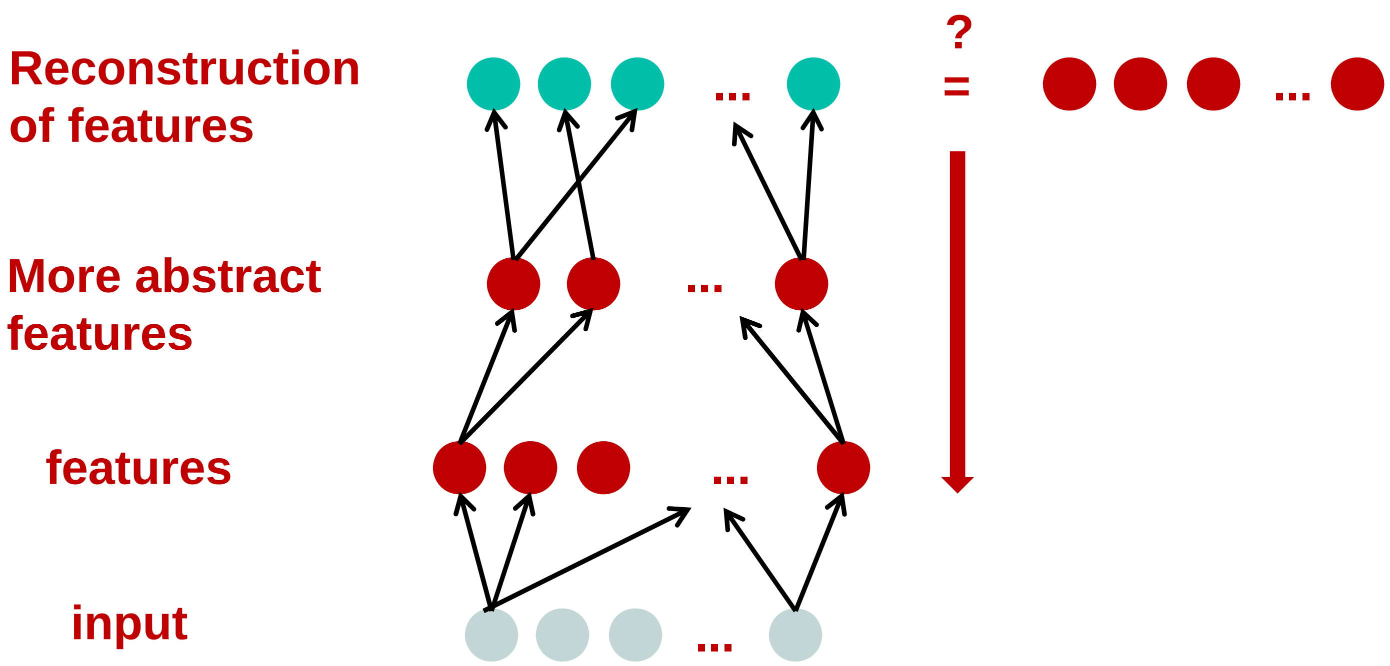
Layer-wise Unsupervised Pre-training



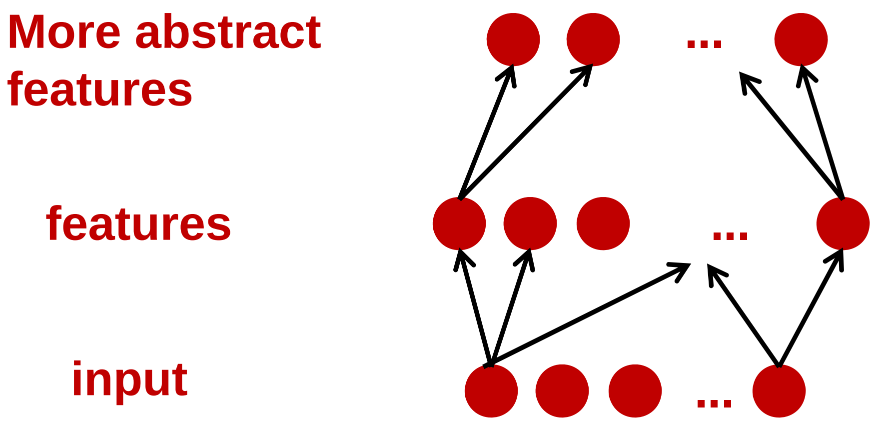
Layer-wise Unsupervised Pre-training



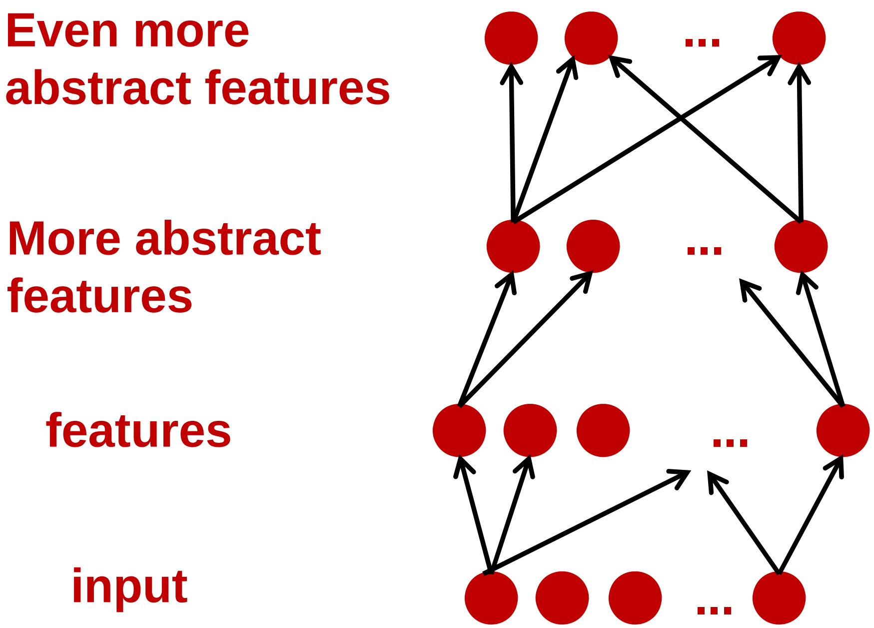
Layer-wise Unsupervised Pre-training



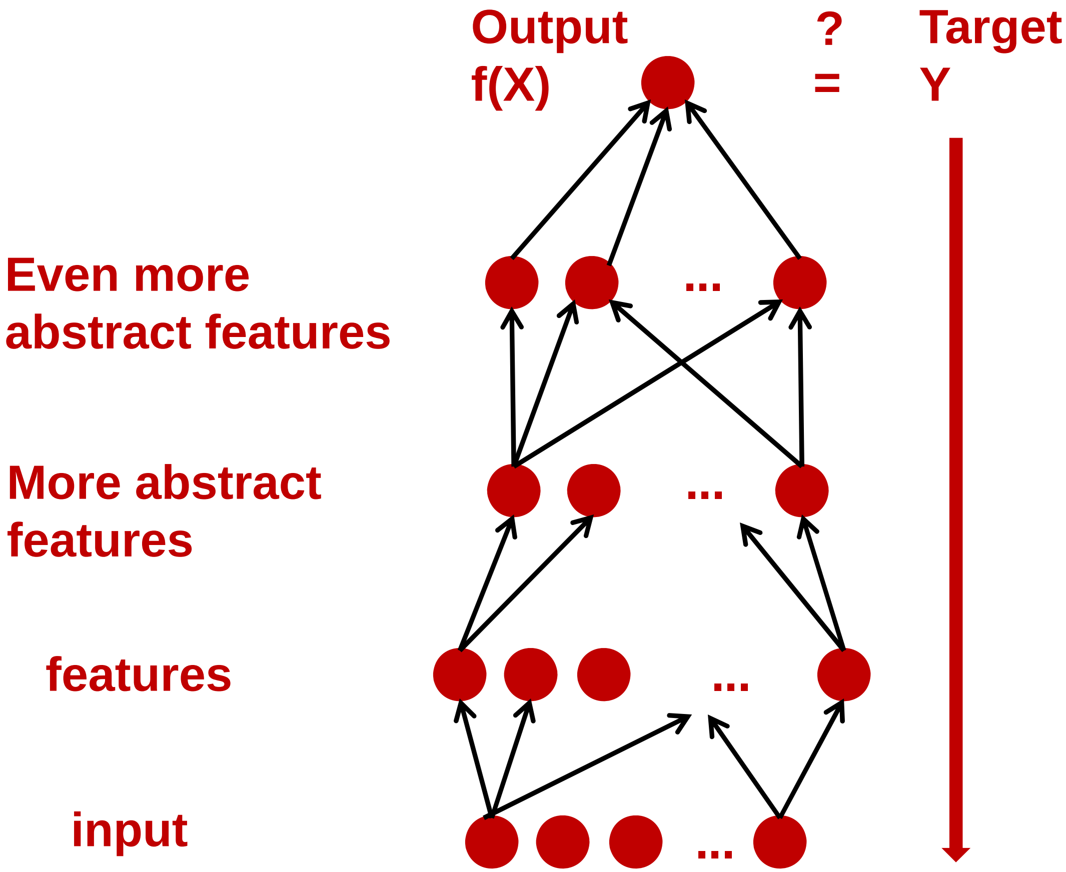
Layer-wise Unsupervised Pre-training



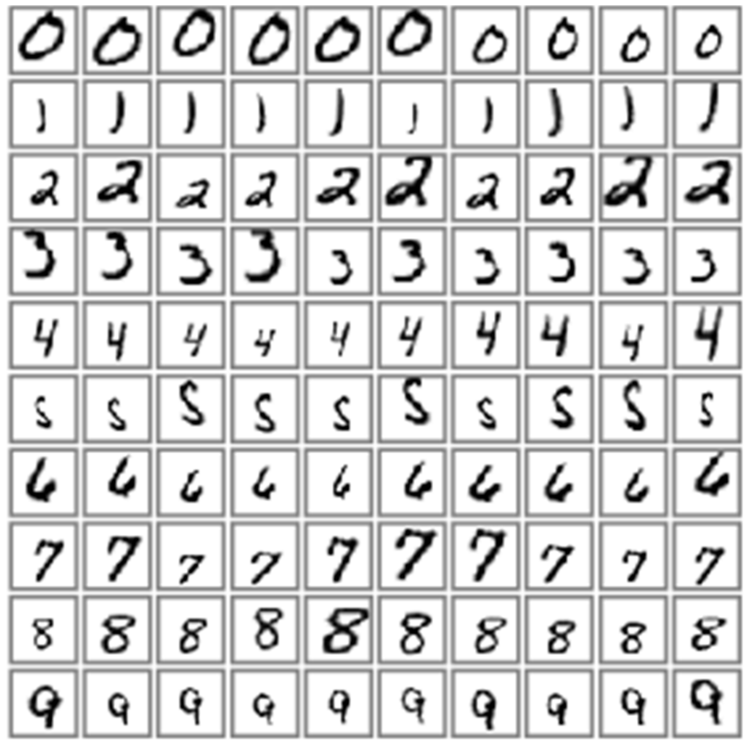
Layer-wise Unsupervised Pre-training



Layer-wise Unsupervised Pre-training



Application:
MNIST Handwritten Digit Dataset

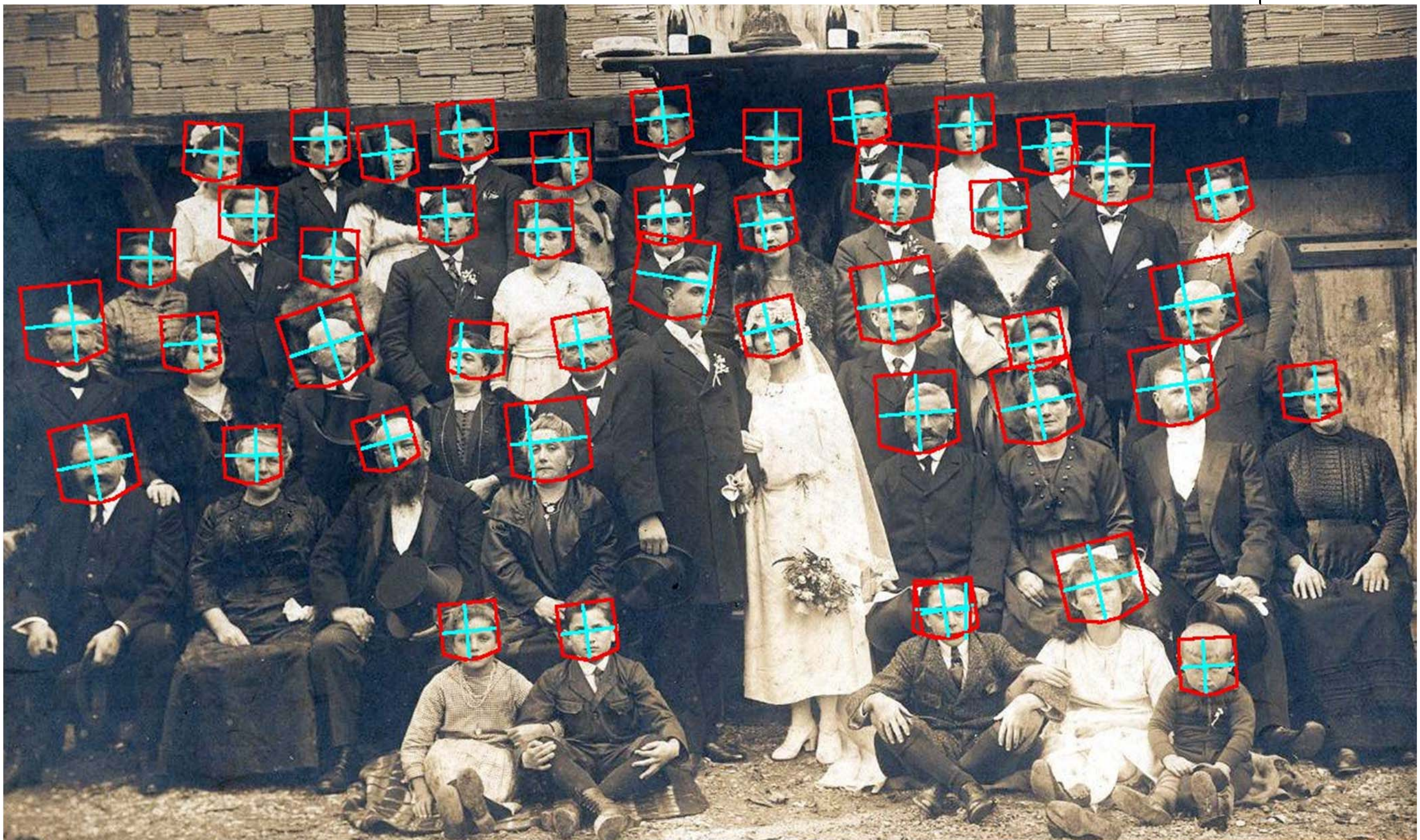


Handwritten Digit Dataset MNIST: 60,000 training samples, 10,000 test samples

Results on MNIST Handwritten Digits

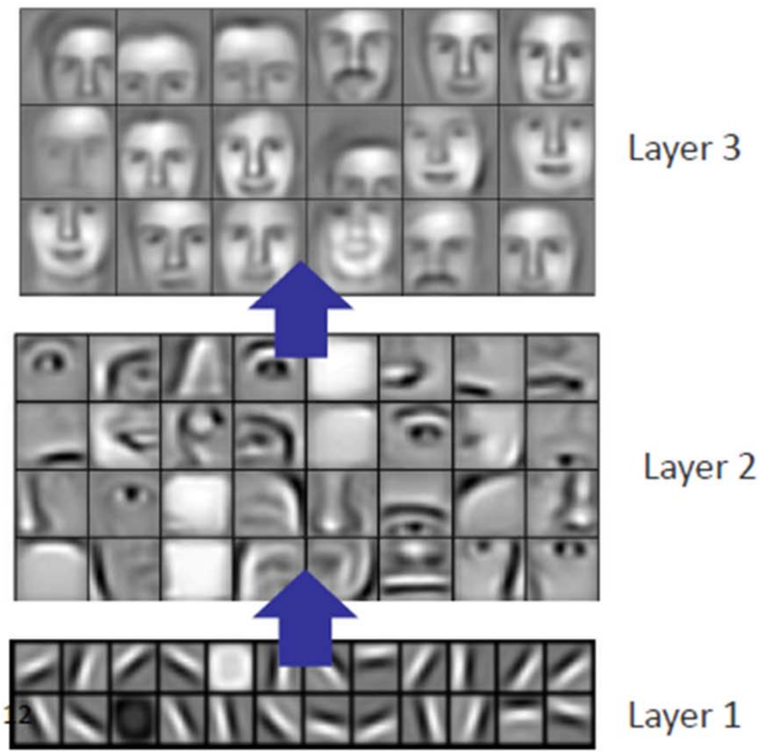
