DUE: 1/21/22 at 1:00 p.m. SHARP on Gradescope. No extensions.

NAME: ________________________ ANDREW ID: ________________________

WHAT SHOULD WE CALL YOU? ____

ANYTHING ELSE YOU WANT US TO KNOW? ________________________________

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This homework reviews the **prerequisites** for this class. Please solve all the problems below, and turn in your solutions via Gradescope. If you find that you've forgotten some topics, please take the time NOW to review these. Every single thing in this homework will come up repeatedly in the class, so please **commit these facts to memory**.

Feel free to get help from office hours or fellow classmates, but try to get help in the form of "hints" rather than "answers." You should cite anyone who helped you, and of course write up your own work.

Show your work. Feel free to use as many additional pages as needed to clearly show your work.

(1.1) Derive a closed-form expression for S where $S = 1 + x + x^2 + x^3 + \cdots + x^n$

(1.2) Derive: $S = 1 + x + x^2 + x^3 + \cdots, \quad \text{where } |x| < 1$

(1.3) Derive: $S = 1 + 2x + 3x^2 + 4x^3 + \cdots + nx^{n-1}$

(1.4) Derive: $S = 1 + 2x + 3x^2 + 4x^3 + \cdots, \quad \text{where } |x| < 1$

Please evaluate these integrals WITHOUT the aid of a calculator. Show all steps.

(2.1) Evaluate: $\int_0^\infty 5xe^{-5x}dx$

(2.2) Evaluate: $\int_0^{10}\int_0^{y} xdxdy$

Do this in 2 different ways. For each way, draw the region of integration:

(a) Without changing the order of integration

(b) By changing the order of integration

(2.3) Evaluate: $\int_1^{10}\int_1^{x^2} dydx$

Do this in 2 different ways. For each way, draw the region of integration:

(a) Without changing the order of integration

(b) By changing the order of integration

(3.1) What is this called: $\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n$? How can we interpret this expression (think about earning interest on money)?

(3.2) What is this: $\lim_{n\to\infty}\left(1+\frac{x}{n}\right)^n$? (assume x is a constant)

(3.3) Let $0 < x < 1$.

(a) What is bigger, $1 + x$ or e^x?

(b) What is bigger, $1 - x$ or e^{-x}?

[Hint: Think about the Taylor Series Expansion around 0.]

(3.4) Let $f(\epsilon) = \ln(1+\epsilon)$, where $0 < \epsilon \leq 1$. Write the Taylor series expansion of $f(\epsilon)$.

(3.5) What is a good approximation for:

(a) $S = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \cdots + \frac{1}{n}$
Give both an upper and lower bound. [Hint: Think of S as the area covered by certain rectangles. Then upper and lower bound this area via the area under a continuous curve which you can integrate.]

(b) $S = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \cdots$

(4.1) Let $\mathbf{A}$ be an $m \times n$ matrix. Let $\mathbf{B}$ be an $n \times p$ matrix. Let $\mathbf{C} = \mathbf{A} \cdot \mathbf{B}$
Let c_{ij} be the (i, j)th element of $\mathbf{C}$, and likewise for a_{ij} and b_{ij}.

Express c_{ij} as a sum of products.

(4.2) Baskin Robins has n flavors. You are building a cone with $k < n$ scoops.

- Let x = number of different cones that you can make if each flavor can only be used once, and the ordering of the flavors matters.
- Let y = number of different cones that you can make if each flavor can only be used once, and the ordering of the flavors does not matter.

Which is bigger, x or y? How much bigger?

(4.3) Simplify the following sums:

(a) $\binom{n}{0} + \binom{n}{1} + \binom{n}{2} + \cdots + \binom{n}{n}$

(b) $\binom{n}{0} + \binom{n}{1} x + \binom{n}{2} x^2 + \cdots + \binom{n}{n} x^n$

(c) $\binom{n}{0} y^n + \binom{n}{1} xy^{n-1} + \binom{n}{2} x^2 y^{n-2} + \cdots + \binom{n}{n} x^n$

(4.4) There are again n distinct flavors of ice-cream. Jin and Jong each pick 2 distinct flavors. Count the number of possibilities where Jin and Jong share exactly one flavor. In this problem, the order of the flavors does not matter.

An example of a possibility that "counts" is:

Jin gets { Chocolate, Vanila } & Jong gets { Chocolate, Strawberry }

(4.5) You are throwing n identical balls into n distinct (numbered) bins, where each ball is thrown into one bin.

(a) How many ways are there to distribute the n balls among the n bins?

(b) How many ways are there to distribute the n balls among the n bins such that bin 1 has $\geq k$ balls?