10-701: Introduction to Machine Learning Lecture 2 – Decision Trees

Henry Chai

1/22/24

Front Matter

- Announcements:
  - HW1 will be released on Wednesday (1/24), due 2/2 at 11:59 PM
  - Recitations will be held on Fridays, at the same time and place as lecture
    - HW1 recitation this Friday (1/26)
  - Office hours will start this Wednesday (1/24)
- Recommended Readings:
  - Mitchell, Chapter 3: Decision Tree Learning
  - Daumé III, Chapter 1: Decision Trees

Schedule

Lectures are the primary mode of content delivery in this course. Attending lectures is highly recommended; there will be regular in-class activities and polls which will constitute a small portion of your final grade. Engaging in these real-time activities can greatly improve your understanding of the material. Lectures will be recorded and made available to you after the fact. However, the primary purpose of these recordings is to allow you to refer back to the content; watching recordings in lieu of attending lectures is not encouraged.

Date	Topic	Slides	Readings/Resources
Wed, 1/17	Introduction: Notation & Problem Formulation	Lecture 1 (Inked)	
Mon, 1/22	Decision Trees	Lecture 2 (Pre-class)	Mitchell, Chapter 3 Daumé III, Chapter 1
Wed, 1/24	KNNs & Model Selection		
Mon, 1/29	Linear Regression		
Wed, 1/31	MLE/MAP		
Mon, 2/5	Naïve Bayes		

Lecture Schedule

Recall: Our second Machine Learning Classifier

- Alright, let's actually (try to) extract a pattern from the data

x_1 Family History	x_2 Resting Blood Pressure	x_3 Cholesterol	y Heart Disease?
Yes	Low	Normal	No
No	Medium	Normal	No
No	Low	Abnormal	Yes
Yes	Medium	Normal	Yes
Yes	High	Abnormal	Yes

- Decision stump on x_1:

$$h(\boldsymbol{x}') = h(x_1', \ldots, x_D') = \begin{cases} \text{“Yes” if } x_1' = \text{“Yes”} \\ \text{“No” otherwise} \end{cases}$$

Recall: Our second Machine Learning Classifier

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x_1 Family History	x_2 Resting Blood Pressure	x_3 Cholesterol	y Heart Disease?
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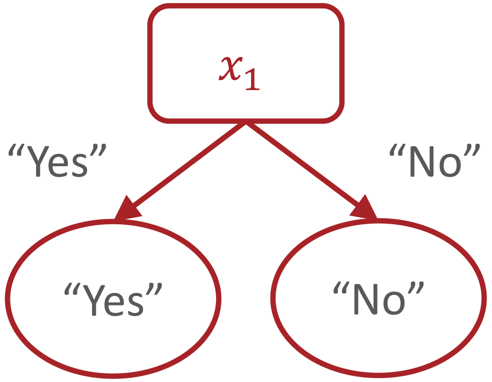


Decision Stumps: Questions

1. How can we pick which feature to split on?

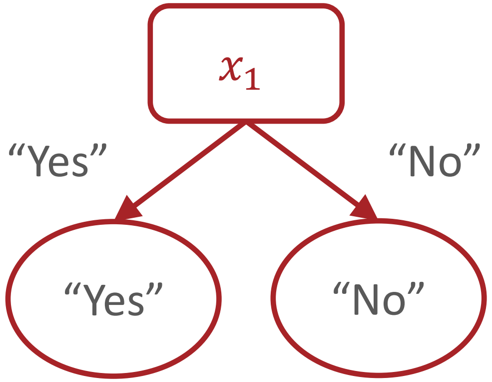
2. Why stop at just one feature?

Splitting Criterion

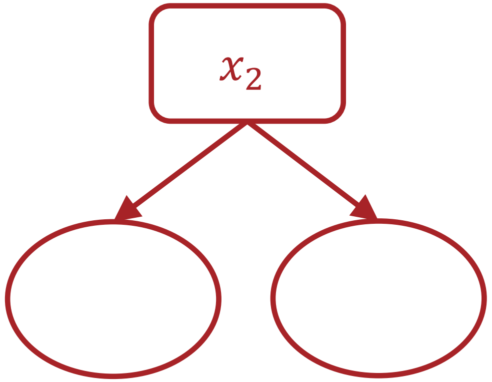
- A **splitting criterion** is a function that measures how good or useful splitting on a particular feature is *for a specified dataset*
- Idea: use the feature that optimizes the splitting criterion for our decision stump.

Training error rate as a Splitting Criterion

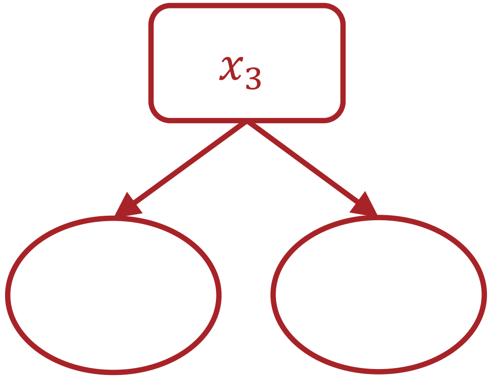
x_1 Family History	x_2 Resting Blood Pressure	x_3 Cholesterol	y Heart Disease?
Yes	Low	Normal	No
No	Medium	Normal	No
No	Low	Abnormal	Yes
Yes	Medium	Normal	Yes
Yes	High	Abnormal	Yes



Training error rate: 2/5



Training error rate:

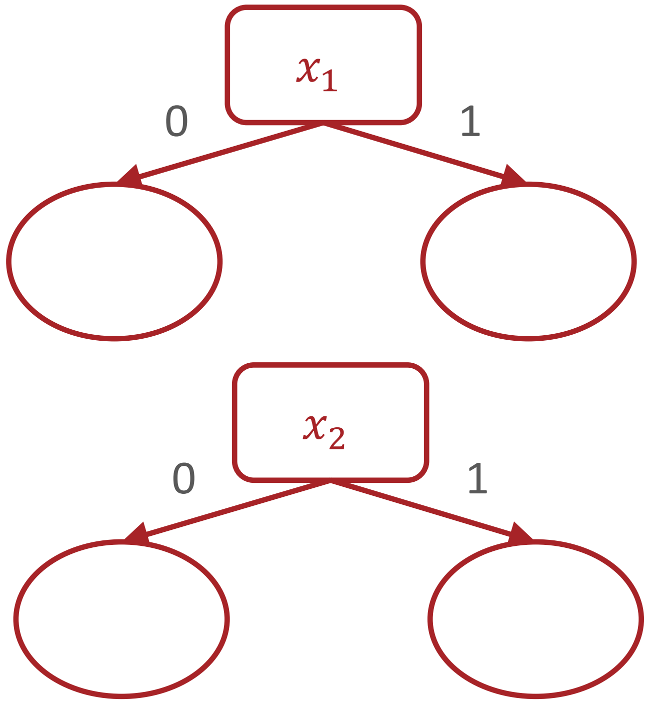


Training error rate:

Training error rate as a Splitting Criterion?

x_1	x_2	y
1	0	0
1	0	0
1	0	1
1	0	1
1	1	1
1	1	1
1	1	1
1	1	1

- Which feature would you split on using training error rate as the splitting criterion?



