10-301/601 Machine Learning **Name:**
Summer 2023 **AndrewID:**
Final Practice Problems
July 27, 2023
Time Limit: N/A

Instructions:

- Fill in your name and Andrew ID above. Be sure to write neatly, or you may not receive credit for your exam.
- Clearly mark your answers in the allocated space **on the front of each page.** If needed, use the back of a page for scratch space, but you will not get credit for anything written on the back of a page. If you have made a mistake, cross out the invalid parts of your solution, and circle the ones which should be graded.
- No electronic devices may be used during the exam.
- Please write all answers in pen.
- You have N/A to complete the exam. Good luck!

Instructions for Specific Problem Types

For "Select One" questions, please fill in the appropriate bubble completely:

Select One: Who taught this course?

● Henry Chai

○ Marie Curie

○ Noam Chomsky

If you need to change your answer, you may cross out the previous answer and bubble in the new answer:

Select One: Who taught this course?

● Henry Chai

○ Marie Curie

~~●~~ Noam Chomsky

For "Select all that apply" questions, please fill in all appropriate squares completely:

Select all that apply: Which are scientists?

■ Stephen Hawking

■ Albert Einstein

■ Isaac Newton

□ I don't know

Again, if you need to change your answer, you may cross out the previous answer(s) and bubble in the new answer(s):

Select all that apply: Which are scientists?

■ Stephen Hawking

■ Albert Einstein

■ Isaac Newton

~~■~~ I don't know

For questions where you must fill in a blank, please make sure your final answer is fully included in the given space. You may cross out answers or parts of answers, but the final answer must still be within the given space.

Fill in the blank: What is the course number?

10-601	10-~~7~~601

1 Learning Theory

1. **True and Sample Errors:** Consider a classification problem with distribution D and target function $c^* : \mathcal{R}^d \mapsto \pm 1$. For any sample S drawn from D, answer whether the following statements are true or false, along with a brief explanation.

    1. **True or False:** For a given hypothesis space $\mathcal{H}$, it is always possible to define a sufficient number of examples in S such that the true error is within a margin of ϵ of the sample error for all hypotheses $h \in H$ with a given probability.

    2. **True or False:** The true error of any hypothesis h is an upper bound on its training error on the sample S.

2. Let X be the feature space and D be a distribution over X. We have a training data set

$$\mathcal{D} = \{(x_1, c^\star(x_1)), \cdots, ((x_N, c^\star(x_N)))\},$$

x_i i.i.d from D. We assume labels $c^\star(x_i) \in \{-1, 1\}$.

Let $\mathcal{H}$ be a hypothesis class and let $h \in \mathcal{H}$ be a hypothesis. In this question we restrict ourselves to $\mathcal{H}$. We use

$$err_S(h) = \frac{1}{N} \sum_{i=1}^{N} \mathbb{I}(h(x_i) \neq c^\star(x_i))$$

to denote the training error and

$$err_D(h) = P_{x \sim D}(h(x) \neq c^\star(x))$$

to denote the true error. Recall that if the concept class is finite, in the realizable case

$$m \geq \frac{1}{\epsilon}\left[\ln(|\mathcal{H}|) + \ln\left(\frac{1}{\delta}\right)\right]$$

labeled examples are sufficient so that with probability at least $1 - \delta$, all $h \in \mathcal{H}$ with $err_D(h) \geq \epsilon$ have $err_S(h) > 0$; in the agnostic case,

$$m \geq \frac{1}{2\epsilon^2}\left[\ln(|\mathcal{H}|) + \ln\left(\frac{2}{\delta}\right)\right]$$

labeled examples are sufficient such that with probability at least $1 - \delta$, all $h \in \mathcal{H}$ have $|err_D(h) - err_S(h)| < \epsilon$.

1. Briefly describe the difference between the realizable case and agnostic case.

2. What is the full name of PAC learning? How do ϵ and δ tie into the name?

3. **True or False:** Consider two finite hypothesis sets $\mathcal{H}_1$ and $\mathcal{H}_2$ such that $\mathcal{H}_1 \subset \mathcal{H}_2$. Let $h_1 = \arg\min_{h \in \mathcal{H}_1} err_S(h)$ and $h_2 = \arg\min_{h \in \mathcal{H}_2} err_S(h)$. Because $|\mathcal{H}_2| \geq |\mathcal{H}_1|$, $err_D(h_2) \geq err_D(h_1)$.

3. **Fill in the Blanks:** Complete the following sentence by circling one option in each square (options are separated by "/"s):

   In order to prove that the VC-dimension of a hypothesis set $\mathcal{H}$ is D, you must show that $\mathcal{H}$ $\boxed{\text{can / cannot}}$ shatter $\boxed{\text{any set / some set / multiple sets}}$ of D data points and $\boxed{\text{can / cannot}}$ shatter $\boxed{\text{any set / some set / multiple sets}}$ of $D+1$ data points.

4. Consider the hypothesis set $\mathcal{H}$ consisting of all positive intervals in $\mathbb{R}$, i.e. all hypotheses of the form $h(x; a, b) = \begin{cases} +1 & \text{if } x \in [a, b] \\ -1 & \text{if } x \notin [a, b] \end{cases}$

   1. **Short Answer:** In 1-2 sentences, briefly justify why the VC dimension of $\mathcal{H}$ is less than 3.

2. **Select one:** What is the VC dimension of $\mathcal{H}$?

