

Optimized Compilation of Multiset Rewriting with Comprehensions

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Outline

- 1 Introduction
- 2 Comprehensions in CHR^{cp}
- 3 Compilation
- 4 Implementation
- 5 Conclusion

Constraint Handling Rules (CHR)

- A rule-based programming language:
 - Pure committed choice forward chaining
 - Declarative
 - Concurrent
- CHR is a specific instance of
 - Multiset rewriting
 - Constraint logic programming

Constraint Handling Rules (CHR)

$$r @ \bar{H} \iff g \mid \bar{B}$$

- \bar{H} and \bar{B} are multisets of *atomic constraint patterns*: $p(\vec{t})$
 - r : Rule name
 - \bar{H} : Head constraints (LHS)
 - g : Guard conditions
 - \bar{B} : Body constraints (RHS)

- A *program* CHR \mathcal{P} is a set of rules
- CHR programs \mathcal{P} are executed on constraint stores:

$$\mathcal{P} \triangleright St \mapsto_{\alpha}^* St'$$

- The *stores* St and St' are multiset of constraints

Adding Comprehension Patterns to CHR (CHR^{cp})

$$r @ \bar{H} \iff g \mid \bar{B}$$

- Programming in CHR is great*!
 - declarative and concise
 - high-level
- but not perfect...
 - performing aggregated operation
 - rewrite dynamic numbers of facts
- This work:
 - CHR + Comprehension Patterns (CHR^{cp})
 - Optimized compilation scheme for CHR^{cp}
 - Implementation and preliminary experimental results

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Example: Pivoted Swapping

- Entities X and Y want to swap data D w.r.t. pivot P
 - All X 's data $\geq P$ to Y
 - All Y 's data $\leq P$ to X
- Constraints:
 - $data(X, D)$ represents data D belonging to X
 - $swap(X, Y, P)$ represents an intend to swap data between X and Y w.r.t pivot P .

Example: Pivoted Swapping (in “Vanilla” CHR)

- LHS cannot match a dynamic numbers of constraints
- Standard CHR implementation:

<i>init</i> @ <i>swap</i> (<i>X</i> , <i>Y</i> , <i>P</i>)	\iff	<i>grabGE</i> (<i>X</i> , <i>P</i> , <i>Y</i> , []), <i>grabLE</i> (<i>Y</i> , <i>P</i> , <i>X</i> , [])
<i>ge1</i> @ <i>grabGE</i> (<i>X</i> , <i>P</i> , <i>Y</i> , <i>Ds</i>), <i>data</i> (<i>X</i> , <i>D</i>)	\iff	$D \geq P \mid$ <i>grabGE</i> (<i>X</i> , <i>P</i> , <i>Y</i> , [<i>D</i> <i>Ds</i>])
<i>ge2</i> @ <i>grabGE</i> (<i>X</i> , <i>P</i> , <i>Y</i> , <i>Ds</i>)	\iff	<i>unrollData</i> (<i>Y</i> , <i>Ds</i>)
<i>le1</i> @ <i>grabLE</i> (<i>Y</i> , <i>P</i> , <i>X</i> , <i>Ds</i>), <i>data</i> (<i>Y</i> , <i>D</i>)	\iff	$D \leq P \mid$ <i>grabLE</i> (<i>Y</i> , <i>P</i> , <i>X</i> , [<i>D</i> <i>Ds</i>])
<i>le2</i> @ <i>grabLE</i> (<i>Y</i> , <i>P</i> , <i>X</i> , <i>Ds</i>)	\iff	<i>unrollData</i> (<i>X</i> , <i>Ds</i>)
<i>unroll1</i> @ <i>unrollData</i> (<i>L</i> , [<i>D</i> <i>Ds</i>])	\iff	<i>unrollData</i> (<i>L</i> , <i>Ds</i>), <i>data</i> (<i>L</i> , <i>D</i>)
<i>unroll2</i> @ <i>unrollData</i> (<i>L</i> , [])	\iff	<i>true</i>

- Verbose: 7 rules and 3 auxiliary constraints

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- LHS cannot match a dynamic numbers of constraints
- Standard CHR implementation:

