Cryptographic Protocol Analysis via Strand Spaces: Authentication Tests

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Arrows between Nodes

$n_1 \to n_2$	means	n_1, n_2 are nodes with term$(n_1) = +t$, term$(n_2) = -t$	
	i.e.	n_2 receives t from n_1	
$n_1 \Rightarrow n_2$	means	$n_1 = s \downarrow i$, $n_2 = s \downarrow i + 1$	
$n_1 \Rightarrow^* n_2$	means	$n_1 = s \downarrow i$, $n_2 = s \downarrow j$,	where $i \leq j$
$n_1 \Rightarrow^+ n_2$	means	$n_1 = s \downarrow i$, $n_2 = s \downarrow j$,	where $i < j$

- $n_1 \to n_2$ and $n_1 \Rightarrow n_2$ express a *causal* dependence of n_2 on n_1
- Nodes and edges $\to$, $\Rightarrow$ in Σ form a graph G_Σ

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Bundles

- A subgraph $\mathcal{C}$ of G_Σ is a *bundle* if $\mathcal{C}$ is finite and causally well-grounded, which means:
  1. If $n_2 \in \mathcal{C}$ negative, there is a unique $n_1 \rightarrow n_2$ in $\mathcal{C}$ (everything heard was said)
  2. If $s \downarrow i+1 \in \mathcal{C}$, then $s \downarrow i \Rightarrow s \downarrow i+1$ in $\mathcal{C}$ (everyone starts at the beginning)
  3. $\mathcal{C}$ is acyclic (time never flows backward)
- Causal partial ordering $n_1 \preceq_\mathcal{C} n_2$ means n_2 reachable from n_1 via arrows in $\mathcal{C}$

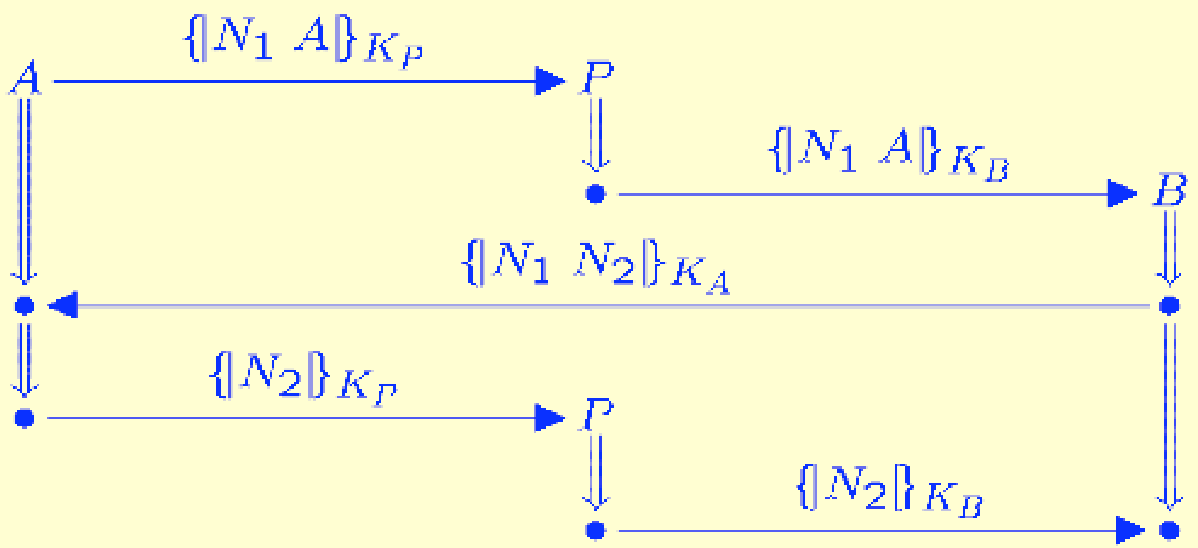
  Induction: Every non-empty set $S \subset \mathcal{C}$ of nodes contains $\preceq_\mathcal{C}$-minimal members

Representing Protocols

- Problem: Given a picture, define some sets of traces
- Associate keys with principals
  - The public key of. . .
  - The long term key shared between server and. . .
  - Ranges disjoint from potential session keys
- Define the behavior
  - Identify roles
  - Note terms that cannot be checked
  - Find parameters for each role
  - List signed terms
- Identify uniquely originating values
  - Nonces
  - Session keys emitted by server

Authentication Tests, I: Outgoing Tests

Needham-Schroeder: Undesirable Run



Due to Gavin Lowe (1995)

Example: NS

- Roles: Initiator, responder; Parameters: A, B, N_a, N_b
  - All terms can be checked
  - Uses K_A to mean "The public key of A"
  - List of terms: (signs depend on role)