   ◯ 0

   ◯ 1

   ◯ 2

3. **Numerical Answer:** Now, consider hypothesis sets $\mathcal{H}_k$ indexed by k, such that $\mathcal{H}_k$ consists of all hypotheses formed by k **non-overlapping** positive intervals in $\mathbb{R}$. Give an expression for the VC dimension of $\mathcal{H}_k$ in terms of k.

   Hint: Think about how to repeatedly apply the result you found in Part (b).

5. **Select one:** Your friend, who is taking an introductory ML course, is preparing to train a model for binary classification. Having just learned about PAC Learning, she informs you that the model is in the finite, agnostic case.

   Now she wants to know how changing certain values will change the number of labelled training data points required to satisfy the PAC criterion. For each of the following changes, determine whether the sample complexity will increase, decrease, or stay the same.

   1. Using a simpler model (decreasing $|\mathcal{H}|$)

      ◯ Sample complexity will increase

      ◯ Sample complexity will decrease

      ◯ Sample complexity will stay the same

   2. Choosing a new hypothesis set $\mathcal{H}^*$, such that $|\mathcal{H}^*| = |\mathcal{H}|$

      ◯ Sample complexity will increase

      ◯ Sample complexity will decrease

      ◯ Sample complexity will stay the same

   3. Decreasing δ

      ◯ Sample complexity will increase

      ◯ Sample complexity will decrease

      ◯ Sample complexity will stay the same

   4. Decreasing ϵ

      ◯ Sample complexity will increase

      ◯ Sample complexity will decrease

      ◯ Sample complexity will stay the same

6. **Errors, Errors Everywhere,** In this question we will consider the effect of training set size and regularization on the performance of a classifier. We will use err_D to denote the true error rate of the classifier and err_S to denote the error on the training set.

   Please provide a **one line justification** to your answer.

   1. **T or F**: PAC learning theory allows us to determine with 100% certainty the maximum possible true error of a classifier.

   2. Which one of the following is a guaranteed consequence of regularization in linear regression? (the error metric is mean square error)

      (a) Training error will increase or remain the same as the non-regularized model

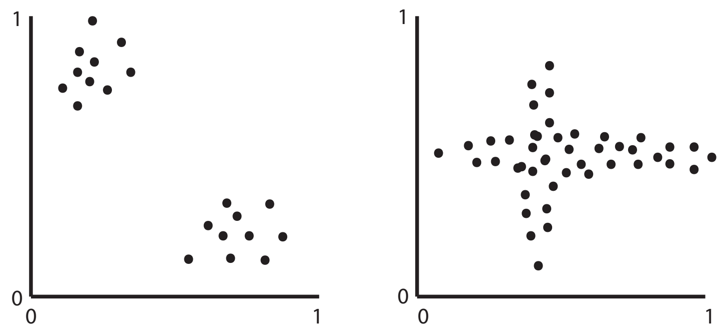
      (b) Training error will decrease or remain the same as the non-regularized model

      (c) True error will increase or remain the same as the non-regularized model

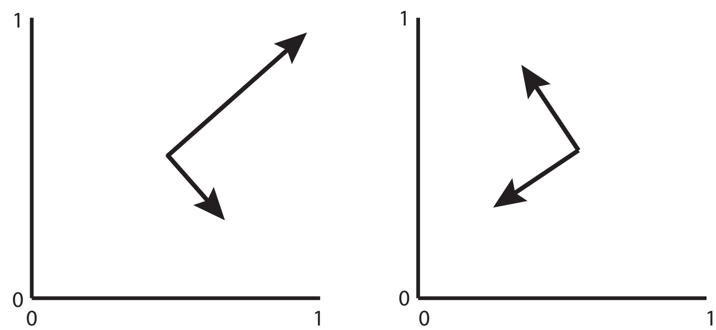
      (d) True error will decrease or remain the same as the non-regularized model

2 Principal Component Analysis

1. 1. Consider the following two plots of data. Draw arrows from the mean of the data to denote the direction and relative magnitudes of the principal components.



2. Now consider the following two plots, where we have drawn only the principal components. Draw the data ellipse or place data points that could yield the given principal components for each plot. Note that for the right hand plot, the principal components are of equal magnitude.



2. Circle one answer and explain.

In the following two questions, assume that using PCA we factorize $X \in \mathbb{R}^{n \times m}$ as $Z^T U \approx X$, for $Z \in \mathbb{R}^{m \times n}$ and $U \in \mathbb{R}^{m \times m}$, where the rows of X contain the data points, the rows of U are the prototypes/principal components, and $Z^T U = \hat{X}$.

1. Removing the last row of U and Z will still result in an approximation of X, but this will never be a better approximation than $\hat{X}$.

   ◯ True

   ◯ False

   Justify your answer:

2. $\hat{X}\hat{X}^T = Z^T Z$.

   ◯ True

   ◯ False

   Justify your answer:

3. The goal of PCA is to interpret the underlying structure of the data in terms of the principal components that are best at predicting the output variable.

   ◯ True

   ◯ False

   Justify your answer:

4. The output of PCA is a new representation of the data that is always of lower dimensionality than the original feature representation.

   ◯ True

   ◯ False

   Justify your answer:

3 K-Means

1. For **True or False** questions, circle your answer and justify it; for **QA** questions, write down your answer.

    1. For a particular dataset and a particular k, k-means will always produce the same result, if the initialized centers are the same. Assume there is no tie when assigning the clusters.