Splitting Criterion

- A **splitting criterion** is a function that measures how good or useful splitting on a particular feature is *for a specified dataset*
- Insight: use the feature that optimizes the splitting criterion for our decision stump.
- Potential splitting criteria:
  - Training error rate (minimize)
  - Gini impurity (minimize) $\rightarrow$ CART algorithm
  - Mutual information (maximize) $\rightarrow$ ID3 algorithm

Splitting Criterion

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Entropy

- Entropy of a (discrete) random variable X that takes on values in $\mathcal{X}$:

$$H(X) = -\sum_{x \in \mathcal{X}} p(x) \log_2\big(p(x)\big)$$

Entropy

- Entropy of a collection of values S:

$$H(S) = -\sum_{v \in V(S)} \frac{|S_v|}{|S|} \log_2\left(\frac{|S_v|}{|S|}\right)$$

where $V(S)$ is the set of unique values in S

S_v is the collection of elements in S with value v

- If all the elements in S are the same, then

$$H(S) = -1 \log_2(1) = 0$$

Entropy

- Entropy of a collection of values S:

$$H(S) = -\sum_{v \in V(S)} \frac{|S_v|}{|S|} \log_2\left(\frac{|S_v|}{|S|}\right)$$

where $V(S)$ is the set of unique values in S

S_v is the collection of elements in S with value v

- If S is split fifty-fifty between two values, then

$$H(S) = -\frac{1}{2}\log_2\left(\frac{1}{2}\right) - \frac{1}{2}\log_2\left(\frac{1}{2}\right) = -\log_2\left(\frac{1}{2}\right) = 1$$

Mutual Information

- Mutual information between two random variables X and Y describes how much clarity about the value of one variable is gained by observing the other

$$I(Y; X) = H(Y) - H(Y|X)$$

Mutual Information

- Mutual information can be used to compute how much information or clarity a particular feature provides about the label

$$I(Y; x_d) = H(Y) - \sum_{v \in V(x_d)} (f_v)\left(H\left(Y_{x_d=v}\right)\right)$$

where x_d is a feature

Y is the collection of all labels

$V(x_d)$ is the set of unique values of x_d

f_v is the fraction of inputs where $x_d = v$

$Y_{x_d=v}$ is the collection of labels where $x_d = v$

Mutual Information: Example

x_d	y
1	1
1	1
0	0
0	0

$$I(x_d, Y) = H(Y) - \sum_{v \in V(x_d)} (f_v)\left(H\left(Y_{x_d=v}\right)\right)$$

$$= 1 - \frac{1}{2} H\left(Y_{x_d=0}\right) - \frac{1}{2} H\left(Y_{x_d=1}\right)$$

$$= 1 - \frac{1}{2}(0) - \frac{1}{2}(0) = 1$$

Mutual Information: Example

x_d	y
1	1
0	1
1	0
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$$I(x_d, Y) = H(Y) - \sum_{v \in V(x_d)} (f_v)\left(H\left(Y_{x_d=v}\right)\right)$$

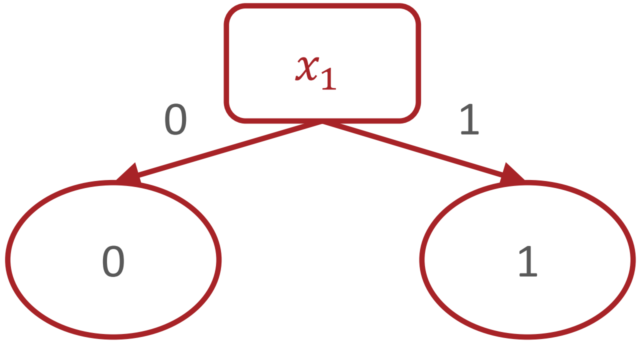
$$= 1 - \frac{1}{2} H\left(Y_{x_d=0}\right) - \frac{1}{2} H\left(Y_{x_d=1}\right)$$

$$= 1 - \frac{1}{2}(1) - \frac{1}{2}(1) = 0$$

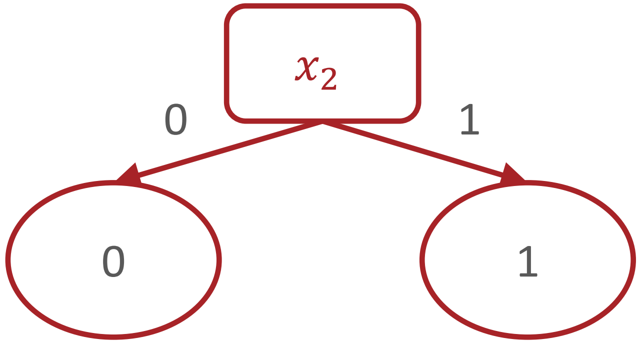
Mutual Information as a Splitting Criterion

x_1	x_2	y
1	0	0
1	0	0
1	0	1
1	0	1
1	1	1
1	1	1
1	1	1
1	1	1

- Which feature would you split on using mutual information as the splitting criterion?



Mutual Information: 0



Mutual Information: $\left(-\frac{2}{8}\log_2\frac{2}{8}-\frac{6}{8}\log_2\frac{6}{8}\right)-\left(\frac{1}{2}(1)-\frac{1}{2}(0)\right)\approx 0.31$

Decision Stumps: Questions

1. How can we pick which feature to split on?

2. Why stop at just one feature?

From Decision Stump …

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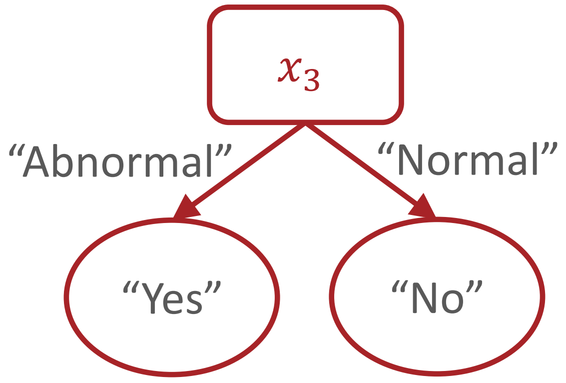