$init @ swap(X, Y, P)$	\iff	$grabGE(X, P, Y, []), grabLE(Y, P, X, [])$
$ge1 @ grabGE(X, P, Y, Ds), data(X, D)$	\iff	$D \geq P \mid grabGE(X, P, Y, [D \mid Ds])$
$ge2 @ grabGE(X, P, Y, Ds)$	\iff	$unrollData(Y, Ds)$
$le1 @ grabLE(Y, P, X, Ds), data(Y, D)$	\iff	$D \leq P \mid grabLE(Y, P, X, [D \mid Ds])$
$le2 @ grabLE(Y, P, X, Ds)$	\iff	$unrollData(X, Ds)$
$unroll1 @ unrollData(L, [D \mid Ds])$	\iff	$unrollData(L, Ds), data(L, D)$
$unroll2 @ unrollData(L, [])$	\iff	$true$

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<i>ge1</i> @ <i>grabGE</i> (<i>X</i> , <i>P</i> , <i>Y</i> , <i>Ds</i>), <i>data</i> (<i>X</i> , <i>D</i>)	\iff	$D \geq P \mid \text{grabGE}(\text{X}, \text{P}, \text{Y}, [\text{D} \mid \text{Ds}])$
<i>ge2</i> @ <i>grabGE</i> (<i>X</i> , <i>P</i> , <i>Y</i> , <i>Ds</i>)	\iff	<i>unrollData</i> (<i>Y</i> , <i>Ds</i>)
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<i>unroll2</i> @ <i>unrollData</i> (<i>L</i> , [])	\iff	<i>true</i>

- Verbose: 7 rules and 3 auxiliary constraints
- Uses accumulators
- Relies on rule priorities: apply *ge1* before *ge2*, *le1* before *le2*

CHR with Comprehension Patterns

- Additional form of constraint patterns:
 - *Comprehension patterns* $m: \{p(\vec{x}) \mid g\}_{\vec{x} \in t}$
 - Collects *all* $p(\vec{x})$ in the store that satisfy g
 - t is the multiset of all bindings to \vec{x}

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 - Collects *all* $p(\vec{x})$ in the store that satisfy g
 - t is the multiset of all bindings to \vec{x}
- Pivoted Swapping in CHR^{cp} :

$$\begin{array}{l}
 \text{swap}(X, Y, P) \\
 \text{pivotSwap} @ \quad \{data(X, D) \mid D \geq P\}_{D \in Xs} \iff \{data(Y, D)\}_{D \in Xs} \\
 \quad \quad \quad \{data(Y, D) \mid D \leq P\}_{D \in Ys} \quad \quad \quad \{data(X, D)\}_{D \in Ys}
 \end{array}$$

- Xs and Ys built from the store — *output*
- Xs and Ys used to unfold the comprehensions — *input*

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- An example of CHR^{cp} rule application:

$\{ \text{swap}(a, b, 5), \text{data}(a, 1), \text{data}(a, 6), \text{data}(a, 7), \text{data}(b, 2), \text{data}(b, 8) \}$

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- An example of CHR^cp rule application:

$$\mapsto_{\alpha} \begin{array}{l} \{ \textcolor{green}{swap}(a, b, 5), \textcolor{brown}{data}(a, 1), \textcolor{brown}{data}(a, 6), \textcolor{brown}{data}(a, 7), \textcolor{blue}{data}(b, 2), \textcolor{brown}{data}(b, 8) \} \\ \{ \textcolor{brown}{data}(a, 1), \textcolor{red}{data}(b, 6), \textcolor{red}{data}(b, 7), \textcolor{blue}{data}(a, 2), \textcolor{brown}{data}(b, 8) \} \end{array}$$

- applying *pivotSwap* with $\{a/X, b/Y, 5/P, \{6, 7\}/Xs, \{2\}/Ys\}$

CHR with Comprehension Patterns

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- Semantics of CHR^{cp} guarantees *maximality of comprehension*:

$$\{ \text{swap}(a, b, 5), \text{data}(a, 1), \boxed{\text{data}(a, 6)}, \text{data}(a, 7), \text{data}(b, 2), \text{data}(b, 8) \}$$

CHR with Comprehension Patterns

- Pivoted Swapping in CHR^{cp} :

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– applying *pivotSwap* with $\{a/X, b/Y, 5/P, \{6, 7\}/Xs, \{2\}/Ys\}$

- Semantics of CHR^{cp} guarantees *maximality of comprehension*:

$$\not\mapsto_{\alpha} \begin{array}{l} \{ \text{swap}(a, b, 5), \text{data}(a, 1), \boxed{\text{data}(a, 6)}, \text{data}(a, 7), \text{data}(b, 2), \text{data}(b, 8) \} \\ \{ \text{data}(a, 1), \boxed{\text{data}(a, 6)}, \text{data}(b, 7), \text{data}(a, 2), \text{data}(b, 8) \} \end{array}$$

– not valid! $\boxed{\text{data}(a, 6)}$ is left behind!