        ◯ True

        ◯ False

        Justify your answer:

    2. k-means can always converge to the global optimum.

        ◯ True

        ◯ False

        Justify your answer:

    3. k-means is not sensitive to outliers.

        ◯ True

        ◯ False

        Justify your answer:

    4. k in k-nearest neighbors and k-means have the same meaning.

        ◯ True

        ◯ False

        Justify your answer:

    5. What's the biggest difference between k-nearest neighbors and k-means?

        Write your answer in one sentence:

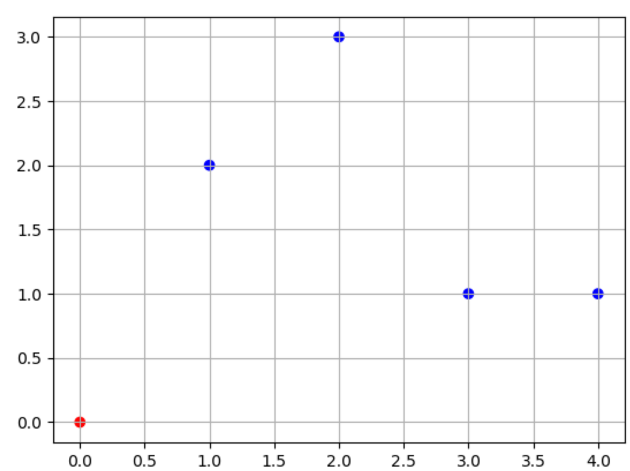
2. In k-means, random initialization could possibly lead to a local optimum with very bad performance. To alleviate this issue, instead of initializing all of the centers completely randomly, we decide to use a smarter initialization method. This leads us to k-means++.

   The only difference between k-means and k-means++ is the initialization strategy, and all of the other parts are the same. The basic idea of k-means++ is that instead of simply choosing the centers to be random points, we sample the initial centers iteratively, each time putting higher probability on points that are far from any existing center. Formally, the algorithm proceeds as follows.

   Given: Data set $x^{(i)}, i = 1, \dots, N$
   Initialize:
   $\mu^{(1)} \sim \text{Uniform}(\{x^{(i)}\}_{i=1}^N)$
   For $j = 2, \dots, k$
   Computing probabilities of selecting each point
   $$p_i = \frac{\min_{j'<j} \|\mu^{(j')} - x^{(i)}\|_2^2}{\sum_{i'=1}^N \min_{j'<j} \|\mu^{(j')} - x^{(i')}\|_2^2}$$
   Select next center given the appropriate probabilities
   $\mu^{(j)} \sim \text{Categorical}(\{x^{(i)}\}_{i=1}^N, \mathbf{p}_{1:N})$

   Note: n is the number of data points, k is the number of clusters. For cluster 1's center, you just randomly choose one data point. For the following centers, every time you initialize a new center, you will first compute the distance between a data point and the center closest to this data point. After computing the distances for all data points, perform a normalization and you will get the probability. Use this probability to sample for a new center.

   Now assume we have 5 data points (n=5): (0, 0), (1, 2), (2, 3), (3, 1), (4, 1). The number of clusters is 3 ($k = 3$). The center of cluster 1 is randomly chosen as (0, 0). These data points are shown in the figure below.



1. What is the probability of every data point being chosen as the center for cluster 2? (The answer should contain 5 probabilities, each for every data point)

2. Which data point is mostly liken chosen as the center for cluster 2?

3. Assume the center for cluster 2 is chosen to be the most likely one as you computed in the previous question. Now what is the probability of every data point being chosen as the center for cluster 3? (The answer should contain 5 probabilities, each for every data point)

4. Which data point is mostly liken chosen as the center for cluster 3?

5. Assume the center for cluster 3 is also chosen to be the most likely one as you computed in the previous question. Now we finish the initialization for all 3 centers. List the data points that are classified into cluster 1, 2, 3 respectively.

6. Based on the above clustering result, what's the new center for every cluster?

7. According to the result of (ii) and (iv), explain how does k-means++ alleviate the local optimum issue due to initialization?

3. Consider a dataset with seven points $\{x_1, \ldots, x_7\}$. Given below are the distances between all pairs of points.

	x_1	x_2	x_3	x_4	x_5	x_6	x_7
x_1	0	5	3	1	6	2	3
x_2	5	0	4	6	1	7	8
x_3	3	4	0	4	3	5	6
x_4	1	6	4	0	7	1	2
x_5	6	1	3	7	0	8	9
x_6	2	7	5	1	8	0	1
x_7	3	8	6	2	9	1	0

Assume that $k = 2$, and the cluster centers are initialized to x_3 and x_6. Which of the following shows the two clusters formed at the end of the first iteration of k-means? Circle the correct option.

