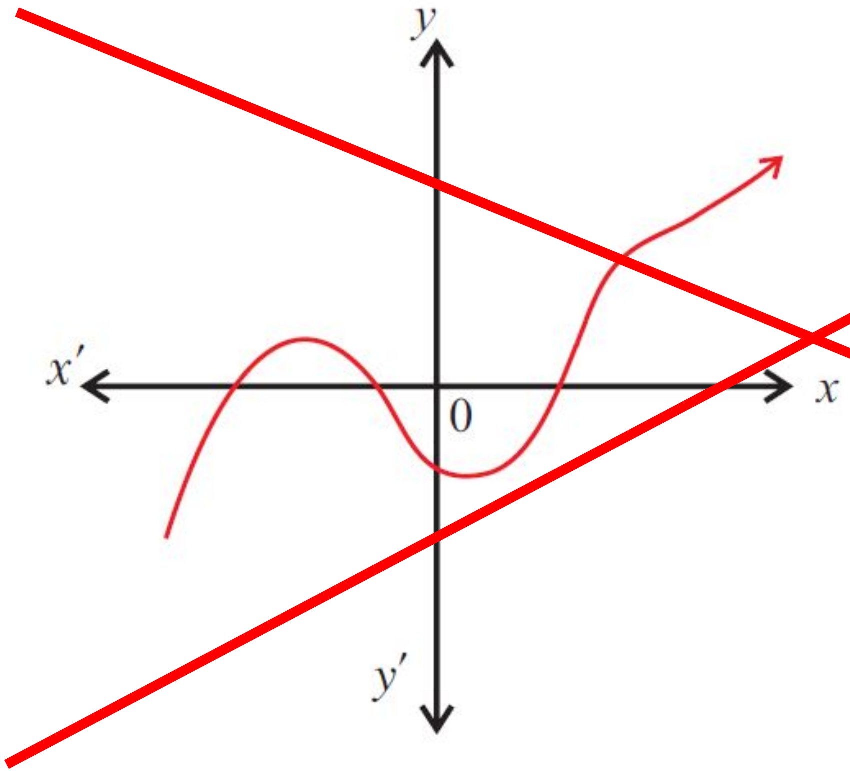
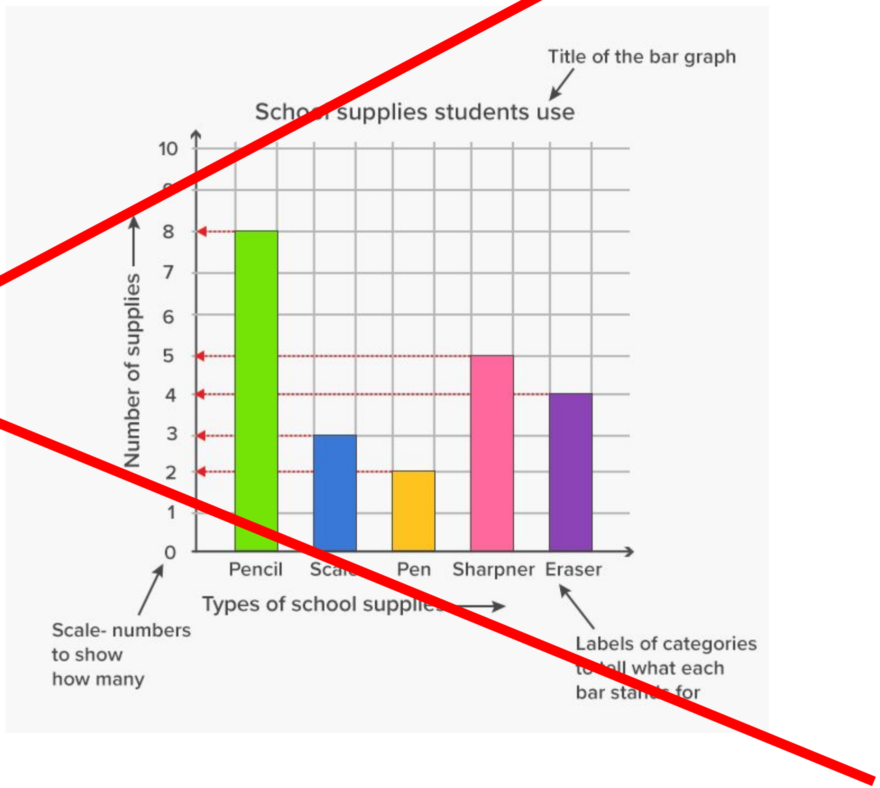
Graph Algorithms

July 24th, 2021 (Class #3)

What is a graph?