From Decision Stump to Decision Tree

x_1 Family History	x_2 Resting Blood Pressure	x_3 Cholesterol	y Heart Disease?
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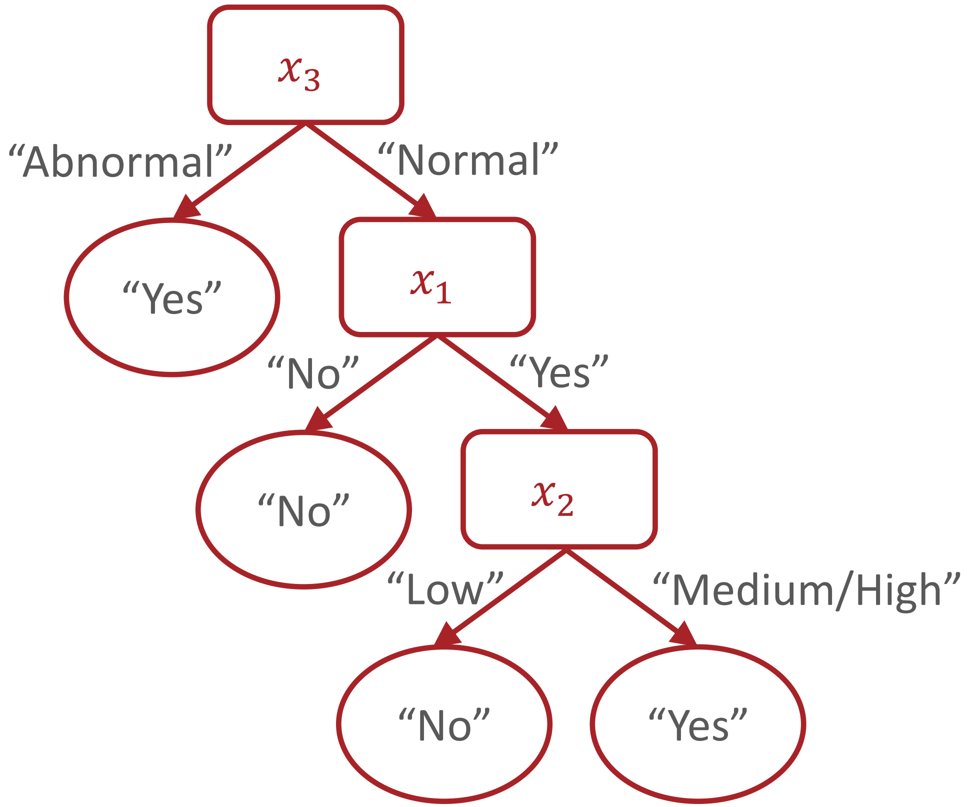

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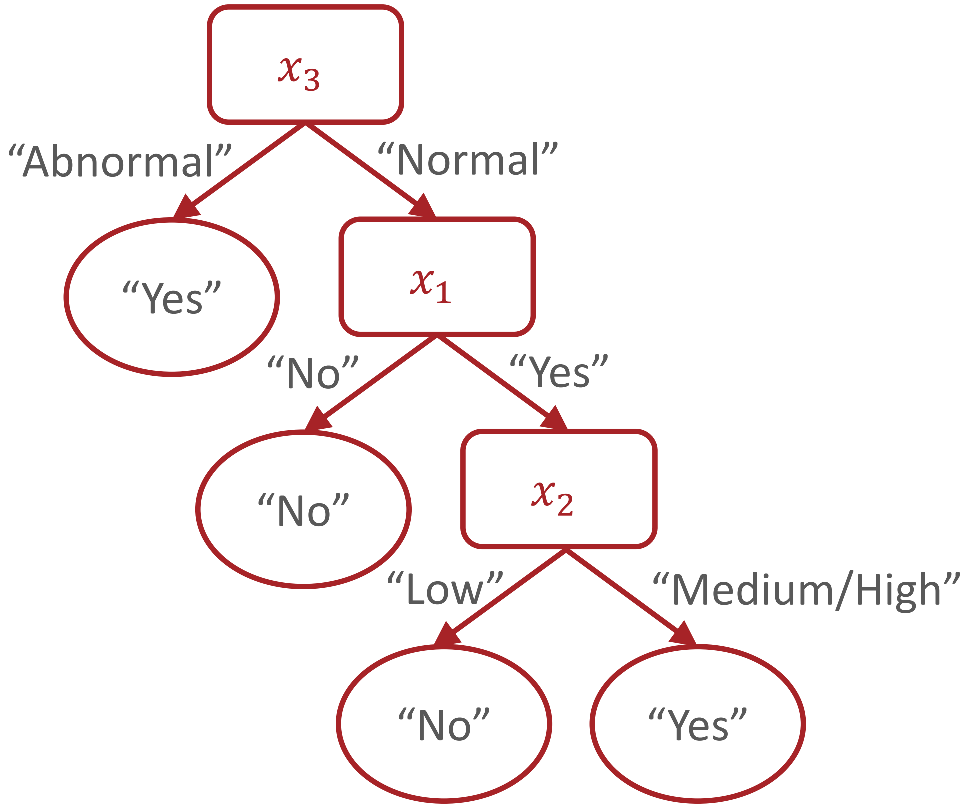

From Decision Stump to Decision Tree

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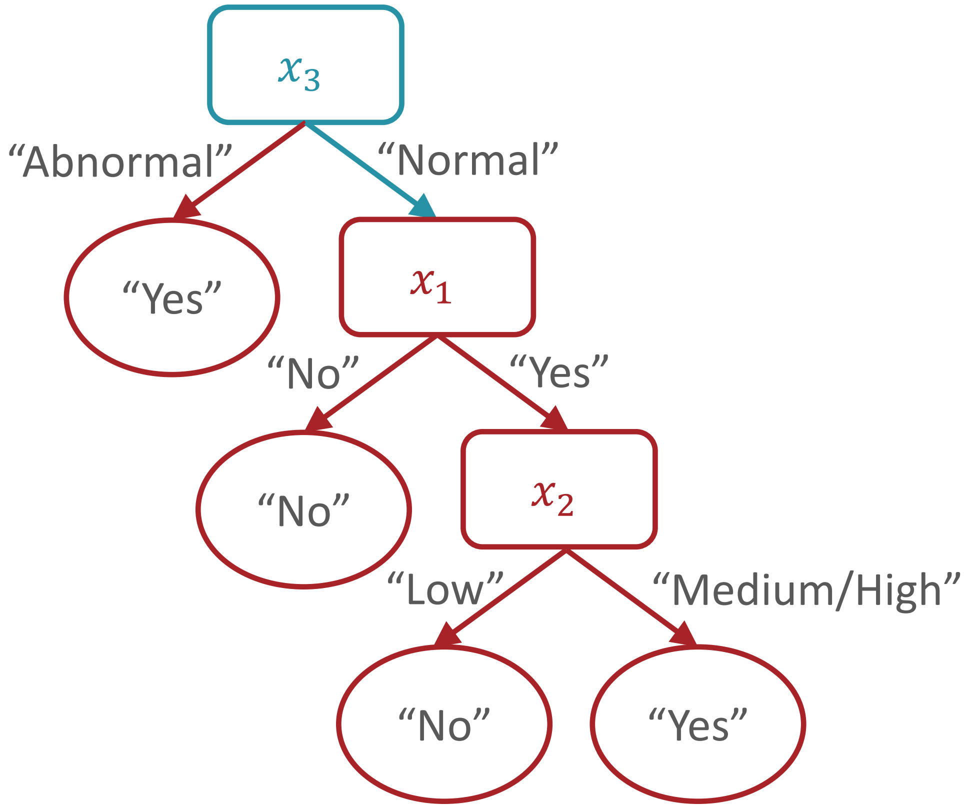