(a) $\{x_1, x_2, x_3, x_4\}$, $\{x_5, x_6, x_7\}$

(b) $\{x_2, x_3, x_5\}$, $\{x_1, x_4, x_6, x_7\}$

(c) $\{x_1, x_2, x_3, x_5\}$, $\{x_4, x_6, x_7\}$

(d) $\{x_2, x_3, x_4, x_7\}$, $\{x_1, x_5, x_6\}$

4 Naive Bayes

1. Consider the following data. It has 4 features $\mathbf{X} = (x_1, x_2, x_3, x_4)$ and 3 labels $y \in \{+1, 0, -1\}$. Assume that the probabilities $p(\mathbf{X}|y)$ and $p(y)$ are both Bernoulli distributions. Answer the questions that follow under the Naive Bayes assumption.

x_1	x_2	x_3	x_4	y
1	1	0	1	+1
0	1	1	0	+1
1	0	1	1	0
0	1	1	1	0
0	1	0	0	-1
1	0	0	1	-1
0	0	1	1	-1

   1. Compute the Maximum Likelihood Estimates for $p(x_i = 1|y), \forall i \in \{1, 2, 3, 4\}$ and $\forall y \in \{+1, 0, -1\}$.

	$y = +1$	$y = 0$	$y = -1$
$x_1 = 1$			
$x_2 = 1$			
$x_3 = 1$			
$x_4 = 1$			

   2. Compute the Maximum Likelihood Estimates for the prior probabilities $p(y = +1), p(y = 0), p(y = -1)$.

   3. Use the values computed in the above two parts to classify the data point $(x_1 = 1, x_2 = 1, x_3 = 1, x_4 = 1)$ as belonging to class $+1, 0$ or -1.

2. You are given a dataset of 10,000 students with their sex, height, and hair color. You are trying to build a machine learning classifier to predict the sex of a student, so you randomly split the data into a training set and a testing set. Here are the specifications of the data set:

   - sex $\in$ {male,female}
   - height $\in$ [0,300] centimeters
   - hair $\in$ {brown, black, blond, red, green}
   - 3240 men in the data set
   - 6760 women in the data set

   Under only the assumptions necessary for Naïve Bayes (not the distributional assumptions you might naturally or intuitively make about the data set), answer True or False and provide a one sentence justification of your answer.

1. **True or False:** Height is a continuous valued variable. Therefore, Naïve Bayes is not appropriate since it cannot handle continuous valued variables.

2. **True or False:** Since there aren't similar numbers of men and women in the data set, Naïve Bayes will have high test error.

3. **True or False:** $p(\texttt{height}|\texttt{sex}, \texttt{hair}) = p(\texttt{height}|\texttt{sex})$.

4. **True or False:** $p(\texttt{height}, \texttt{hair}|\texttt{sex}) = p(\texttt{height}|\texttt{sex}) * p(\texttt{hair}|\texttt{sex})$.

5. Suppose you wish to learn $P(Y|X_1, X_2, X_3)$, where Y, X_1, X_2 and X_3 are all boolean-valued random variables. For the questions below, answer True or False and provide a one sentence justification for your answer.

   (a) **True or False:** In this case, a good choice for Naïve Bayes would be to implement a Gaussian Naïve Bayes classifier.

   (b) **True or False:** To learn $P(Y|X_1, X_2, X_3)$ using Naïve Bayes, you must make conditional independence assumptions, including the assumption that Y is conditionally independent of X_1 given X_2.

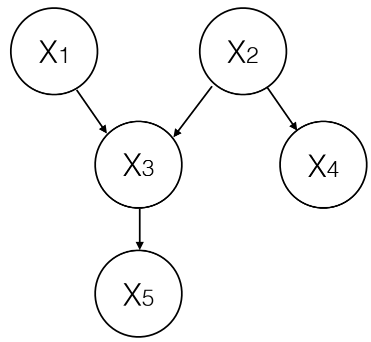
   (c) **True or False:** We can train Naïve Bayes using maximum likelihood estimates for each parameter, but not MAP estimates.

6. Suppose we add a numeric, real-valued variable X_4 to our problem. Note we now have a mix of some discrete-valued X_i and one continuous X_i. Explain why we can no longer use Naïve Bayes, or if we can, how we would modify our original solution.

5 Bayesian Networks

1. Consider the following Bayesian network.

    1. Determine whether the following conditional independencies are true.



(a) $X_1 \perp X_2 \mid X_3$?
Circle one: Yes No

(b) $X_1 \perp X_4$?
Circle one: Yes No

(c) $X_5 \perp X_2 \mid X_3$?
Circle one: Yes No

2. Write out the joint probability in a form that utilizes as many independence/conditional independence assumptions contained in the graph as possible. Answer: $P(X_1, X_2, X_3, X_4, X_5) =$

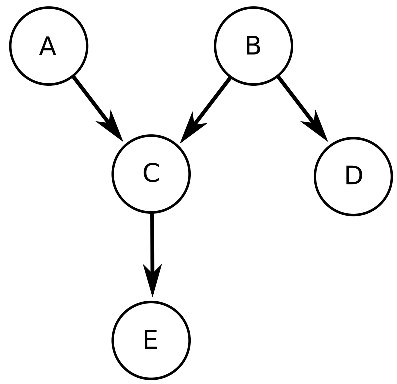
3. In a Bayesian network, if $X_1 \perp X_2$, then $X_1 \perp X_2 | Y$ for every node Y in the graph.

Circle one: True False

4. In a Bayesian network, if $X_1 \perp X_2 | Y$ for some node Y in the graph, it is always true that $X_1 \perp X_2$.

Circle one: True False

2. Consider the Bayesian network shown below for the following questions (a)-(f). Assume all variables are boolean-valued.