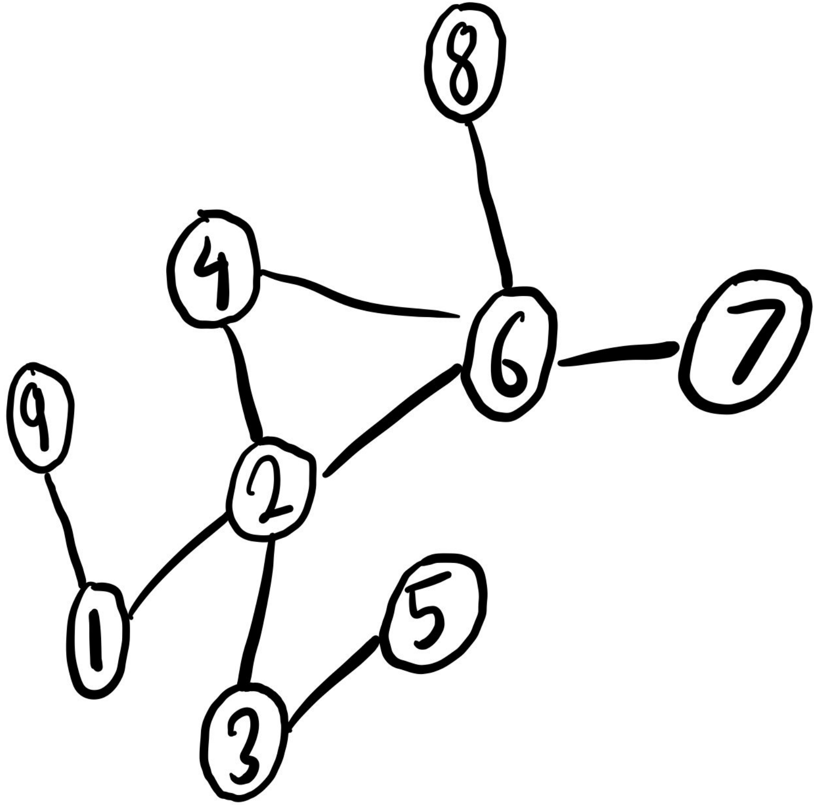
What actually are graphs?

A graph shows connections between objects.

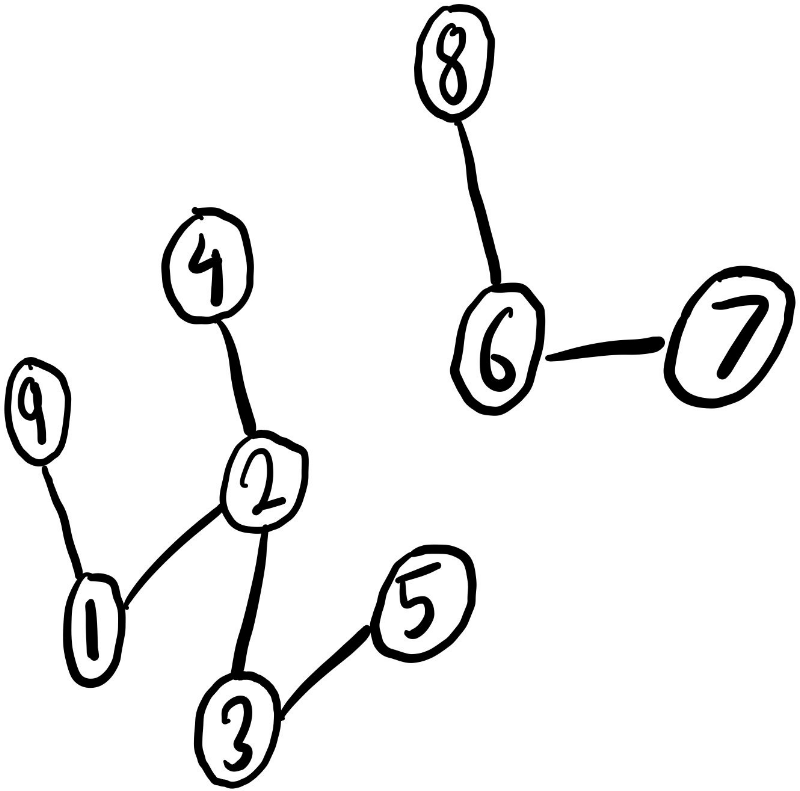
edges

nodes

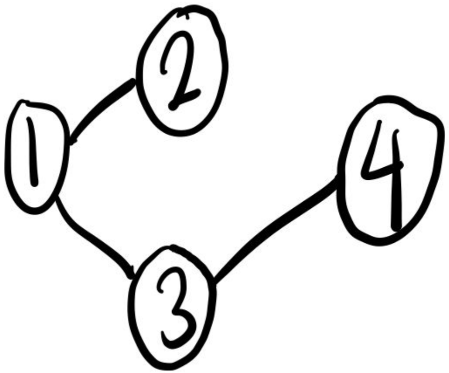
What are some interesting questions to ask about graphs?