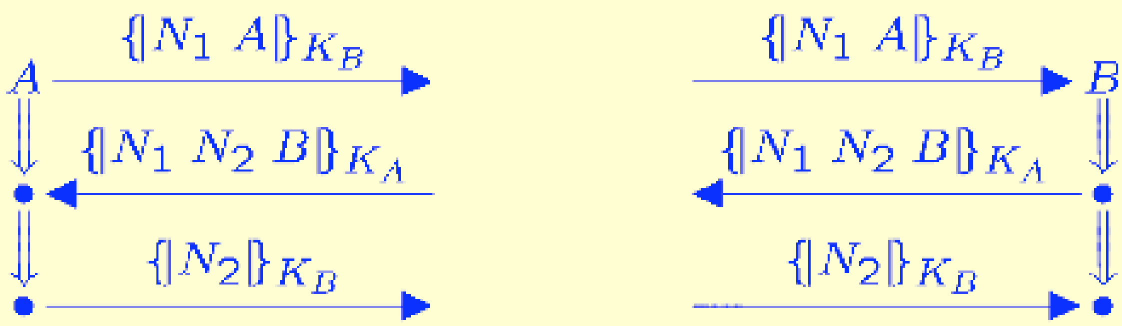
    $\{|N_a\ A|\}_{K_B}, \quad \{|N_a\ N_b|\}_{K_A}, \quad \{|N_b|\}_{K_B}$
  - Values intended to originate uniquely:

    $N_a,\ N_b$
- NSInit$[A, B, N_a, N_b]$: set of strands with trace

  $+\{|N_a\ A|\}_{K_B}, \quad -\{|N_a\ N_b|\}_{K_A}, \quad +\{|N_b|\}_{K_B}$
- NSLResp$[A, B, N_a, N_b]$: set of strands with trace

  $-\{|N_a\ A|\}_{K_B}, \quad +\{|N_a\ N_b|\}_{K_A}, \quad -\{|N_b|\}_{K_B}$

Needham-Schroeder-Lowe Protocol



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NSL: Responder's Guarantee

- Suppose:

  K_A^{-1} uncompromised

  N_2 uniquely originating
- Responder's edge $\{\!|N_1\ N_2\ B|\!\}_{K_A} \Rightarrow \{\!|N_2|\!\}_{K_B}$ is a test
  - Penetrator can't decrypt $\{\!|N_1\ N_2\ B|\!\}_{K_A}$
  - Super-encrypting does no good
  - Penetrator's only choice: discard it or deliver it?
- If responder receives $\{\!|N_2|\!\}_{K_B}$ then it was delivered
  - But to whom? Which regular strands will receive, change $\{\!|N_1\ N_2\ B|\!\}_{K_A}$?
  - Only regular strand NSInit$[A, B, N_1, N_2]$, at node 2

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Original NS Responder's Guarantee

- Suppose again:
  K_A^{-1} uncompromised
  N_2 uniquely originating
- Responder's edge $\{\!|N_1\ N_2|\!\}_{K_A} \Rightarrow \{\!|N_2|\!\}_{K_B}$ is a test
  - Penetrator can't decrypt $\{\!|N_1\ N_2|\!\}_{K_A}$
  - Super-encrypting does no good
  - Penetrator's only choice: discard it or deliver it?
- If responder receives $\{\!|N_2|\!\}_{K_B}$ then it was delivered
  - But to whom?
  - Only regular strand NSInit$[A, *, N_1, N_2]$ can receive $\{\!|N_1\ N_2|\!\}_{K_A}$ and change it
- Whoops: What if $* \neq B$?

The Algebra of Terms: Components

- Term t_0 is a component of t, written $\boxed{t_0} \sqsubset t$

  If $t \in T \cup K$, then $\boxed{t} \sqsubset t$

  If $t = \{\!|h|\!\}_K$, then $\boxed{t} \sqsubset t$

  $\boxed{t_0} \sqsubset g$ implies $\boxed{t_0} \sqsubset g\ h$

  $\boxed{t_0} \sqsubset h$ implies $\boxed{t_0} \sqsubset g\ h$

- A component is a "largest non-concatenated part"
  Penetrator fully controls concatenation anyway