1. (Short answer) Write down the factorization of the joint probability $P(A, B, C, D, E)$ for the above graphical model, as a product of the five distributions associated with the five variables.
2. **True or False**: Is C conditionally independent of D given B (i.e. is $(C \perp D)|B$)?
3. **True or False**: Is A conditionally independent of D given C (i.e. is $(A \perp D)|C$)?
4. **True or False**: Is A independent of B (i.e. is $A \perp B$)?
5. Write an expression for $P(C = 1|A = 1, B = 0, D = 1, E = 0)$ in terms of the parameters of Conditional Probability Distributions associated with this graphical model.

6 Hidden Markov Models

1. Recall that both the Hidden Markov Model (HMM) can be used to model sequential data with local dependence structures. In this question, let Y_t be the hidden state at time t, X_t be the observation at time t, $\mathbf{Y}$ be all the hidden states, and $\mathbf{X}$ be all the observations.

   1. Draw the HMM as a Bayesian network where the observation sequence has length 3 (i.e., $t = 1, 2, 3$), labelling nodes with Y_1, Y_2, Y_3 and X_1, X_2, X_3.

   2. Write out the factorized joint distribution of $P(\mathbf{X}, \mathbf{Y})$ using the independencies/-conditional independencies assumed by the HMM graph, using terms Y_1, Y_2, Y_3 and X_1, X_2, X_3.
   $P(\mathbf{X}, \mathbf{Y}) =$

   3. True or False: In general, we should not include unobserved variables in a graphical model because we cannot learn anything useful about them without observations.
   True **False**

2. Consider an HMM with states $Y_t \in \{S_1, S_2, S_3\}$, observations $X_t \in \{A, B, C\}$ and parameters $\boldsymbol{\pi} = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$, transition matrix $\boldsymbol{B} = \begin{bmatrix} 1/2 & 1/4 & 1/4 \\ 0 & 1/2 & 1/2 \\ 0 & 0 & 1 \end{bmatrix}$, and emission matrix $\boldsymbol{A} = \begin{bmatrix} 1/2 & 1/2 & 0 \\ 1/2 & 0 & 1/2 \\ 0 & 1/2 & 1/2 \end{bmatrix}$.

   1. What is $P(Y_5 = S_3)$?

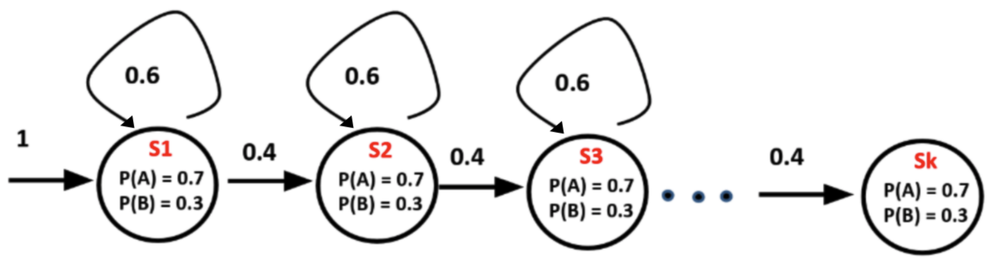
   2. What is $P(Y_5 = S_3 | X_{1:7} = AABCABC)$?

   3. Fill in the following table assuming the observation $AABCABC$. The α's are values obtained during the forward algorithm: $\alpha_t(i) = P(X_1, ..., X_t, Y_t = i)$.

t	$\alpha_t(1)$	$\alpha_t(2)$	$\alpha_t(3)$
1			
2			
3			
4			
5			
6			
7			

   4. Write down the sequence of $Y_{1:7}$ with the maximal posterior probability assuming the observation $AABCABC$. What is that posterior probability?

3. Consider the HMM in the figure below.



The HMM has k states $(s_1, ..., s_k)$. s_k is the terminal state. All states have the same emission probabilities (shown in the figure). The HMM always starts at s_1 as shown, and can move to either the next greater-number state or stay in the current state. Transition probabilities for all states except s_k are also the same as shown. More formally:

1. $P(Y_i = S_t \mid Y_{i-1} = S_{t-1}) = 0.4$
2. $P(Y_i = S_t \mid Y_{i-1} = S_t) = 0.6$
3. $P(Y_i = S_t \mid Y_{i-1} = S_j) = 0$ for all $j \in [k] \setminus \{t, t-1\}$

Once a run reaches s_k it outputs a symbol based on the s_k state emission probability and terminates.

1. Assume we observed the output AABAABBA from the HMM. Select all answers below that COULD be correct.

   ◯ $k > 8$

   ◯ $k < 8$

   ◯ $k > 6$

   ◯ $k < 6$

   ◯ $k = 7$

2. Now assume that $k = 4$. Let $P('AABA')$ be the probability of observing AABA from a full run of the HMM. For the following equations, fill in the box with $>, <, =$ or ? (? implies it is impossible to tell).

   (a) $P('AAB')$ $\boxed{}$ $P('BABA')$

   (b) $P('ABAB')$ $\boxed{}$ $P('BABA')$

   (c) $P('AAABA')$ $\boxed{}$ $P('BBAB')$

7 Reinforcement Learning

7.1 Markov Decision Process

Environment Setup (may contain spoilers for Shrek 1)

Lord Farquaad is hoping to evict all fairytale creatures from his kingdom of Duloc, and has one final ogre to evict: Shrek. Unfortunately all his previous attempts to catch the crafty ogre have fallen short, and he turns to you, with your knowledge of Markov Decision Processes (MDP's) to help him catch Shrek once and for all.