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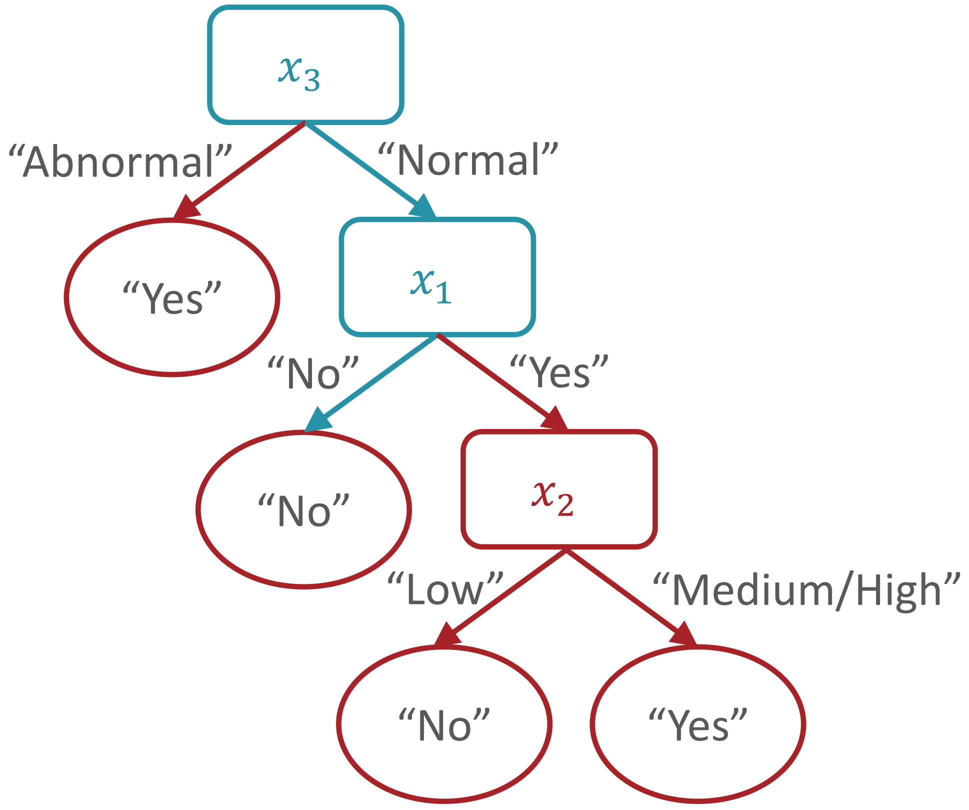

From Decision Stump to Decision Tree

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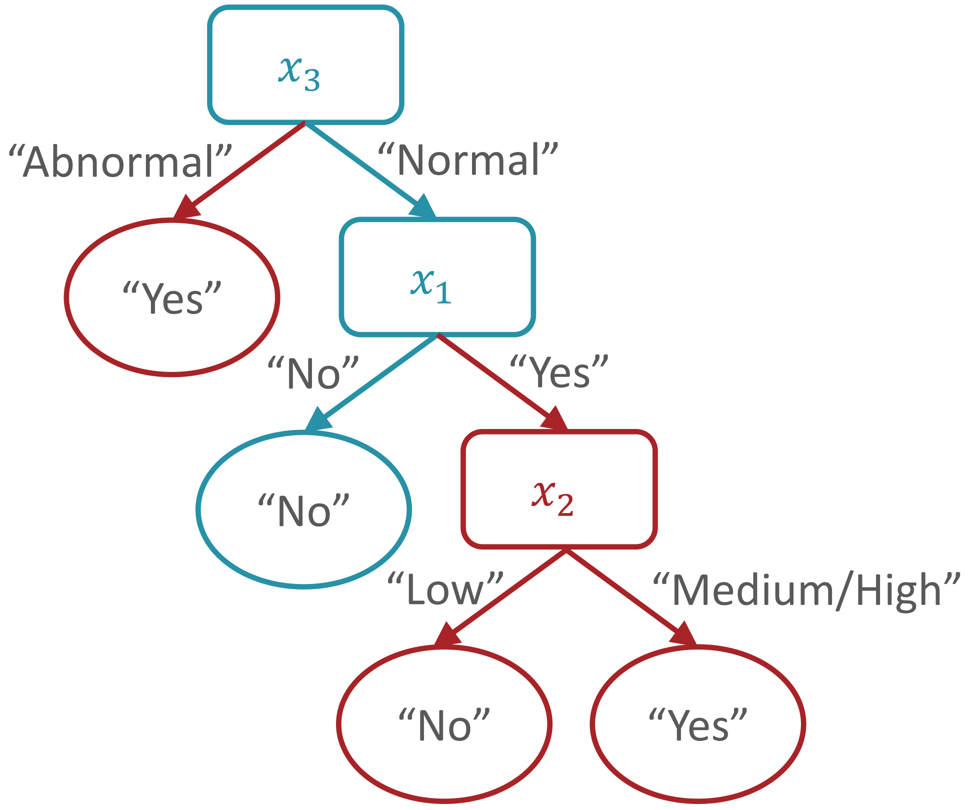

Decision Tree Prediction: Pseudocode

```
def predict(x'):
  - walk from root node to a leaf node
    while(true):
      if current node is internal (non-leaf):
          check the associated attribute, x_d
          go down branch according to x'_d
      if current node is a leaf node:
          return label stored at that leaf
```

Decision Tree Learning: Pseudocode

```
def train(𝒟):
    store root = tree_recurse(𝒟)
def tree_recurse(𝒟′):
    q = new node()
    base case – if (SOME CONDITION):
    recursion – else:
        find best attribute to split on, x_d
        q.split = x_d
        for v in V(x_d), all possible values of x_d:
            𝒟_v = {(x^(n), y^(n)) ∈ 𝒟 | x_d^(n) = v}
            q.children(v) = tree_recurse(𝒟_v)
    return q
```

$$\mathcal{D}_v = \left\{\left(x^{(n)}, y^{(n)}\right) \in \mathcal{D} \mid x_d^{(n)} = v\right\}$$