Semantics of CHR^{cp}

- Abstract semantics of CHR^{cp} (\mapsto_α):
 - High level specification of CHR^{cp}
 - Formalizes “maximality of comprehensions”
- Operational semantics of CHR^{cp} (\mapsto_ω):
 - Extends from [?]
 - Defines systematic execution scheme that avoids recomputation of matches
- \mapsto_ω is sound w.r.t \mapsto_α
- See paper and technical report for details!

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Highlights of CHR^{cp} optimized compilation

- Compile CHR^{cp} rules into procedural operations:
 - Implements the operational semantics (\mapsto_ω)
 - with various optimizations
- Existing CHR optimizations are still applicable:
 - Optimal join ordering + index selection (**OJO**)
- Optimizations specific to comprehension patterns:
 - Incrementally store monotone constraints (**Mono**)
 - Selective enforcement of unique match (**Uniq**)
 - Bootstrapping active comprehension patterns (**Bt**)

Optimal Join-Ordering and Index Selection (OJO)

Given a rule: $p_1, \dots, p_i, \dots, p_n \iff \dots$

- OJO in standard CHR:
 - Find candidates p_j in an optimal sequence
 - Compute optimal indexing structures on p_j
- We extend (**OJO**) to handle comprehension patterns:

$swap(X, Y, P), \lambda data(X, D) \mid D \geq P \int_{D \in X_s}, \lambda data(Y, D) \mid D < P \int_{D \in Y_s} \iff \dots$

- Given $swap(john, jack, 40)$, how do we find all $data(john, D)$ such that $\lambda data(john, D) \mid D \geq 40 \int_{D \in X_s}$?
- Heuristics to compute optimal orderings
- Compute indexing structures on comprehension patterns

Incrementally store monotone constraints (**Mono**) 1/3

- Standard CHR is monotonic:
 $\mathcal{P} \triangleright St \mapsto_{\alpha} St'$ implies $\mathcal{P} \triangleright \{St, St''\} \mapsto_{\alpha} \{St', St''\}$
 - We can *incrementally* store all constraints
 - Benefits? Performance improvements!

Incrementally store monotone constraints (**Mono**) 1/3

- Standard CHR is monotonic:
 $\mathcal{P} \triangleright St \mapsto_{\alpha} St'$ implies $\mathcal{P} \triangleright \{St, St''\} \mapsto_{\alpha} \{St', St''\}$
 - We can *incrementally* store all constraints
 - Benefits? Performance improvements!
- But CHR^{cp} is not monotonic! Recall:

$$\not\mapsto_{\alpha} \left\{ \begin{array}{l} \textcolor{green}{swap(a, b, 5)}, \textcolor{blue}{data(a, 1)}, \boxed{\textcolor{black}{data(a, 6)}}, \textcolor{orange}{data(a, 7)}, \textcolor{blue}{data(b, 2)}, \textcolor{black}{data(b, 8)} \\ \textcolor{black}{data(a, 1)}, \boxed{\textcolor{black}{data(a, 6)}}, \textcolor{red}{data(b, 7)}, \textcolor{blue}{data(a, 2)}, \textcolor{black}{data(b, 8)} \end{array} \right\}$$

- Not valid! Because we want maximality of comprehensions!
- Naive solution: Store ALL constraints IMMEDIATELY!

Incrementally store monotone constraints (**Mono**) 2/3

- Conditional monotonicity [?]:

If St'' contains only constraints *monotone w.r.t.* \mathcal{P}
 then $\mathcal{P} \triangleright St \mapsto_{\alpha} St'$ implies $\mathcal{P} \triangleright \{St, St''\} \mapsto_{\alpha} \{St', St''\}$

- A constraint c is *monotone* w.r.t. \mathcal{P} (denoted $\mathcal{P} \triangleq_{\text{unf}}^{\neg} c$), iff c cannot participate in any comprehension pattern m such that $(r@..., m, ... \iff ...) \in \mathcal{P}$
- (Mono) optimization: Store ONLY non-monotone constraints IMMEDIATELY!

Incrementally store monotone constraints (**Mono**) 3/3

$$\begin{array}{l}
 \text{pivotSwap} @ \text{swap}(X, Y, P) \\
 \{ \text{data}(X, D) \mid D \geq P \}_{D \in X_s} \iff \{ \text{data}(Y, D) \}_{D \in X_s} \\
 \{ \text{data}(Y, D) \mid D < P \}_{D \in Y_s} \iff \{ \text{data}(X, D) \}_{D \in Y_s}
 \end{array}$$

- *swap* is monotone, but not *data*
- “Glass is half-full!”: incrementally store *swap* constraints!
- We statically compute $\mathcal{P} \triangleq_{\text{unf}}^{\neg} c$ for each RHS constraint
- In experiments, we compare performances of
 - Naive approach: store ALL IMMEDIATELY!
 - (Mono) optimization: ONLY store non-monotone constraints IMMEDIATELY!

Selective Enforcement of Unique Match (**Uniq**) 1/2

- Consider a rule with more than one comprehension pattern:

$$r@..., m_i, ..., m_j, ... \iff ...$$

- m_i and m_j potentially match with the same constraints.
 - E.g., $m_i = \{p(X) \mid X < 4\}$ and $m_j = \{p(Y) \mid Y > 2\}$
 - Notice that $p(3)$ can be matched to either m_i OR m_j
- In general, we need match m_i and m_j to non-overlapping multisets of constraints – a potentially expensive operation

Selective Enforcement of Unique Match (**Uniq**) 2/2

- But sometimes...

$$\begin{array}{l} \text{swap}(X, Y, P) \\ m_i = \{ \text{data}(X, D) \mid D \geq P \} \\ m_j = \{ \text{data}(Y, D) \mid D < P \} \end{array} \iff \dots$$

- Notice that $m_i \cap m_j \neq \emptyset$ is unsatisfiable!
- (Uniq) Optimization:
 - Statically test $m_i \cap m_j \neq \emptyset$ for each pair m_i and m_j
 - Avoid redundant runtime operations
- In experiments, we compare performances of:
 - Naive approach: Enforce uniqueness for ALL m_i, m_j pairs.
 - (Uniq) optimization: Only for m_i, m_j if $m_i \cap m_j \neq \emptyset$ is satisfiable

Reasoning about Comprehensions

- Wait! So, how do we statically compute $\mathcal{P} \stackrel{\Delta}{=}_{\text{unf}}^{\neg} c$ (from **Mono**)?
- We implemented a library extension to Microsoft's Z3 solver:
 - to reason about set comprehensions.
- Details in “Reasoning about Set Comprehensions”, SMT'2014
- Download at: <https://github.com/sllam/pysetcomp>

Bootstrapping active comprehension patterns (**Bt**)

$$\text{pivotSwap} @ \begin{array}{l} \text{swap}(X, Y, P) \\ \lambda \text{data}(X, D) \mid D \geq P \int_{D \in X_s} \\ \lambda \text{data}(Y, D) \mid D \leq P \int_{D \in Y_s} \end{array} \iff \begin{array}{l} \lambda \text{data}(Y, D) \int_{D \in X_s} \\ \lambda \text{data}(X, D) \int_{D \in Y_s} \end{array}$$

- Suppose that we are processing a $\text{data}(\text{john}, 5)$ and attempting to match it to $\lambda \text{data}(X, D) \mid D \geq P \int_{D \in X_s}$.
- Naive approach:
 - 1 Retrieve ALL $\text{data}(\text{john}, _)$ (since P is unknown)
 - 2 Filter away bindings in X_s after we found a $\text{swap}(\text{john}, Y, P)$
- Bootstrapping active comprehension pattern (**Bt**):
 - 1 Match $\text{data}(\text{john}, 5)$ to $\text{data}(X, D)$ (treat as atomic pattern)
 - 2 Complete the comprehension pattern AFTER (and ONLY IF) we actually found $\text{swap}(\text{john}, Y, P)$

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Implementation

- Prototype implementation
 - Available at:
 - <https://github.com/sllam/msre> for download
 - <http://rise4fun.com/msre> for online demo
 - Compiler and type-checker implemented in Python
 - Generates C++ codes that implements CHR^{cp} run-time
 - Utilizes Z3 SMT solver for type checking and other analysis [?]

Preliminary Experimental Results

Program	Standard rules only			With comprehensions			Code reduction (lines)
Swap	5 preds	7 rules	21 lines	2 preds	1 rule	10 lines	110%
GHS	13 preds	13 rules	47 lines	8 preds	5 rules	35 lines	34%
HQSort	10 preds	15 rules	53 lines	7 preds	5 rules	38 lines	39%

Program	Input Size	Orig	+OJO	+OJO +Bt	+OJO +Mono	+OJO +Uniq	All	Speedup
Swap	(40, 100)	241 vs 290	121 vs 104	vs 104	vs 103	vs 92	vs 91	33%
	(200, 500)	1813 vs 2451	714 vs 681	vs 670	vs 685	vs 621	vs 597	20%
	(1000, 2500)	8921 vs 10731	3272 vs 2810	vs 2651	vs 2789	vs 2554	vs 2502	31%
GHS	(100, 200)	814 vs 1124	452 vs 461	vs 443	vs 458	vs 437	vs 432	5%
	(500, 1000)	7725 vs 8122	3188 vs 3391	vs 3061	vs 3290	vs 3109	vs 3005	6%
	(2500, 5000)	54763 vs 71650	15528 vs 16202	vs 15433	vs 16097	vs 15835	vs 15214	2%
HQSort	(8, 50)	1275 vs 1332	1117 vs 1151	vs 1099	vs 1151	vs 1081	vs 1013	10%
	(16, 100)	5783 vs 6211	3054 vs 2980	vs 2877	vs 2916	vs 2702	vs 2661	15%
	(32, 150)	13579 vs 14228	9218 vs 8745	vs 8256	vs 8617	vs 8107	vs 8013	15%

Execution times (ms) for various optimizations on programs with increasing input size.

n vs m : n is execution time for standard rules, m is execution time for comprehensions

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- Its preliminary, but promising

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	(1000, 2500)	8921 vs 10731	3272 vs 2810	vs 2651	vs 2789	vs 2554	vs 2502	31%
GHS	(100, 200)	814 vs 1124	452 vs 461	vs 443	vs 458	vs 437	vs 432	5%
	(500, 1000)	7725 vs 8122	3188 vs 3391	vs 3061	vs 3290	vs 3109	vs 3005	6%
	(2500, 5000)	54763 vs 71650	15528 vs 16202	vs 15433	vs 16097	vs 15835	vs 15214	2%
HQSort	(8, 50)	1275 vs 1332	1117 vs 1151	vs 1099	vs 1151	vs 1081	vs 1013	10%
	(16, 100)	5783 vs 6211	3054 vs 2980	vs 2877	vs 2916	vs 2702	vs 2661	15%
	(32, 150)	13579 vs 14228	9218 vs 8745	vs 8256	vs 8617	vs 8107	vs 8013	15%

Execution times (ms) for various optimizations on programs with increasing input size.

n vs m : n is execution time for standard rules, m is execution time for comprehensions

vs m : m is execution time for comprehensions

- Code reduction + performance improvement
- Its preliminary, but promising

Outline

- 1 Introduction
- 2 Comprehensions in CHR^{cp}
- 3 Compilation
- 4 Implementation
- 5 Conclusion

Conclusion

- CHR^{cp} : Multiset rewriting + **comprehension patterns**
- Optimized compilation scheme:
 - Implements operational semantics of CHR^{cp}
 - Optimization schemes (OJO), (Bt), (Mono) and (Uniq)
 - Promising preliminary results (code reduction + improved performance)

Future Works

- Decentralized execution [?]:
 - over traditional computer networks
 - P2P applications on mobile devices (Android SDK)
- Logical interpretation of comprehensions:
 - Logical proof system for CHR^{cp}
 - Linear logic with appropriate extensions

Thank you for your attention!

Questions please?

Bibliography