- Components of $N_b\ \{\!|K\ N_b\ A|\!\}_{K_B}\ \{\!|N_a\ B\ K|\!\}_{K_A}$

  N_b
  $\{\!|K\ N_b\ A|\!\}_{K_B}$
  $\{\!|N_a\ B\ K|\!\}_{K_A}$

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The Anatomy of the Case, I

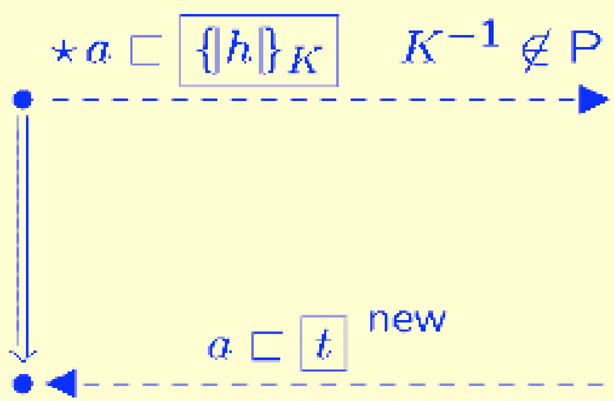
- Find values uniquely originating on $s_r \in \mathsf{NSLResp}[A, B, N_a, N_b]$
  - N_b only, in $\{\!|N_a\ N_b\ B|\!\}_{K_A}$ on node $n_0 = s_r \downarrow 2$
- Find negative (receiving) nodes containing value N_b

$$n_1 = s_r \downarrow 3 \text{ with term } \{\!|N_b|\!\}_{K_B}$$

check: K_A^{-1} is not "penetrable" $(*)$
$\{\!|N_a\ N_b\ B|\!\}_{K_A}$ not a subterm of a regular node
N_b occurs in only one component of n_0

- Therefore, $n_0 \Rightarrow n_1$ is an "outgoing test"

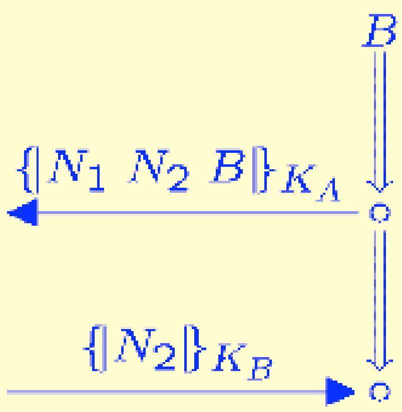
Outgoing Test



a uniquely originates at $\star$

$\boxed{t}$ means a component

NSL Responder Test



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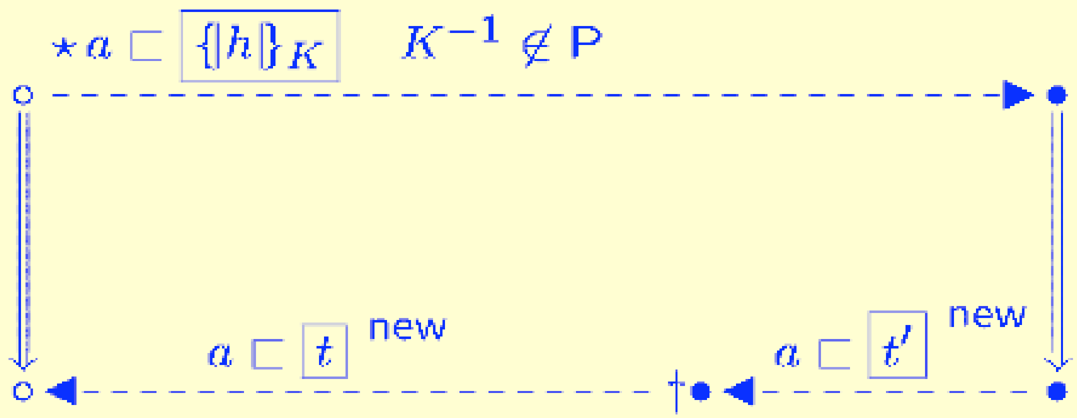
The Anatomy of the Case, II

- Since $n_0 \Rightarrow n_1$ is an outgoing test, there's a regular transforming edge $m_0 \Rightarrow m_1$ such that
  - $\boxed{\{|N_a\ N_b\ B|\}_{K_A}} \sqsubset \mathsf{term}(m_0)$
  - m_0 negative (receiving)
  - m_1 contains N_b in a new component
- Inspecting protocol, $m_0 = s_i \downarrow 2$, where $s_i \in \mathsf{NSLInit}[A, B, N_a, N_b]$, so
  - $m_1 = s_i \downarrow 3$
  - s_i has $\mathcal{C}$-height 3
- This is the NSL responder's guarantee