Decision Tree: Pseudocode

```
def train(𝒟):
    store root = tree_recurse(𝒟)
def tree_recurse(𝒟′):
    q = new node()
    base case – if (𝒟′ is empty OR
        all labels in 𝒟′ are the same OR
        all features in 𝒟′ are identical OR
        some other stopping criterion):
        q.label = majority_vote(𝒟′)


    recursion – else:
    return q
```

Decision Tree: Example (Iteratively)

- How is Henry getting to work?
- Label: mode of transportation
  - $y \in \mathcal{Y} = \{\text{Bike}, \text{Drive}, \text{Bus}\}$
- Features: 4 categorial features
  - Is it raining? $x_1 \in \{\text{Rain}, \text{No Rain}\}$
  - When am I leaving (relative to rush hour)? $x_2 \in \{\text{Before}, \text{During}, \text{After}\}$
  - What am I bringing? $x_3 \in \{\text{Backpack}, \text{Lunchbox}, \text{Both}\}$
  - Am I tired? $x_4 \in \{\text{Tired}, \text{Not Tired}\}$

Data

x_1	x_2	x_3	x_4	y
Rain	Before	Both	Tired	Drive
Rain	During	Both	Not Tired	Bus
Rain	During	Both	Tired	Drive
Rain	After	Backpack	Not Tired	Bus
Rain	After	Backpack	Tired	Bus
Rain	After	Lunchbox	Tired	Drive
No Rain	Before	Backpack	Tired	Bike
No Rain	Before	Lunchbox	Not Tired	Bus
No Rain	Before	Lunchbox	Tired	Drive
No Rain	During	Backpack	Not Tired	Bus
No Rain	During	Both	Tired	Drive
No Rain	After	Backpack	Not Tired	Bike
No Rain	After	Backpack	Tired	Bike
No Rain	After	Both	Not Tired	Bus
No Rain	After	Both	Tired	Drive
No Rain	After	Lunchbox	Not Tired	Bus

Which feature would we split on first using mutual information as the splitting criterion?

x_1	x_2	x_3	x_4	y
Rain	Before	Both	Tired	Drive
Rain	During	Both	Not Tired	Bus
Rain	During	Both	Tired	Drive
Rain	After	Backpack	Not Tired	Bus
Rain	After	Backpack	Tired	Bus
Rain	After	Lunchbox	Tired	Drive
No Rain	Before	Backpack	Tired	Bike
No Rain	Before	Lunchbox	Not Tired	Bus
No Rain	Before	Lunchbox	Tired	Drive
No Rain	During	Backpack	Not Tired	Bus
No Rain	During	Both	Tired	Drive
No Rain	After	Backpack	Not Tired	Bike
No Rain	After	Backpack	Tired	Bike
No Rain	After	Both	Not Tired	Bus
No Rain	After	Both	Tired	Drive
No Rain	After	Lunchbox	Not Tired	Bus

Recall:

$$H(S) = -\sum_{v \in V(S)} \frac{|S_v|}{|S|} \log_2\left(\frac{|S_v|}{|S|}\right)$$

$H(Y)$

x_1	x_2	x_3	x_4	y
Rain	Before	Both	Tired	Drive
Rain	During	Both	Not Tired	Bus
Rain	During	Both	Tired	Drive
Rain	After	Backpack	Not Tired	Bus
Rain	After	Backpack	Tired	Bus
Rain	After	Lunchbox	Tired	Drive
No Rain	Before	Backpack	Tired	Bike
No Rain	Before	Lunchbox	Not Tired	Bus
No Rain	Before	Lunchbox	Tired	Drive
No Rain	During	Backpack	Not Tired	Bus
No Rain	During	Both	Tired	Drive
No Rain	After	Backpack	Not Tired	Bike
No Rain	After	Backpack	Tired	Bike
No Rain	After	Both	Not Tired	Bus
No Rain	After	Both	Tired	Drive
No Rain	After	Lunchbox	Not Tired	Bus

Recall:

$$H(S) = -\sum_{v \in V(S)} \frac{|S_v|}{|S|} \log_2\left(\frac{|S_v|}{|S|}\right)$$

$$H(Y) = -\frac{3}{16}\log_2\left(\frac{3}{16}\right) - \frac{6}{16}\log_2\left(\frac{6}{16}\right) - \frac{7}{16}\log_2\left(\frac{7}{16}\right) \approx 1.5052$$

x_1	x_2	x_3	x_4	y
Rain	Before	Both	Tired	Drive
Rain	During	Both	Not Tired	Bus
Rain	During	Both	Tired	Drive
Rain	After	Backpack	Not Tired	Bus
Rain	After	Backpack	Tired	Bus
Rain	After	Lunchbox	Tired	Drive
No Rain	Before	Backpack	Tired	Bike
No Rain	Before	Lunchbox	Not Tired	Bus
No Rain	Before	Lunchbox	Tired	Drive
No Rain	During	Backpack	Not Tired	Bus
No Rain	During	Both	Tired	Drive
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No Rain	After	Backpack	Tired	Bike
No Rain	After	Both	Not Tired	Bus
No Rain	After	Both	Tired	Drive
No Rain	After	Lunchbox	Not Tired	Bus

Recall: $I(x_d; Y) = H(Y) - \sum_{v \in V(x_d)} (f_v)\left(H\left(Y_{x_d=v}\right)\right)$

$I(x_1, Y) \approx 1.5052$

$-\frac{6}{16}(1)$

$-\frac{10}{16}\left(-\frac{3}{10}\log_2\left(\frac{3}{10}\right)\right.$

$\left.-\frac{3}{10}\log_2\left(\frac{3}{10}\right)-\frac{4}{10}\log_2\left(\frac{4}{10}\right)\right)$

x_1	x_2	x_3	x_4	y
Rain	Before	Both	Tired	Drive
Rain	During	Both	Not Tired	Bus
Rain	During	Both	Tired	Drive
Rain	After	Backpack	Not Tired	Bus
Rain	After	Backpack	Tired	Bus
Rain	After	Lunchbox	Tired	Drive
No Rain	Before	Backpack	Tired	Bike
No Rain	Before	Lunchbox	Not Tired	Bus
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No Rain	During	Backpack	Not Tired	Bus
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No Rain	After	Backpack	Tired	Bike
No Rain	After	Both	Not Tired	Bus
No Rain	After	Both	Tired	Drive
No Rain	After	Lunchbox	Not Tired	Bus