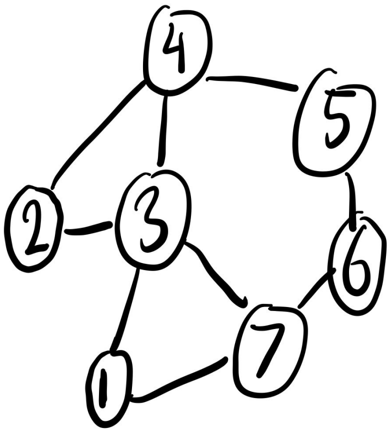


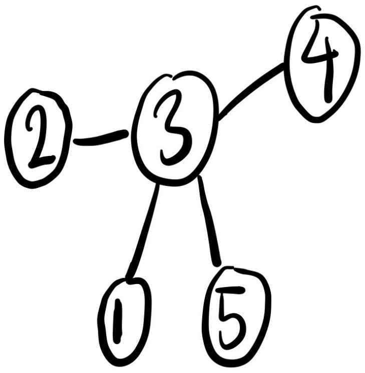
Are they connected?

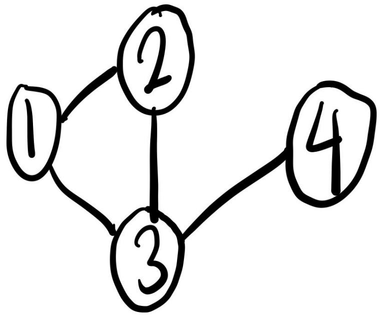


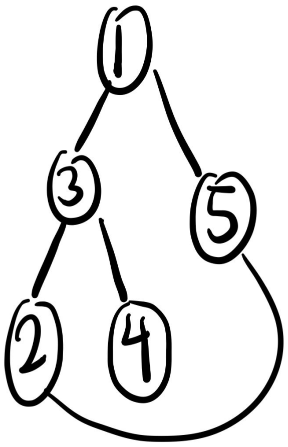
Do they have...