Consider the following MDP environment where the agent is Lord Farquaad:

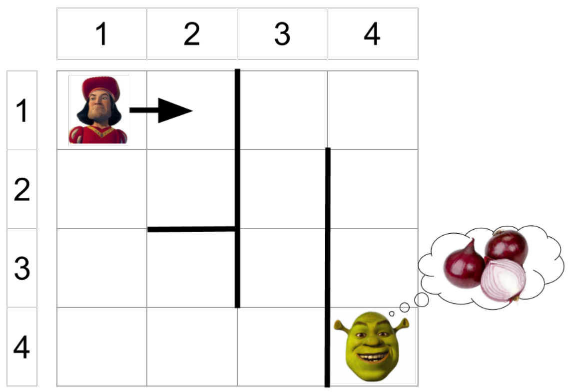


Figure 1: Kingdom of Duloc, circa 2001

Here's how we will define this MDP:

- S **(state space):** a set of states the agent can be in. In this case, the agent (Farquaad) can be in any location (row, col) and also in any orientation $\in \{N, E, S, W\}$. Therefore, state is represented by a three-tuple (row, col, dir), and $S =$ all possible of such tuples. Farquaad's start state is $(1, 1, E)$.
- A **(action space):** a set of actions that the agent can take. Here, we will have just three actions: turn right, turn left, and move forward (turning does not change row or col, just dir). So our action space is $\{R, L, M\}$. Note that Farquaad is debilitatingly short, so he cannot travel through (or over) the walls. Moving forward when facing a wall results in no change in state (but counts as an action).
- $R(s, a)$ **(reward function):** In this scenario, Farquaad gets a reward of 5 by moving into the swamp (the cell containing Shrek), and a reward of 0 otherwise.
- $p(s'|s, a)$ **(transition probabilities):** We'll use a deterministic environment, so this will bee 1 if s' is reachable from s and by taking a, and 0 if not.

1. What are $|S|$ and $|A|$ (size of state space and size of action space)?

2. Why is it called a "Markov" decision process? (Hint: what is the assumption made with p?)

3. What are the following transition probabilities?

$$p((1,1,N)|(1,1,N),M) =$$
$$p((1,1,N)|(1,1,E),L) =$$
$$p((2,1,S)|(1,1,S),M) =$$
$$p((2,1,E)|(1,1,S),M) =$$

4. Given a start position of $(1,1,E)$ and a discount factor of $\gamma = 0.5$, what is the expected discounted future reward from $a = R$? For $a = L$? (Fix $\gamma = 0.5$ for following problems).

5. What is the optimal action from each state, given that orientation is fixed at E? (if there are multiple options, choose any)

6. Farquaad's chief strategist (Vector from Despicable Me) suggests that having $\gamma = 0.9$ will result in a different set of optimal policies. Is he right? Why or why not?

7. Vector then suggests the following setup: $R(s,a) = 0$ when moving into the swamp, and $R(s,a) = -1$ otherwise. Will this result in a different set of optimal policies? Why or why not?

8. Vector now suggests the following setup: $R(s, a) = 5$ when moving into the swamp, and $R(s, a) = 0$ otherwise, but with $\gamma = 1$. Could this result in a different optimal policy? Why or why not?

9. Surprise! Elsa from Frozen suddenly shows up. Vector hypnotizes her and forces her to use her powers to turn the ground into ice. The environment is now stochastic: since the ground is now slippery, when choosing the action M, with a 0.2 chance, Farquaad will slip and move two squares instead of one. What is the expected future-discounted rewards from $s = (2, 4, S)$?

7.2 Value and Policy Iteration

1. **Select all that apply:** Which of the following environment characteristics would increase the computational complexity per iteration for a value iteration algorithm? Choose all that apply:

   ☐ Large Action Space

   ☐ A Stochastic Transition Function

   ☐ Large State Space

   ☐ Unknown Reward Function

   ☐ None of the Above

2. **Select all that apply:** Which of the following environment characteristics would increase the computational complexity per iteration for a policy iteration algorithm? Choose all that apply:

   ☐ Large Action Space

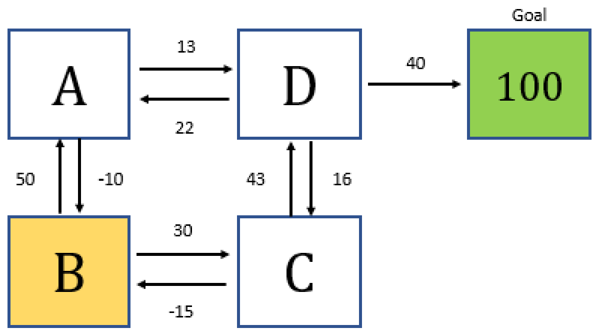
   ☐ A Stochastic Transition Function

   ☐ Large State Space

   ☐ Unknown Reward Function

   ☐ None of the Above

3. In the image below is a representation of the game that you are about to play. There are 5 states: A, B, C, D, and the goal state. The goal state, when reached, gives 100 points as reward (that is, you can assume $R(\texttt{D}, \texttt{right}) = 140$). In addition to the goal's points, you also get points by moving to different states. The amount of points you get are shown next to the arrows. You start at state B. To figure out the best policy, you use asynchronous value iteration with a decay (γ) of 0.9. You should initialize the value of each state to 0.



(a) When you first start playing the game, what action would you take (up, down, left, right) at state B?