Recall: $I(x_d; Y) = H(Y)$

$$- \sum_{v \in V(x_d)} (f_v)\left(H\left(Y_{x_d=v}\right)\right)$$

$I(x_1, Y) \approx 1.5052$

$$-\frac{6}{16}(1)$$

$$-\frac{10}{16}(1.5710)$$

$$\approx 0.1482$$

x_1	x_2	x_3	x_4	y
Rain	Before	Both	Tired	Drive
Rain	During	Both	Not Tired	Bus
Rain	During	Both	Tired	Drive
Rain	After	Backpack	Not Tired	Bus
Rain	After	Backpack	Tired	Bus
Rain	After	Lunchbox	Tired	Drive
No Rain	Before	Backpack	Tired	Bike
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No Rain	After	Both	Tired	Drive
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Recall: $I(x_d; Y) = H(Y) - \sum_{v \in V(x_d)} (f_v)\left(H\left(Y_{x_d=v}\right)\right)$

$I(x_d, Y)$	
x_1	0.1482
x_2	0.1302
x_3	0.5358
x_4	0.5576

x_1	x_2	x_3	x_4	y
Rain	Before	Both	Tired	Drive
Rain	During	Both	Not Tired	Bus
Rain	During	Both	Tired	Drive
Rain	After	Backpack	Not Tired	Bus
Rain	After	Backpack	Tired	Bus
Rain	After	Lunchbox	Tired	Drive
No Rain	Before	Backpack	Tired	Bike
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No Rain	During	Backpack	Not Tired	Bus
No Rain	During	Both	Tired	Drive
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Recall: $I(x_d; Y) = H(Y) - \sum_{v \in V(x_d)} (f_v)\left(H\left(Y_{x_d=v}\right)\right)$

$I(x_d, Y)$	
x_1	0.1482
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x_1	x_2	x_3	x_4	y
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Recall: $I(x_d; Y) = H(Y) - \sum_{v \in V(x_d)} (f_v)\left(H(Y_{x_d=v})\right)$

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No Rain	After	Both	Not Tired	Bus
No Rain	After	Lunchbox	Not Tired	Bus
Rain	Before	Both	Tired	Drive
Rain	During	Both	Tired	Drive
Rain	After	Backpack	Tired	Metro
Rain	After	Lunchbox	Tired	Drive
No Rain	Before	Backpack	Tired	Bike
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$$\text{Recall: } I(x_d; Y) = H(Y) - \sum_{v \in V(x_d)} (f_v)\left(H(Y_{x_d=v})\right)$$

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No Rain	During	Both	Tired	Drive
No Rain	After	Backpack	Tired	Bike
No Rain	After	Both	Tired	Drive

Decision Tree: Example

x_4

Not Tired | Tired

Not Tired:

x_1	x_2	x_3	x_4	y
Rain	During	Both	Not Tired	Bus
Rain	After	Backpack	Not Tired	Bus
No Rain	Before	Lunchbox	Not Tired	Bus
No Rain	During	Backpack	Not Tired	Bus
No Rain	After	Backpack	Not Tired	Bike
No Rain	After	Both	Not Tired	Bus
No Rain	After	Lunchbox	Not Tired	Bus

Tired:

x_1	x_2	x_3	x_4	y
Rain	Before	Both	Tired	Drive
Rain	During	Both	Tired	Drive
Rain	After	Backpack	Tired	Bus
Rain	After	Lunchbox	Tired	Drive
No Rain	Before	Backpack	Tired	Bike
No Rain	Before	Lunchbox	Tired	Drive
No Rain	During	Both	Tired	Drive
No Rain	After	Backpack	Tired	Bike
No Rain	After	Both	Tired	Drive

x_4

Not Tired

x_1	x_2	x_3	x_4	y
Rain	During	Both	Not Tired	Bus
Rain	After	Backpack	Not Tired	Bus
No Rain	Before	Lunchbox	Not Tired	Bus
No Rain	During	Backpack	Not Tired	Bus
No Rain	After	Backpack	Not Tired	Bike
No Rain	After	Both	Not Tired	Bus
No Rain	After	Lunchbox	Not Tired	Bus

Tired

x_1	x_2	x_3	x_4	y
Rain	Before	Both	Tired	Drive
Rain	During	Both	Tired	Drive
Rain	After	Backpack	Tired	Bus
Rain	After	Lunchbox	Tired	Drive
No Rain	Before	Backpack	Tired	Bike
No Rain	Before	Lunchbox	Tired	Drive
No Rain	During	Both	Tired	Drive
No Rain	After	Backpack	Tired	Bike
No Rain	After	Both	Tired	Drive

x_4

Not Tired

x_1	x_2	x_3	x_4	y
Rain	During	Both	Not Tired	Bus
Rain	After	Backpack	Not Tired	Bus
No Rain	Before	Lunchbox	Not Tired	Bus
No Rain	During	Backpack	Not Tired	Bus
No Rain	After	Backpack	Not Tired	Bike
No Rain	After	Both	Not Tired	Bus
No Rain	After	Lunchbox	Not Tired	Bus

Tired

x_1	x_2	x_3	x_4	y
Rain	After	Backpack	Tired	Bus
No Rain	Before	Backpack	Tired	Bike
No Rain	After	Backpack	Tired	Bike
Rain	Before	Both	Tired	Drive
Rain	During	Both	Tired	Drive
No Rain	During	Both	Tired	Drive
No Rain	After	Both	Tired	Drive
Rain	After	Lunchbox	Tired	Drive
No Rain	Before	Lunchbox	Tired	Drive

$$I(x_1, Y_{x_4=\text{Tired}}) \approx 0.3244$$

$$I(x_2, Y_{x_4=\text{Tired}}) \approx 0.2516$$

$$I(x_3, Y_{x_4=\text{Tired}}) \approx \mathbf{0.9183}$$

x_4

Not Tired

x_1	x_2	x_3	x_4	y
Rain	During	Both	Not Tired	Bus
Rain	After	Backpack	Not Tired	Bus
No Rain	Before	Lunchbox	Not Tired	Bus
No Rain	During	Backpack	Not Tired	Bus
No Rain	After	Backpack	Not Tired	Bike
No Rain	After	Both	Not Tired	Bus
No Rain	After	Lunchbox	Not Tired	Bus

Tired

x_3

Backpack

x_1	x_2	y
Rain	After	Bus
No Rain	Before	Bike
No Rain	After	Bike

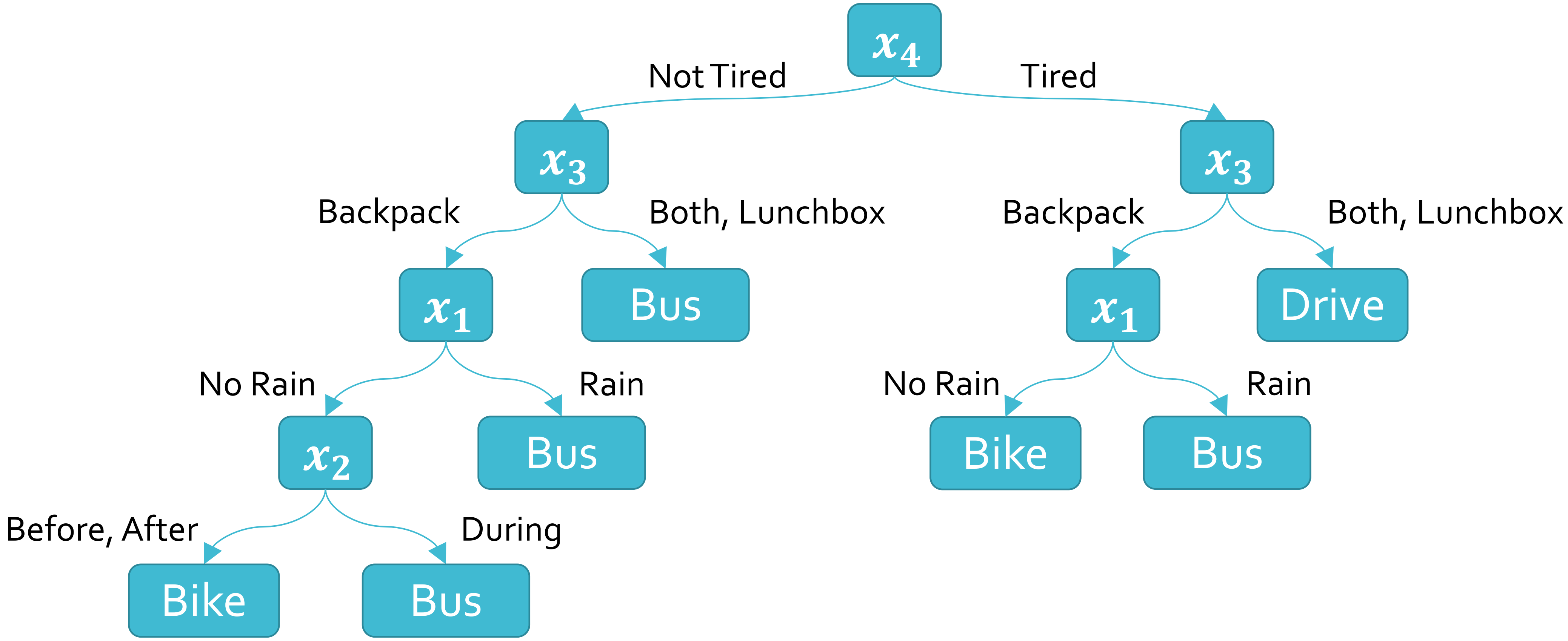
Both, Lunchbox

Drive

$$I\left(x_1, Y_{x_4=\text{Tired}}\right) \approx 0.3244$$

$$I\left(x_2, Y_{x_4=\text{Tired}}\right) \approx 0.2516$$

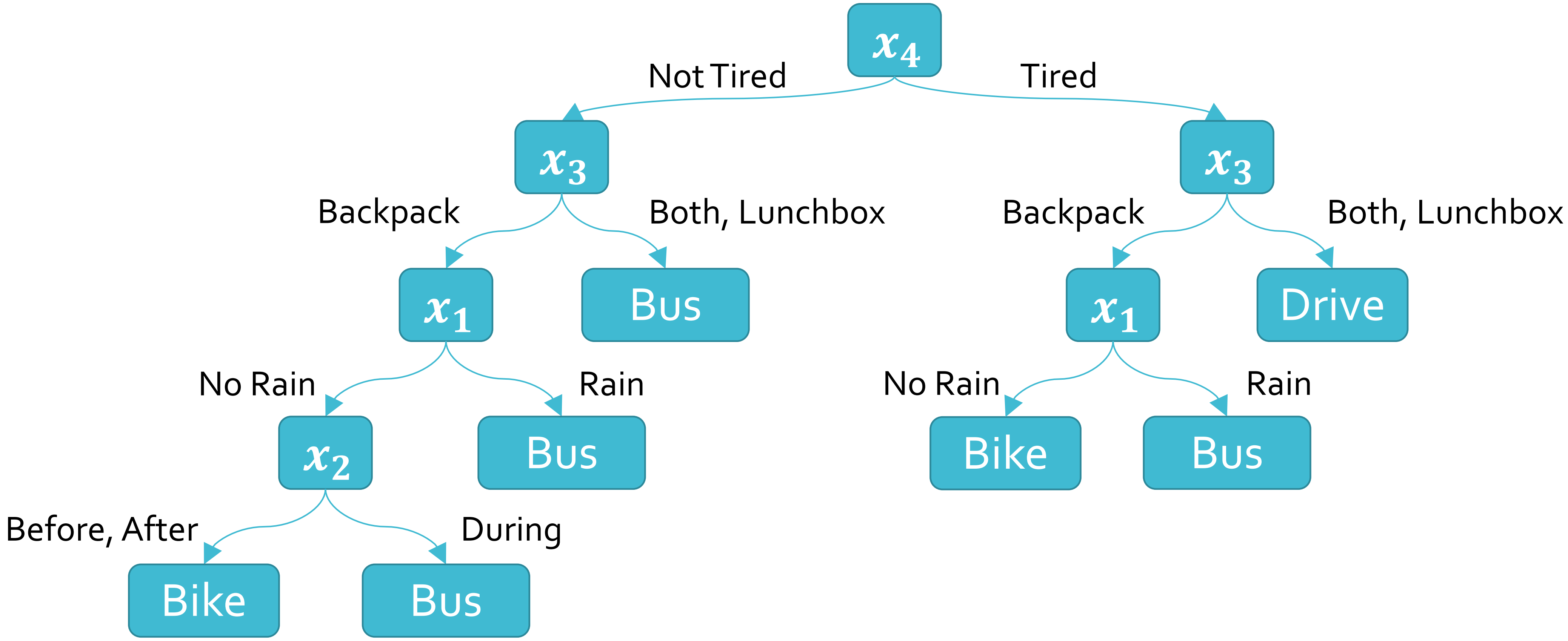
$$I\left(x_3, Y_{x_4=\text{Tired}}\right) \approx \mathbf{0.9183}$$