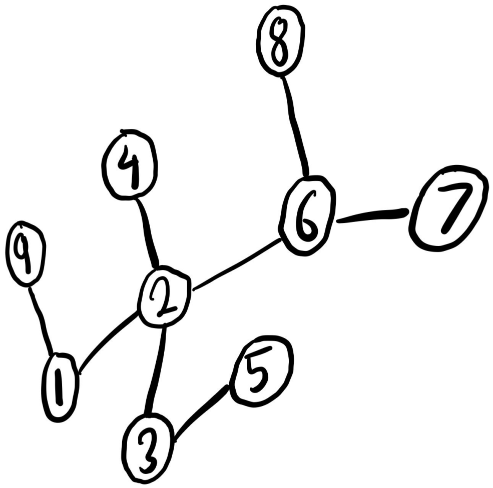


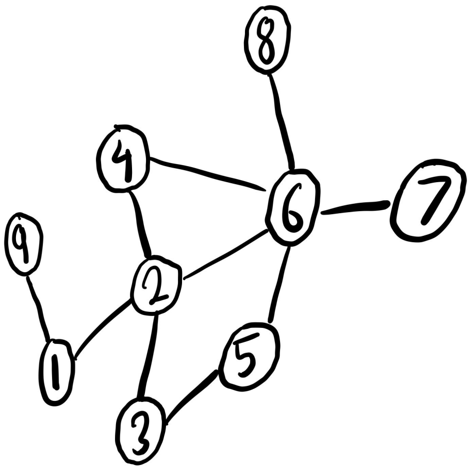




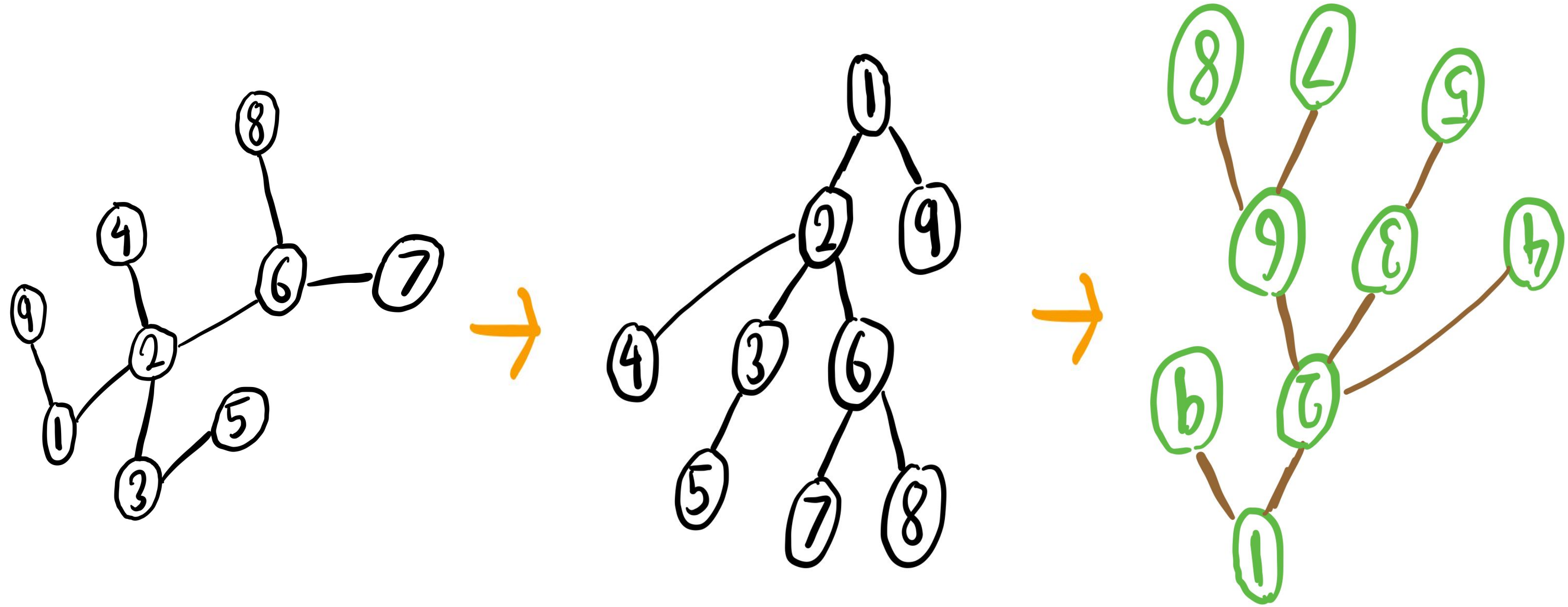




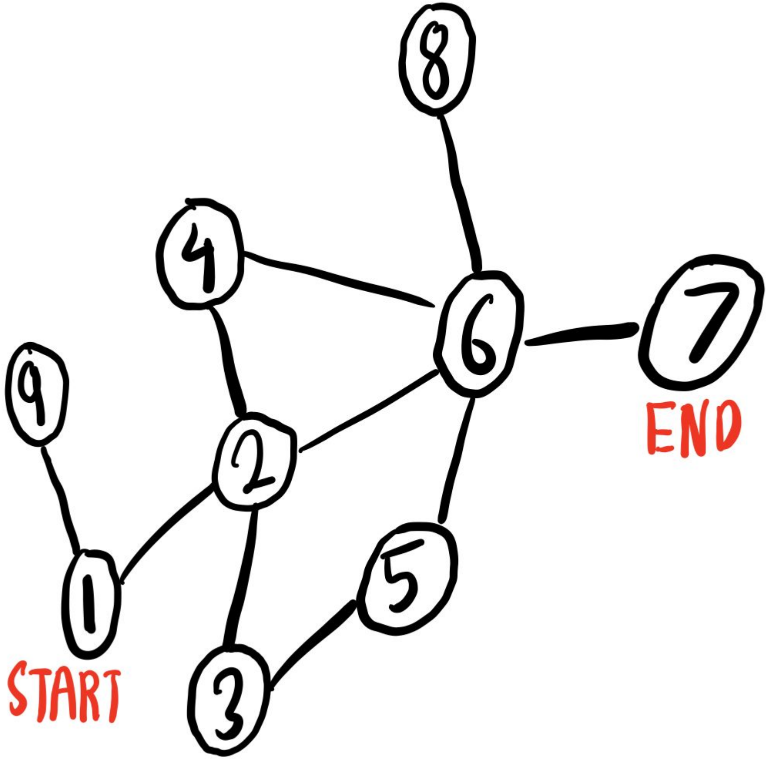




Trees…?



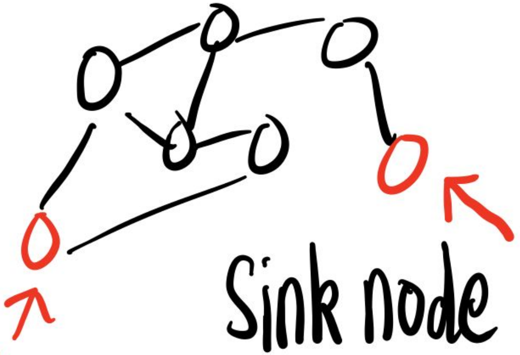
Distance between two nodes?



Shortest possible path

Shortest path in an unweighted graph

Input: A graph...



Start node s

Sink node t

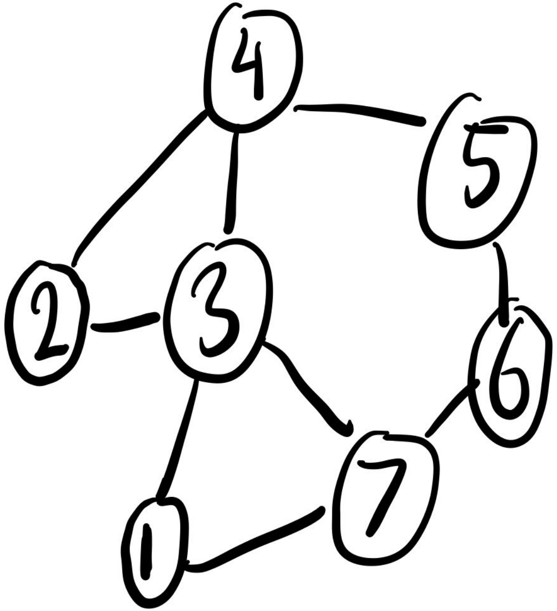
Output: Distance from s to t

(Unweighted) Graph Algorithms...

For **graph connectivity?**

For **cycle detection?**

For the **shortest path problem**?



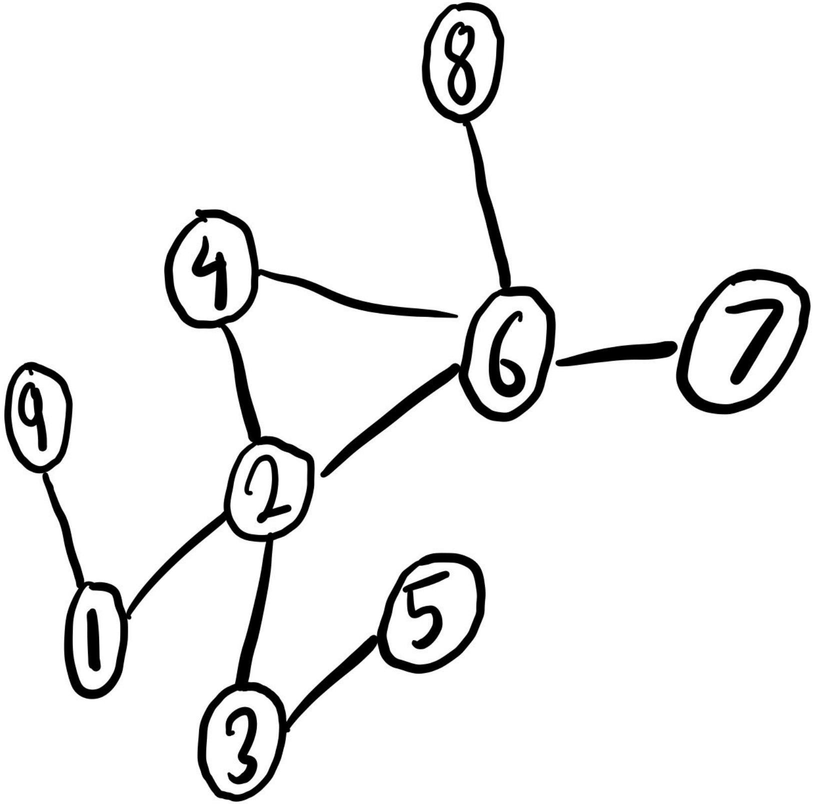
Break for 5 Minutes

Why do we need a “representation” of a graph?

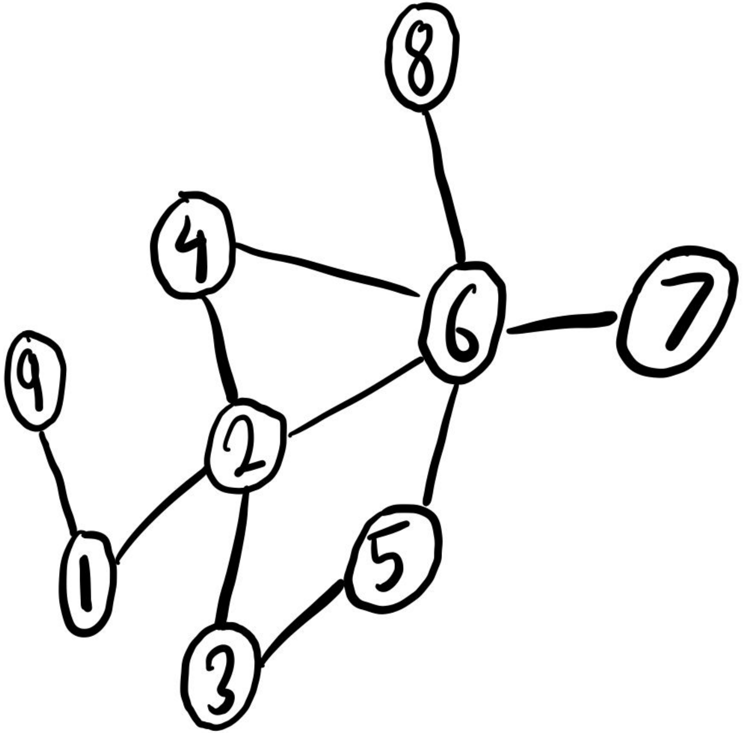
- Input has to be given in a standardized format.

- We need to know how to store the graph.

What’s a good representation?



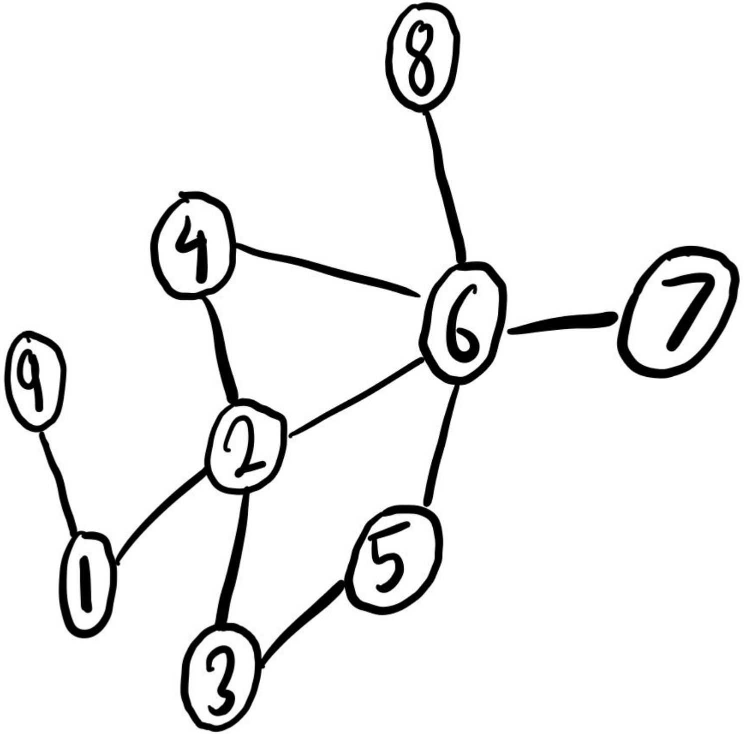
Edge List



Representation

1	9
6	8
4	6
5	3
1	2
5	6
6	2
7	6
3	2
2	4

Adjacency List



Representation

1: [2, 9]

2: [1, 4, 3]

3: [2, 5]

4: [6, 2]

5: [3, 6]

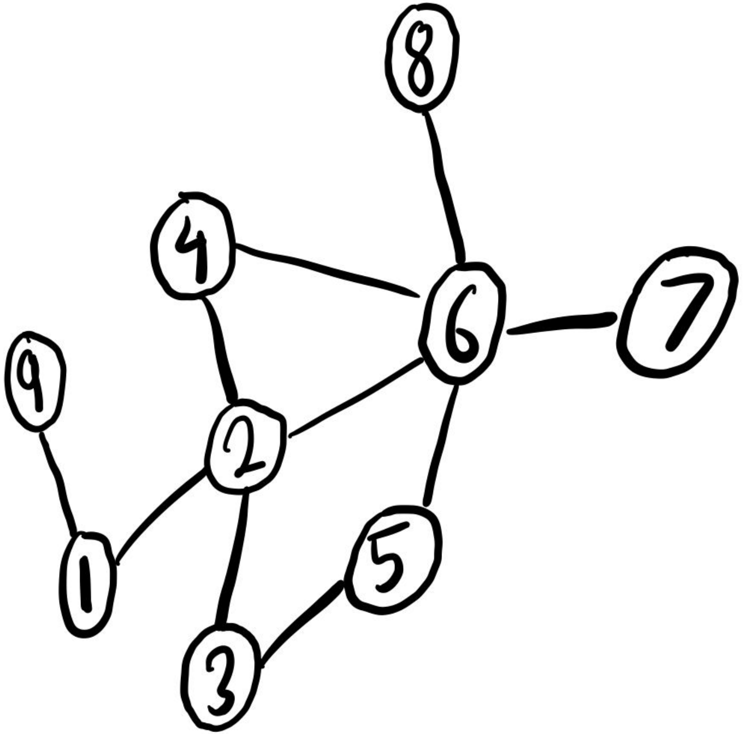
6: [5, 4, 7]

7: [6]

8: [6]

9: [1]

Adjacency Matrix



Representation

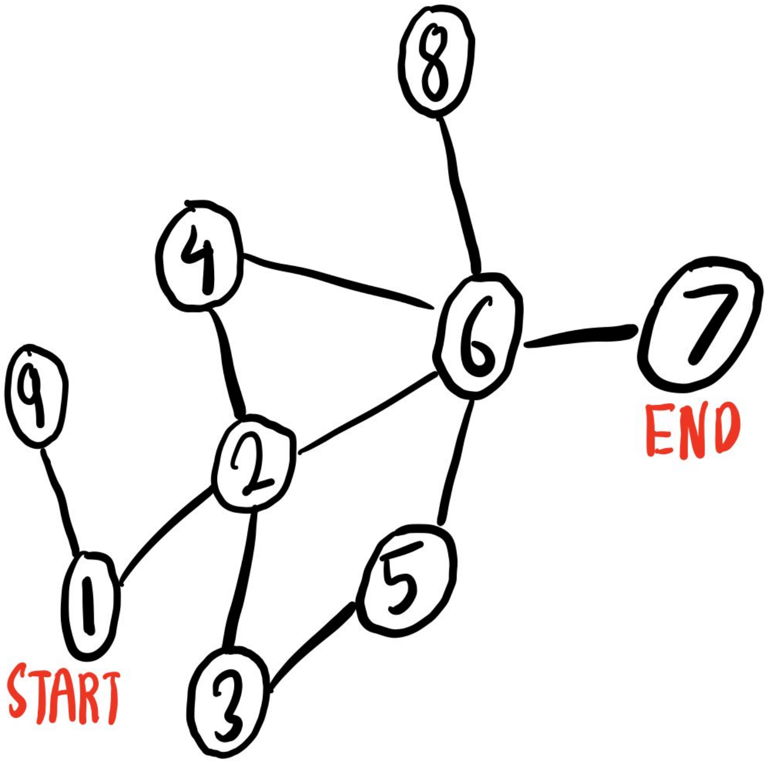
	1	2	3	4	5	6	7	8	9
1	0	1	0	0	0	0	0	0	1
2	1	0	1	1	0	0	0	0	0
3	0	1	0	0	1	0	0	0	0
4	0	1	0	0	0	1	0	0	0
5	0	0	1	0	0	1	0	0	0
6	0	0	0	1	1	0	1	0	0
7	0	0	0	0	0	1	0	0	0
8	0	0	0	0	0	1	0	0	0
9	1	0	0	0	0	0	0	0	0

Shortest Path in an Unweighted Graph

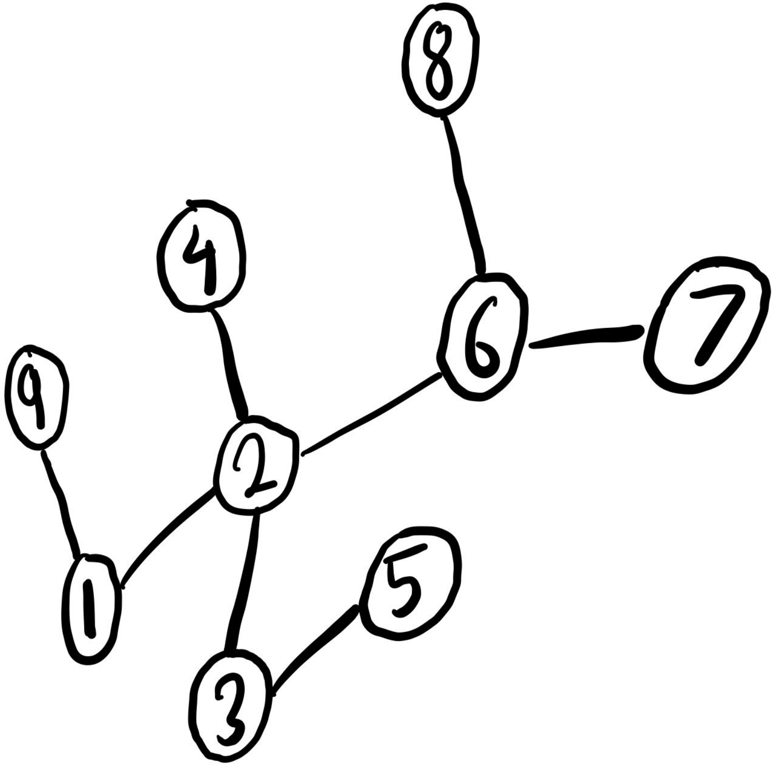
Input:
- A graph
- A start node s
- An end node t

Output: Distance from s to t

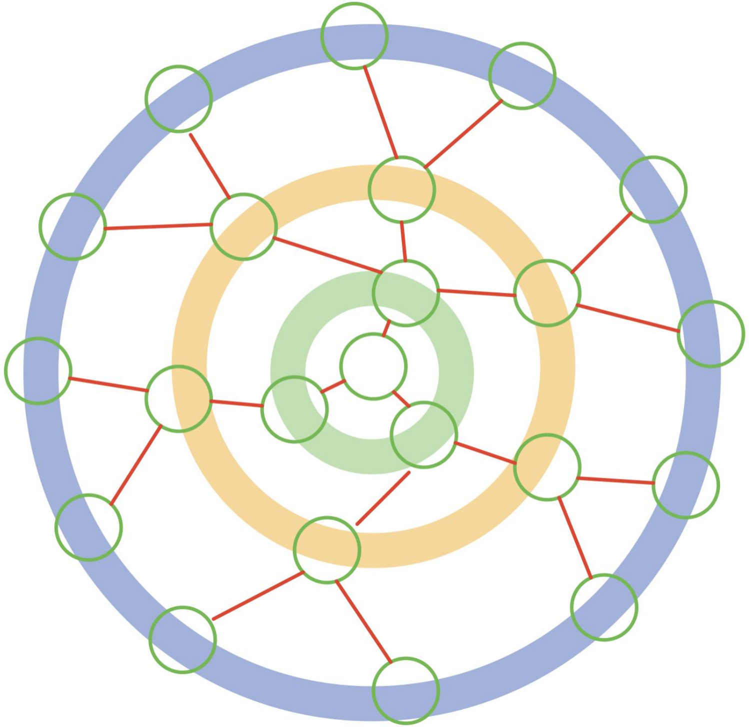
How do we solve shortest path? Is it hard?