CLASSIFIER	DEFORMATION	PREPROCESSING	ERROR (%)	Reference
linear classifier (1-layer NN)		none	12.00	LeCun et al. 1998
linear classifier (1-layer NN)		deskewing	8.40	LeCun et al. 1998
pairwise linear classifier		deskewing	7.60	LeCun et al. 1998
K-nearest-neighbors, (L2)		none	3.09	Kenneth Wilder, U. Chicago
K-nearest-neighbors, (L2)		deskewing	2.40	LeCun et al. 1998
K-nearest-neighbors, (L2)		deskew, clean, blur	1.80	Kenneth Wilder, U. Chicago
K-NN L3, 2 pixel jitter		deskew, clean, blur	1.22	Kenneth Wilder, U. Chicago
K-NN, shape context matching		shape context feature	0.63	Belongie et al. IEEE PAMI 2002
40 PCA + quadratic classifier		none	3.30	LeCun et al. 1998
1000 RBF + linear classifier		none	3.60	LeCun et al. 1998
K-NN, Tangent Distance		subsamp 16x16 pixels	1.10	LeCun et al. 1998
SVM, Gaussian Kernel		none	1.40	
SVM deg 4 polynomial		deskewing	1.10	LeCun et al. 1998
Reduced Set SVM deg 5 poly		deskewing	1.00	LeCun et al. 1998
Virtual SVM deg-9 poly	Affine	none	0.80	LeCun et al. 1998
V-SVM, 2-pixel jittered		none	0.68	DeCoste and Scholkopf, MLJ2002
V-SVM, 2-pixel jittered		deskewing	0.56	DeCoste and Scholkopf, MLJ2002
2-layer NN, 300 HU, MSE		none	4.70	LeCun et al. 1998
2-layer NN, 300 HU, MSE,	Affine	none	3.60	LeCun et al. 1998
2-layer NN, 300 HU		deskewing	1.60	LeCun et al. 1998
3-layer NN, 500+ 150 HU		none	2.95	LeCun et al. 1998
3-layer NN, 500+ 150 HU	Affine	none	2.45	LeCun et al. 1998
3-layer NN, 500+ 300 HU, CE, reg		none	1.53	Hinton, unpublished, 2005
2-layer NN, 800 HU, CE		none	1.60	Simard et al., ICDAR 2003
2-layer NN, 800 HU, CE	Affine	none	1.10	Simard et al., ICDAR 2003
2-layer NN, 800 HU, MSE	Elastic	none	0.90	Simard et al., ICDAR 2003
2-layer NN, 800 HU, CE	Elastic	none	0.70	Simard et al., ICDAR 2003
Convolutional net LeNet-1		subsamp 16x16 pixels	1.70	LeCun et al. 1998
Convolutional net LeNet-4		none	1.10	LeCun et al. 1998
Convolutional net LeNet-5,		none	0.95	LeCun et al. 1998
Conv. net LeNet-5,	Affine	none	0.80	LeCun et al. 1998
Boosted LeNet-4	Affine	none	0.70	LeCun et al. 1998
Conv. net, CE	Affine	none	0.60	Simard et al., ICDAR 2003
Comv net, CE	Elastic	none	0.40	Simard et al., ICDAR 2003

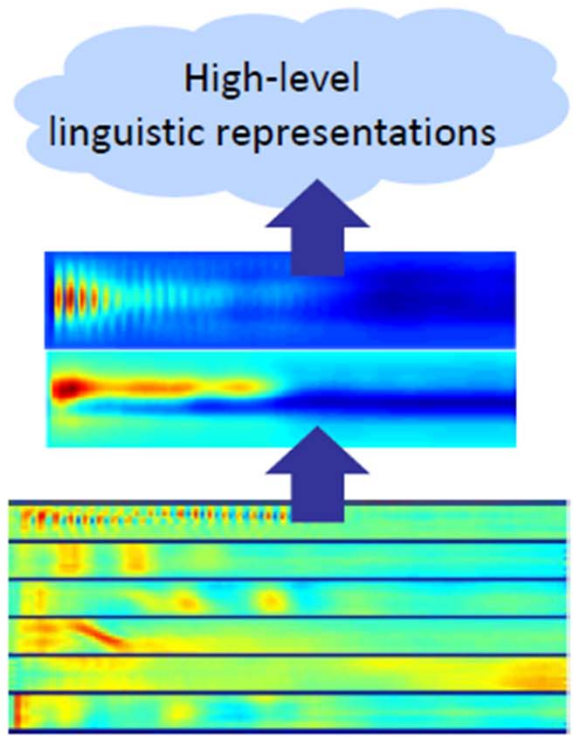
Face Detection with a Convolutional Net



Feature learning

- Successful learning of intermediate representations
  [Lee et al ICML 2009, Lee et al NIPS 2009]





Weaknesses & Criticisms

- Learning everything. Better to encode prior knowledge about structure of images.
- Not clear if an explicit global objective is indeed optimized, making theoretical analysis difficult
  - Many (arbitrary) approximations are introduced
  - Many different loss functions, gate functions, transformation functions are used
  - Many different implementation exist
- Comparison is based on the end empirical results on downstream task, not the actual direct task DNN is designed to compute, make verification and tuning of components of DNN very hard.
  - Imagine using "getting a good tip by the waiter" to evaluate the performance of chef in the kitchen