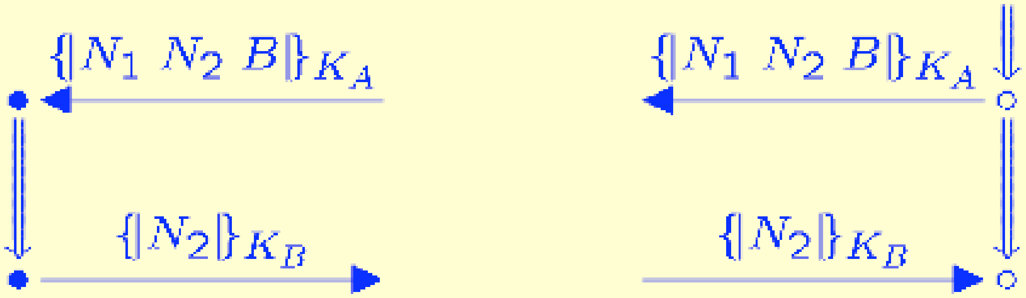
Outgoing test Authentication



"•" means the test shows this regular node exists

† this node depends on extra conditions

NSL Responder Authentication



Outgoing test establishes

- nodes present and non-penetrator

Anatomy of Original NS

- Part I identifies outgoing test, as in NSL
- Since $n_0 \Rightarrow n_1$ is an outgoing test, there's a regular $m_0 \Rightarrow m_1$ such that
  - $\boxed{\{\!|N_a\ N_b|\!\}_{K_A}} \sqsubset \mathsf{term}(m_0)$
  - m_0 negative (receiving)
- Inspecting protocol, $m_0 = s_i \downarrow 2$, where $s_i \in \mathsf{NSInit}[A, *, N_a, N_b]$, so
  - $m_1 = s_i \downarrow 3$
  - s_i has $\mathcal{C}$-height 3
- This is the NS responder's guarantee; B unconstrained

NSL Initiator's Guarantee, I

- Suppose:

  K_B^{-1} uncompromised

  N_1 uniquely originating
- Initiator's edge $\{\!|N_1\ A|\!\}_{K_B} \Rightarrow \{\!|N_1\ N_2\ B|\!\}_{K_A}$ is a test
  - Penetrator can't decrypt $\{\!|N_1\ A|\!\}_{K_B}$
  - Super-encrypting does no good
  - Penetrator's only choice: discard it or deliver it?
- If initiator receives $\{\!|N_1\ N_2\ B|\!\}_{K_A}$ then it was delivered
  - But to whom? Which regular strands will receive, change $\{\!|N_1\ A|\!\}_{K_B}$?
  - Only regular strand $s_r \in \text{NSResp}[A, B, N_1, *]$, at node 1

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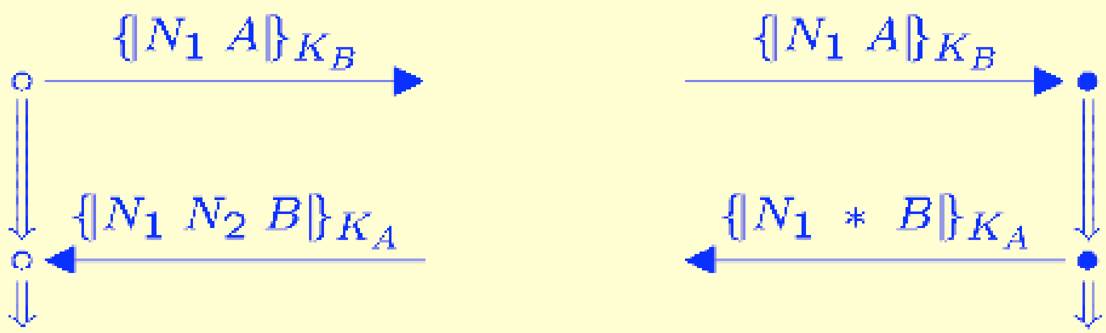
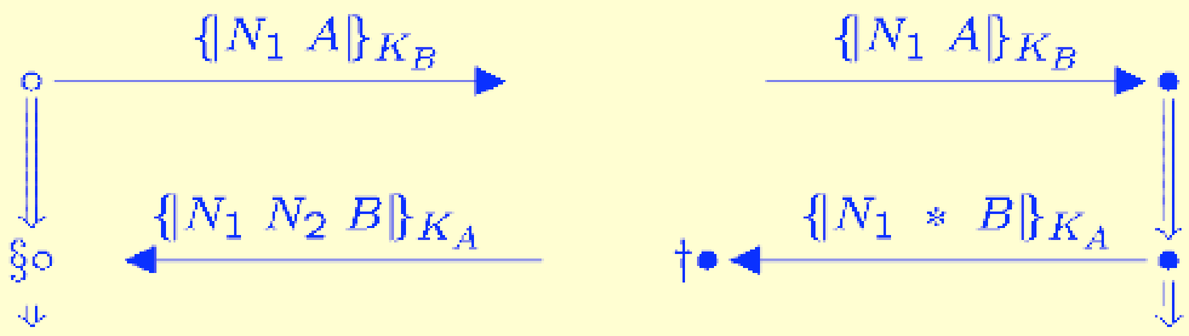
NSL Initiator Authentication, I