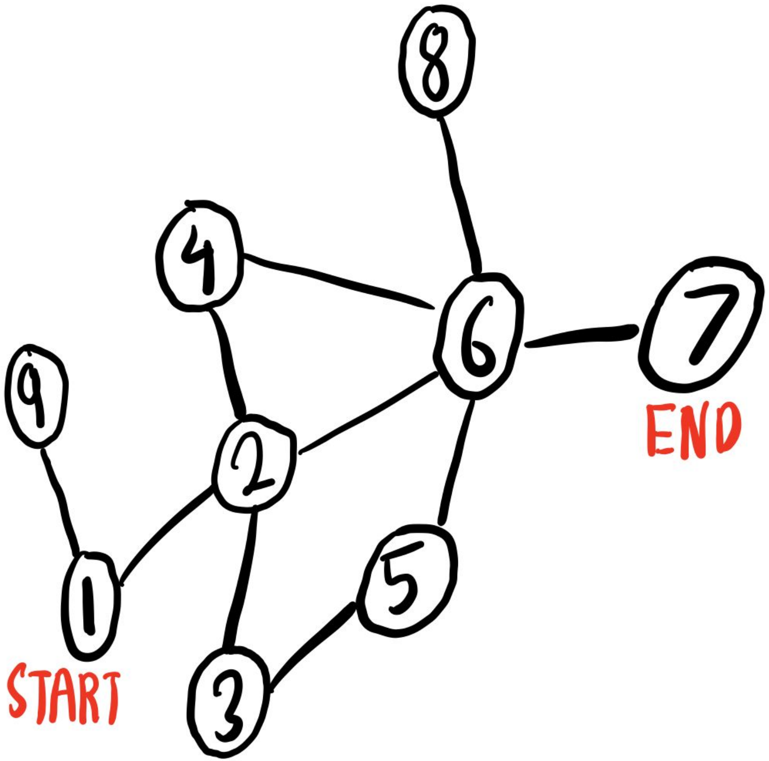
Is it easier on a tree?



Breadth First Search - Concept



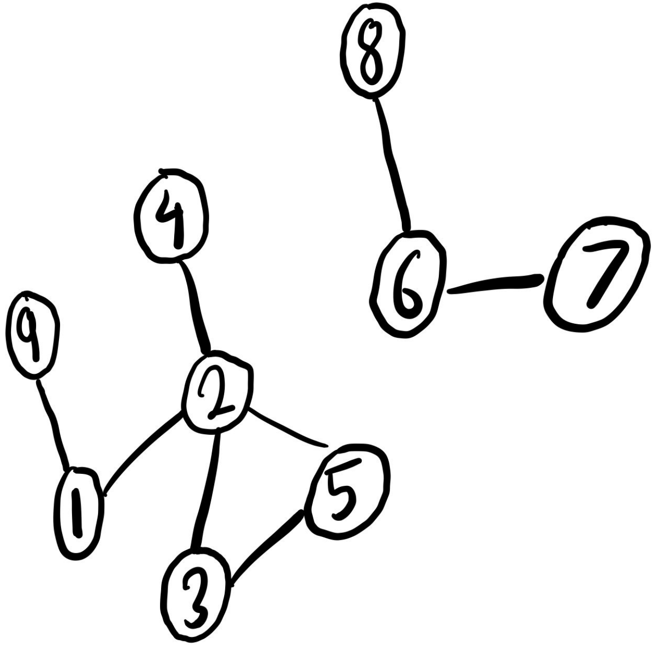
Breadth First Search - Walkthrough



What about the other two problems?

Just use any graph traversal algorithm.

- Start from any node
- Explore the other nodes by following edges.



Connectivity?

cycle detection?

Graph traversal algorithms

BFS: Expand in waves

DFS: Just as simple!
But out of scope :)

Can we solve all graph problems with DFS and BFS?

No! We can solve many, but...

What if the graph is weighted?

→ Lots more algorithms!

Longest path problem?

→ No fast algorithm!