(b) What is the total reward at state B at this time?

(c) Let's say you keep playing until your total values for each state has converged. What action would you take at state B?

(d) What is the total reward at state B at this time?

4. **Select one:** Let $V_k(s)$ indicate the value of state s at iteration k in (synchronous) value iteration. What is the relationship between $V_{k+1}(s)$ and $\sum_{s' \in S} P(s'|s,a)[R(s,a,s') + \gamma V_k(s')]$, for any $a \in A$? Indicate the most restrictive relationship that applies. For example, if $x < y$ always holds, use $<$ instead of $\leq$. Selecting ? means it's not possible to assign any true relationship. Assume $R(s,a,s') \geq 0 \ \forall s, s' \in S,\ a \in A$.

   $V_{k+1}(s) \ \square \ \sum_{s'} P(s'|s,a)[R(s,a,s') + \gamma V_k(s')]$

   ◯ $=$

   ◯ $<$

   ◯ $>$

   ◯ $\leq$

   ◯ $\geq$

   ◯ ?

7.3 Q-Learning

1. For the following true/false, circle one answer and provide a one-sentence explanation:

   (a) One advantage that Q-learning has over Value and Policy iteration is that it can account for non-deterministic policies.

   Circle one: True False

   (b) You can apply Value or Policy iteration to any problem that Q-learning can be applied to.

   Circle one: True False

   (c) Q-learning is guaranteed to converge to the true value Q* for a greedy policy.

   Circle one: True False

2. For the following parts of this problem, recall that the update rule for Q-learning is:

$$\mathbf{w} \leftarrow \mathbf{w} - \alpha \left(q(\mathbf{s}, a; \mathbf{w}) - (r + \gamma \max_{a'} q(\mathbf{s}', a'; \mathbf{w}) \right) \nabla_{\mathbf{w}} q(\mathbf{s}, a; \mathbf{w})$$

   (a) From the update rule, let's look at the specific term $X = (r + \gamma \max_{a'} q(\mathbf{s}', a'; \mathbf{w}))$ Describe in English what is the role of X in the weight update.

   (b) Is this update rule synchronous or asynchronous?

   (c) A common adaptation to Q-learning is to incorporate rewards from more time steps into the term X. Thus, our normal term $r_t + \gamma * max_{a_{t+1}} q(s_{t+1}, a_{t+1}; w)$ would become $r_t + \gamma * r_{t+1} + \gamma^2 \max_{a_{t+2}} q(\mathbf{s}_{t+2}, a_{t+2} : \mathbf{w})$ What are the advantages of using more rewards in this estimation?

3. **Select one:** Let $Q(s, a)$ indicate the estimated Q-value of state-action pair $(s, a) \in |S| \times |A|$ at some point during Q-learning. Suppose you receive reward r after taking action a at state s and arrive at state s'. Before updating the Q values based on this experience, what is the relationship between $Q(s, a)$ and $r + \gamma \max_{a' \in A} Q(s', a')$? Indicate the most restrictive relationship that applies. For example, if $x < y$ always holds, use $<$ instead of $\leq$. Selecting ? means it's not possible to assign any true relationship.

   $Q(s, a) \square r + \gamma \max_{a'} Q(s', a')$

   ○ $=$

   ○ $<$

   ○ $>$

   ○ $\leq$

   ○ $\geq$

   ○ ?

4. During standard (not deep) Q-learning, you get reward r after taking action $North$ from state A and arriving at state B. You compute the sample $r + \gamma Q(B, South)$, where $South = \arg\max_a Q(B, a)$.

   Which of the following Q-values are updated during this step? (Select all that apply)

   ○ Q(A, North)

   ○ Q(A, South)

   ○ Q(B, North)

   ○ Q(B, South)

   ○ None of the above

5. In general, for Q-Learning (standard/tabular Q-learning, not approximate Q-learning) to converge to the optimal Q-values, which of the following are true?

   (a) **True or False:** It is necessary that every state-action pair is visited infinitely often.

   ◯ True

   ◯ False

   (b) **True or False:** It is necessary that the discount γ is less than 0.5.

   ◯ True

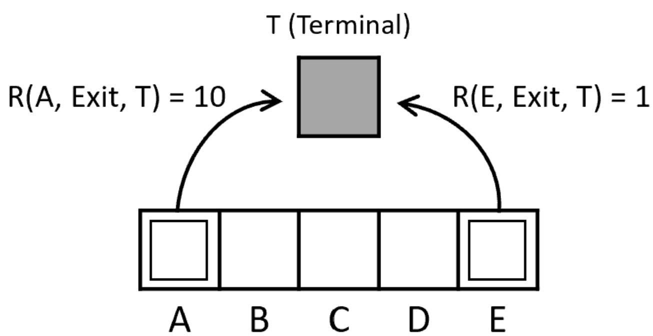
   ◯ False

   (c) **True or False:** It is necessary that actions get chosen according to $\arg\max_a Q(s,a)$.

   ◯ True

   ◯ False

6. Consider training a robot to navigate the following grid-based MDP environment.



- There are six states, A, B, C, D, E, and a terminal state T.
- Actions from states B, C, and D are Left and Right.
- The only action from states A and E is Exit, which leads deterministically to the terminal state

The reward function is as follows:

- $R(A, Exit, T) = 10$
- $R(E, Exit, T) = 1$
- The reward for any other tuple (s, a, s') equals -1

Assume the discount factor is 1. When taking action Left, with probability 0.8, the robot will successfully move one space to the left, and with probability 0.2, the robot will move one space in the opposite direction. When taking action Right, with probability 0.8, the robot will successfully move one space to the right, and with probability 0.2, the robot will move one space in the opposite direction. Run synchronous value iteration on this environment for two iterations. Begin by initializing the value of all states to zero.