NSL Initiator's Guarantee, II

- Suppose also K_A^{-1} uncompromised
- Penetrator choice: discard or deliver $\{\!|N_1 * B|\!\}_{K_A}$
  - Must have delivered it to some regular strand, an initiator strand NSInit$[A, B, N_1, *]$
  - But N_1 originates uniquely
- So $* = N_2$ and $s_r \in$ NSResp$[A, B, N_1, N_2]$
- Uses additional node in Outgoing Test Authentication

NSL Initiator Authentication, II

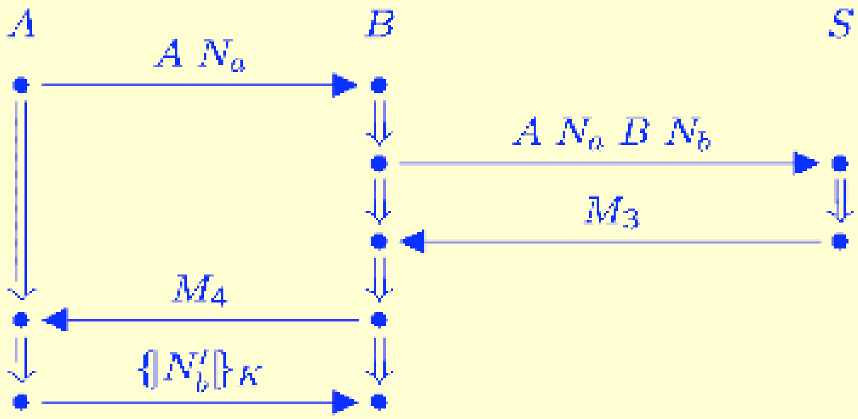


$\S\circ = \dagger\bullet$

because of unique origination of N_a

Authentication Tests, II: Incoming Tests

Carlsen



$$M_3 = \{\!|K\ N_b\ A|\!\}_{K_B}\ \{\!|N_a\ B\ K|\!\}_{K_A}$$
$$M_4 = \{\!|N_a\ B\ K|\!\}_{K_A}\ \{\!|N_a|\!\}_K\ N_b'$$

Example: Carlsen, I

- Roles: Initiator, responder, server;
  Parameters: A, B, N_a, N_b, K, N_b'
  - B cannot check $\{|N_a\ B\ K|\}_{K_A}$, part of M_3
  - Uses K_A to mean "Long term shared key of A"
- Values intended to originate uniquely:
  - Nonces N_a, N_b, N_b'
  - Session key K
- Obligations of key server:
  - Never re-use session key K
  - Never use long-term key K_A as session key
  - Never chooses value known initially to penetrator

Example: Carlsen, II

- CInit$[A, B, N_a, K, N_b']$: set of strands with trace

  $+A\ N_a, \quad -\{\!|N_a\ B\ K|\!\}_{K_A}\ \{\!|N_a|\!\}_K\ N_b', \quad +\{\!|N_b'|\!\}_K$

- CResp$[A, B, N_a, N_b, K, N_b', H]$: set of strands with trace

  $-A\ N_a, \quad +A\ N_a\ B\ N_b, \quad -\{\!|K\ N_b\ A|\!\}_{K_B}\ H,$

  $+H\ \{\!|N_a|\!\}_K\ N_b', \quad -\{\!|N_b'|\!\}_K$

- CServ$[A, B, N_a, N_b, K]$: set of strands with trace

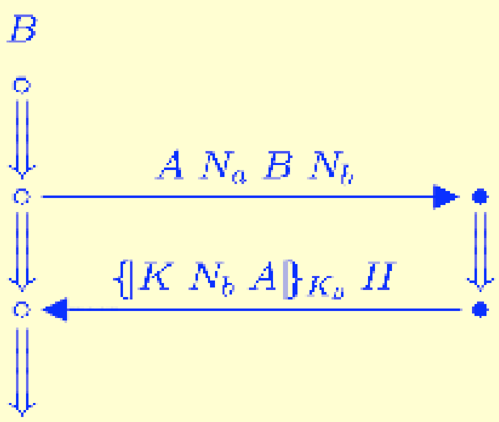
  $-A\ N_a\ B\ N_b, \quad +\{\!|K\ N_b\ A|\!\}_{K_B}\ \{\!|N_a\ B\ K|\!\}_{K_A}$

Incoming Tests Example: Carlsen

- Carlsen protocol uses different pattern
  - Nonce transmitted as plaintext
  - Received back in encrypted form
  - Demonstrates possession of key
- "Incoming test" because transforming edge act creates the encrypted unit received by strand

Responder Authenticates Server

Assume $K_B \notin K_\mathcal{P}$



$\bullet^s$ can only lie on CServ$[A, B, *, N_b, K]$

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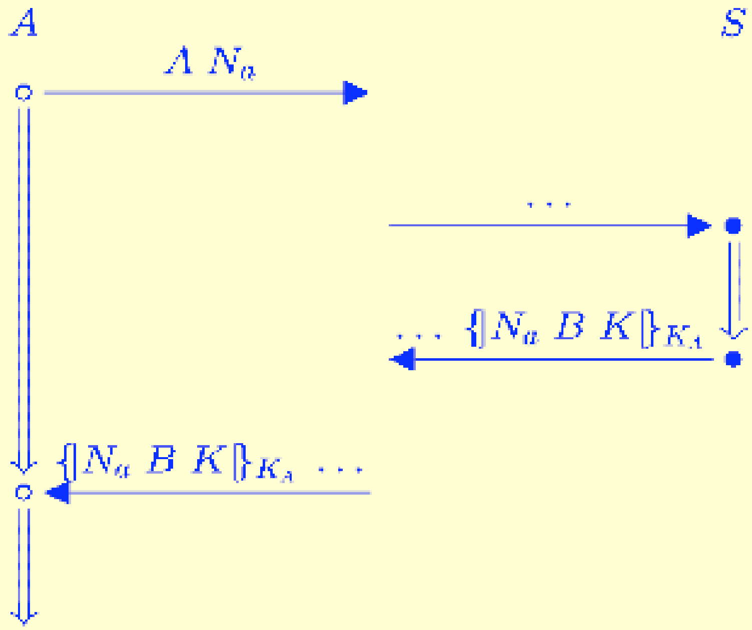
Initiator Authenticates Server

Assume $K_A \notin K_{\mathcal{P}}$



$\bullet^s$ can only lie on CServ$[A, B, N_a, *, K]$

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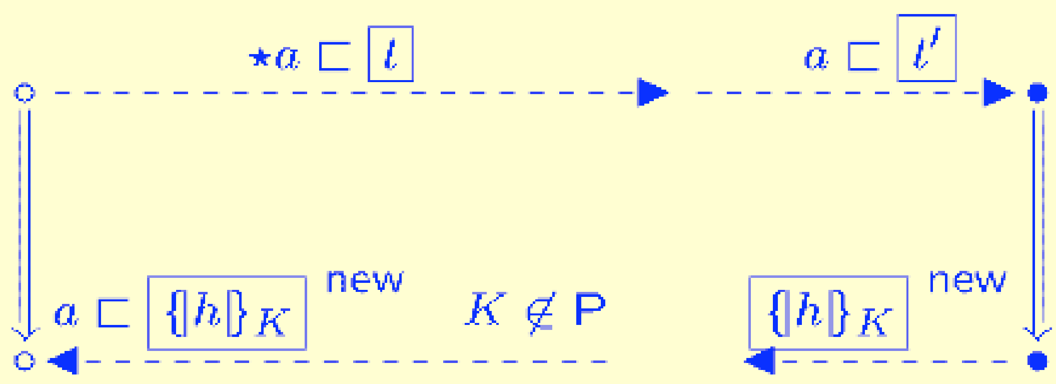
Incoming Test Authentication



Authentication Tests, III: Secrecy

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Penetrable Keys P

- Penetrable keys: the idea
  - If $K \in K_{\mathcal{P}}$, then it's penetrable
  - If K is uttered in $\{\!|\cdots K \cdots|\!\}_{K_0}$ and K_0^{-1} is already penetrable, then K is penetrable
- More rigorously:
  1. $K \in P_0$ if $K \in K_{\mathcal{P}}$
  2. $K \in P_{i+1}$ if exists n regular, positive, with $K \sqsubset_{\mathfrak{K}} \boxed{t}^{new} \sqsubset \text{term}(n)$, and $K_0 \in \mathfrak{K}$ implies $K_0^{-1} \in P_i$
  3. $P = \bigcup_i P_i$
- Carlsen example: $\{\!|K\ N_b\ A|\!\}_{K_B}\ \{\!|N_a\ B\ K|\!\}_{K_A}$

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Safe Keys $\mathcal{S}$, Disjoint from $\mathcal{P}$

Idea: S_0 immediately safe: no one says it new
$K \in S_{i+1}$ if whenever said newly, protected by some K_0 where $K_0^{-1} \in S_i$

1. $K \in S_0$ if $K \notin K_{\mathcal{P}}$ and $K \not\sqsubset \boxed{t}^{new} \sqsubset \mathrm{term}(n)$ for all n positive regular
2. $K \in S_{i+1}$ if $K \notin K_{\mathcal{P}}$ and $t = \cdots \{\!|\cdots K \cdots|\!\}_{K_0} \cdots$ with $K_0^{-1} \in S_i$, whenever $K \sqsubset \boxed{t}^{new} \sqsubset \mathrm{term}(n)$, for all n positive regular
3. $S = \bigcup_i S_i$

$\{\!|K\ N_b\ A|\!\}_{K_B}\ \{\!|N_a\ B\ K|\!\}_{K_A}$

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Safe Keys: Examples

- Needham-Schroeder

  $K_A^{-1} \not\sqsubset \text{term}(n)$ for all n positive regular,

  so $K_A^{-1} \notin K_{\mathcal{P}}$ implies $K_A^{-1} \in S_0$
- Carlsen initiator
  1. $K_A, K_B \notin K_{\mathcal{P}}$ implies $K_A, K_B \in S_0$
  2. $s_s \in \text{CServ}[A, B, N_a, *, K]$ implies K originates

     only on s_s, in $\{\!|K \; * \; A|\!\}_{K_B} \; \{\!|N_a \; B \; K|\!\}_{K_A}$
  3. Hence, $K \in S_1$
- Similar for Carlsen responder with $s_s \in \text{CServ}[A, B, *, N_b, K]$

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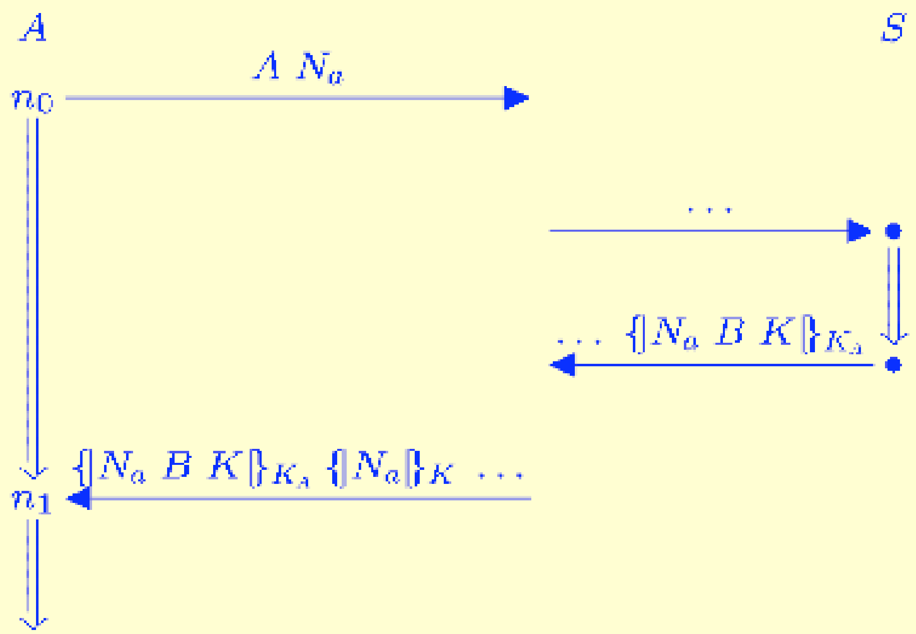
Authentication Tests, IV: Using Safe Keys

Carlsen: Initiator Authenticates Responder

Assume $K_A, K_B \notin K_{\mathcal{P}}$



Since $K \in \mathsf{S}_1$, $n_0 \Rightarrow n_1$ incoming test for N_a in $\{\!|N_a|\!\}_K$

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Carlsen: Initiator Authenticates Responder, I

- Suppose $s_i \in$ CInit$[A, B, N_a, K, *]$ with $\mathcal{C}$-height $\geq$2
  - So there is $s_{s,1} \in$ CServ$[A, B, N_a, *, K]$
  - $K \in S_1$
  - Exists transforming edge $m_0 \Rightarrow^+ m_1$ with $\boxed{\{\!|N_a|\!\}_K}^{new} \sqsubset$ term(m_1)
- Where is m_1? Two cases:
  1. $m_1 = s \downarrow 4$ for $s \in$ CResp$[X, Y, N_a, *, K, *, *]$
  2. $m_1 = s \downarrow 3$ for $s \in$ CInit$[X, Y, *, K, N_a]$

1. Then $s_{s,2} \in$ CServ$[X, Y, *, *, K]$ has $\mathcal{C}$-height 2
   so $s_{s,1} = s_{s,2}$, and $X = A$, $Y = B$
   hence $s \in$ CResp$[A, B, N_a, *, K, *, *]$

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Carlsen: Initiator/Responder, Case 2

- Assuming: $m_1 = s \downarrow 3$ for $s \in$ CInit$[X, Y, *, K, N_a]$
- There's $s_{s,2} \in$ CServ$[X, Y, N, *, K]$,
  so $s_{s,1} = s_{s,2}$, and $X = A$, $Y = B$, $N = N_a$
- Since N_a originates on both s_i and s,
  and N_a originates uniquely,
  $s_i = s$
- Hence $s_i \downarrow 3 \prec_C s_i \downarrow 2$

  contradicting acyclicity clause
  in definition of bundle

  (i.e. Case 2 is impossible)

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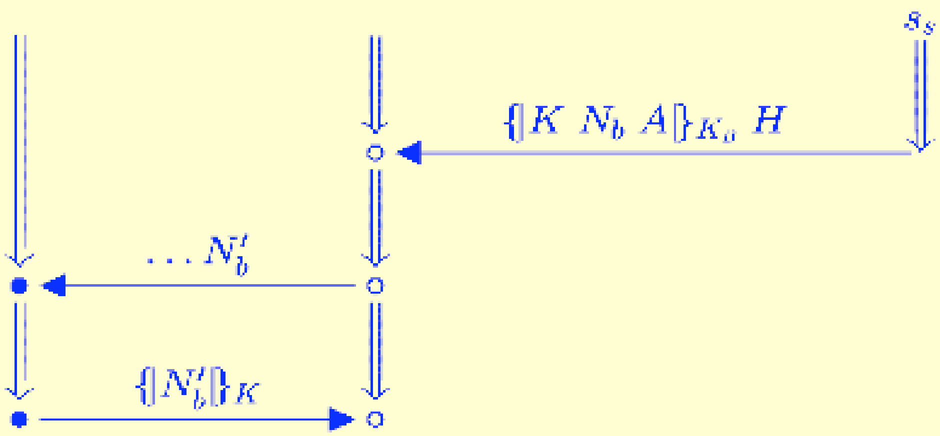
Carlsen: Responder Authenticates Initiator

Assuming $N_a \neq N_b'$, and
$K_A, K_B \notin K_\mathcal{P}$, so $K \in S_1$



Since $K \in S_1$, $n_0 \Rightarrow n_1$ incoming test for N_b' in $\{\!|N_b'|\!\}_K$

Carlsen: Responder Authenticates Initiator, I

- Suppose $s_r \in$ CResp$[A, B, N_a, N_b, K, N_b', *]$ with $\mathcal{C}$-height 5
  - There is $s_{s,1} \in$ CServ$[A, B, *, N_b, K]$ and $K \in S_1$
  - Exists transforming edge $m_0 \Rightarrow^+ m_1$ with

    $\boxed{\{\!|N_b'|\!\}_K}^{new} \sqsubset$ term(m_1)
- Where is m_1? Two cases:
  1. $m_1 = s \downarrow 4$ for $s \in$ CResp$[X, Y, N_b', N, K, *, *]$
  2. $m_1 = s \downarrow 3$ for $s \in$ CInit$[X, Y, *, K, N_b']$

1. Then $s_{s,2} \in$ CServ$[X, Y, *, N, K]$ has $\mathcal{C}$-height 2
   so $s_{s,1} = s_{s,2}$, and $X = A$, $Y = B$, and $N = N_b$
   hence $s \in$ CResp$[A, B, N_b', N_b, K, *, *]$
   Since N_b originates uniquely, $s = s_r$, so $N_a = N_b'$ $\rightarrow\leftarrow$

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Carlsen: Responder/Initiator, Case 2

- Assuming: $m_1 = s \downarrow 3$ for $s \in$ CInit$[X, Y, N, K, N'_b]$
- There's $s_{s,2} \in$ CServ$[X, Y, *, *, K]$,
  so $s_{s,1} = s_{s,2}$, and $X = A$, $Y = B$,
  hence $s \in$ CInit$[A, B, N, K, N'_b]$
- Moreover, $N = N_a$:
  - By Initiator's guarantee,
    $\exists s' \in$ CResp$[A, B, N, *, K, N'_b, *]$
    but N'_b originates uniquely, so $s_r = s'$
- This completes analysis of Carlsen's protocol
- By the way:
  The server gets no guarantees

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