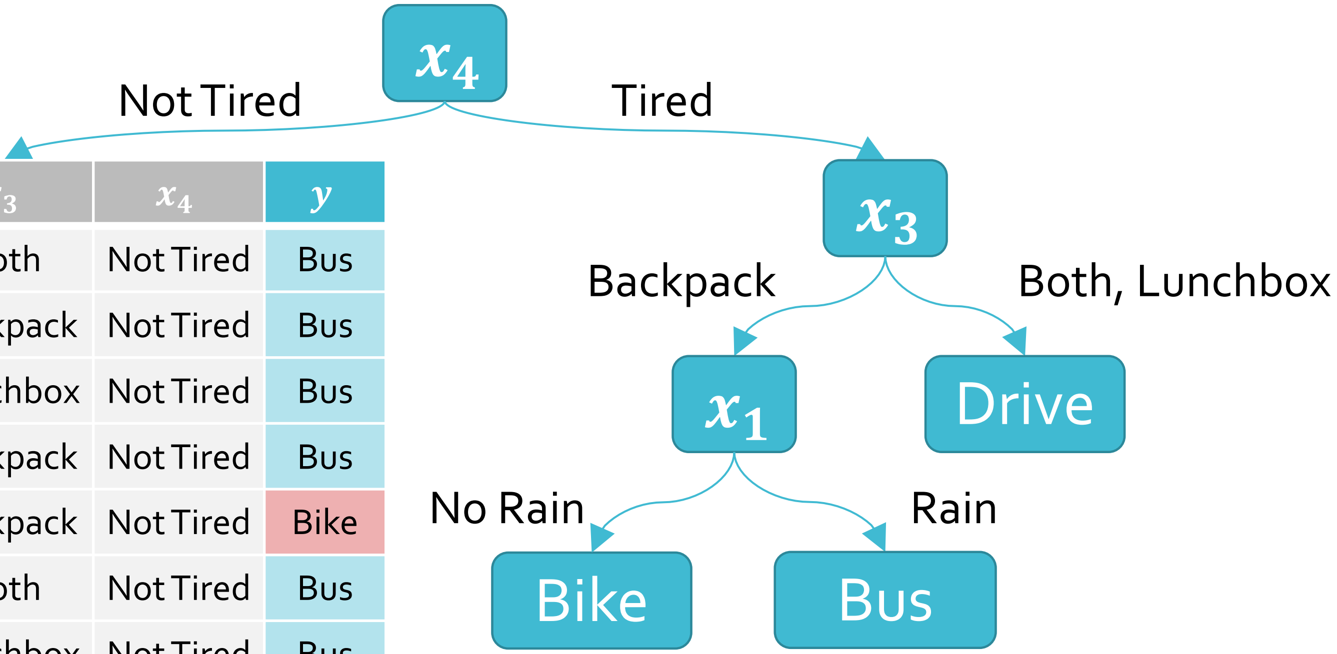
x_4
Not Tired
Tired
x_3
x_3
Backpack
Both, Lunchbox
Backpack
Both, Lunchbox
x_1
Bus
x_1
Drive
No Rain
Rain
No Rain
Rain
x_2
Bus
Bike
Bus
Before, After
During
Bike
Bus

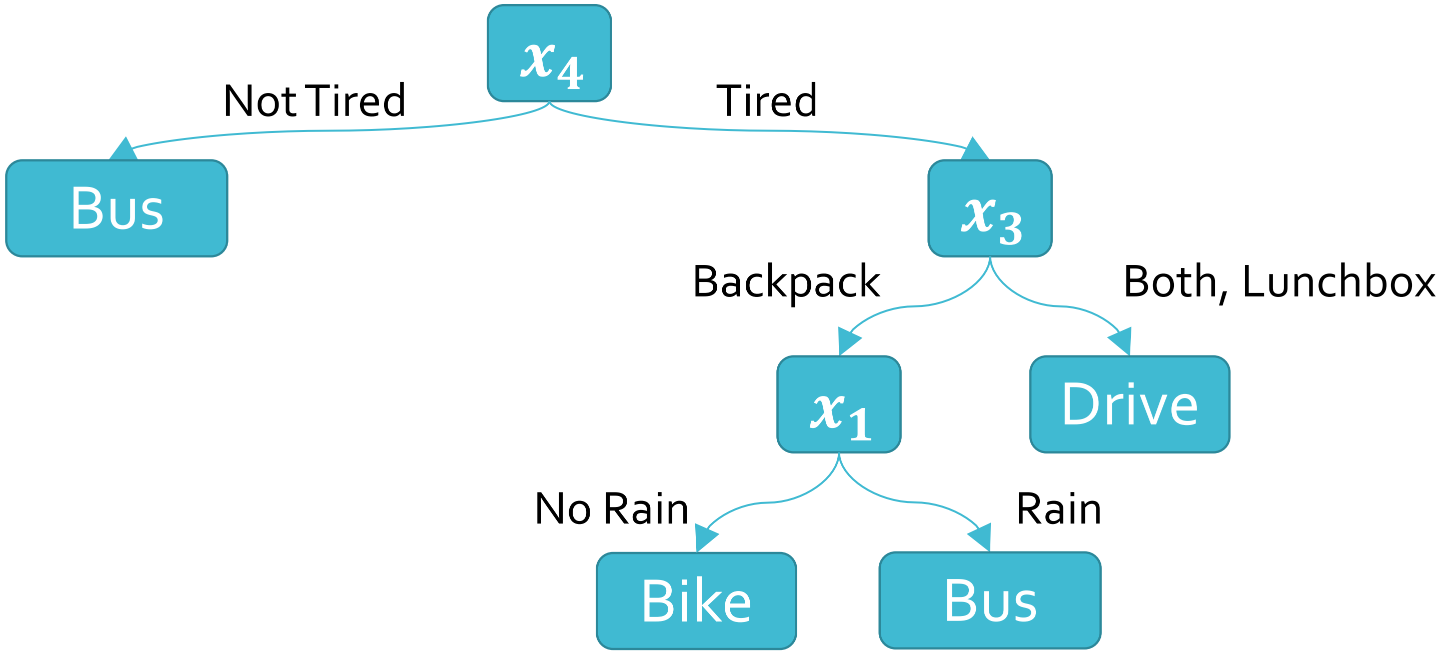
Decision Trees: Inductive Bias

- The **inductive bias** of a machine learning algorithm is the principal by which it generalizes to unseen examples
- What is the inductive bias of the ID3 algorithm i.e., decision tree learning with mutual information maximization as the splitting criterion?
  - Try to find the __________ tree that achieves ________________________________ with ______________________________ features at the top


x_4
Not Tired
Tired
x_3
x_3
Backpack
Both, Lunchbox
Backpack
Both, Lunchbox
x_1
Bus
x_1
Drive
No Rain
Rain
No Rain
Rain
x_2
Bus
Bike
Bus
Before, After
During
Bike
Bus

x_1	x_2	x_3	x_4	y
Rain	During	Both	Not Tired	Bus
Rain	After	Backpack	Not Tired	Bus
No Rain	Before	Lunchbox	Not Tired	Bus
No Rain	During	Backpack	Not Tired	Bus
No Rain	After	Backpack	Not Tired	Bike
No Rain	After	Both	Not Tired	Bus
No Rain	After	Lunchbox	Not Tired	Bus





This tree only misclassifies one training data point!