Write the value of each state after the first ($k = 1$) and the second ($k = 2$) iterations. Write your values as a comma-separated list of 6 numerical expressions in the alphabetical order of the states, specifically $V(A), V(B), V(C), V(D), V(E), V(T)$. Each of the six entries may be a number or an expression that evaluates to a number. Do not include any max operations in your response.

(a) $V_1(A), V_1(B), V_1(C), V_1(D), V_1(E), V_1(T)$ (Values for 6 states):

(b) $V_2(A), V_2(B), V_2(C), V_2(D), V_2(E), V_2(T)$ (values for 6 states):

(c) What is the resulting policy after this second iteration? Write your answer as a comma-separated list of three actions representing the policy for states, B, C, and D, in that order. Actions may be Left or Right.

$\pi(B), \pi(C), \pi(D)$ based on V_2 :

8 Ensemble Methods

1. Consider a random forest ensemble consisting of 5 decision trees DT1, DT2 ... DT5 that has been trained on a dataset consisting of 7 samples. Each tree has been trained on a random subset of the dataset. The following table represents the predictions of each tree on its out-of-bag samples.

Tree	Sample Number	Prediction	Actual
DT1	6	No	Yes
DT1	7	No	Yes
DT2	2	No	No
DT3	1	No	No
DT3	2	Yes	No
DT3	4	Yes	Yes
DT4	2	Yes	No
DT4	7	No	Yes
DT5	3	Yes	Yes
DT5	5	No	No

1. What is the OOB error of the above random forest classifier?

2. In the above random forest classifier, which Decision tree(s) will be given the highest weight in inference? If there are multiple trees, mention them all

3. To reduce the error of each individual decision tree, Neural uses all the features to train each tree. How would this impact the generalisation error of the random forest?

   ○ The generalisation error would decrease as each tree has lower generalisation error

   ○ The generalisation error would increase as each tree has insufficient training data

   ○ The generalisation error would increase as the trees are highly correlated

2. In the AdaBoost algorithm, if the final hypothesis makes no mistakes on the training data, which of the following is correct?

   Select all that apply:

   ☐ Additional rounds of training can help reduce the errors made on unseen data.

   ☐ Additional rounds of training have no impact on unseen data.

   ☐ The individual weak learners also make zero error on the training data.

   ☐ Additional rounds of training always leads to worse performance on unseen data.

3. **True or False:** In AdaBoost weights of the misclassified examples go up by the same multiplicative factor.

   ◯ True

   ◯ False

Round	$D_t(A)$	$D_t(B)$	$D_t(C)$	$D_t(D)$	$D_t(E)$	$D_t(F)$
1	?	?	$\frac{1}{6}$	?	?	?
2	?	?	?	?	?	?
...						
219	?	?	?	?	?	?
220	$\frac{1}{14}$	$\frac{1}{14}$	$\frac{7}{14}$	$\frac{1}{14}$	$\frac{2}{14}$	$\frac{2}{14}$
221	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{7}{20}$	$\frac{1}{20}$	$\frac{1}{4}$	$\frac{1}{10}$
...						
3017	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{16}$	$\frac{1}{16}$	0
...						
8888	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{1}{8}$	$\frac{2}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

4. In the last semester, someone used AdaBoost to train some data and recorded all the weights throughout iterations but some entries in the table are not recognizable. Clever as you are, you decide to employ your knowledge of Adaboost to determine some of the missing information.

   Below, you can see part of table that was used in the problem set. There are columns for the Round # and for the weights of the six training points (A, B, C, D, E, and F) at the start of each round. Some of the entries, marked with "?", are impossible for you to read.

   In the following problems, you may assume that non-consecutive rows are independent of each other, and that a classifier with error less than $\frac{1}{2}$ was chosen at each step.

   1. The weak classifier chosen in Round 1 correctly classified training points A, B, C, and E but misclassified training points D and F. What should the updated weights have been in the following round, Round 2? Please complete the form below.

Round	$D_2(A)$	$D_2(B)$	$D_2(C)$	$D_2(D)$	$D_2(E)$	$D_2(F)$
2						

   2. During Round 219, which of the training points (A, B, C, D, E, F) must have been misclassified, in order to produce the updated weights shown at the start of Round 220? List all the points that were misclassified. If none were misclassified, write 'None'. If it can't be decided, write 'Not Sure' instead.

   3. You observes that the weights in round 3017 or 8888 (or both) cannot possibly be right. Which one is incorrect? Why? Please explain in one or two short sentences.

      ○ Round 3017 is incorrect.

      ○ Round 8888 is incorrect.

      ○ Both rounds 3017 and 8888 are incorrect.

5. What condition must a weak learner satisfy in order for boosting to work?
   Short answer:

6. After an iteration of training, AdaBoost more heavily weights which data points to train the next weak learner? (Provide an intuitive answer with no math symbols.)
   Short answer:

7. **Extra credit** Do you think that a deep neural network is nothing but a case of boosting? Why or why not? Impress us.
   Answer: