15-411: Dynamic Semantics

Jan Hoffmann

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Sign up for code review!

Dynamic Semantics

- **Static semantics:** definition of valid programs
- **Dynamic semantics:** definition of how programs are executed
- So far: Dynamic semantics is given in English on lab handouts
  - This only works since you know how C programs should behave
  - Sometimes needed to consult the reference compiler
- A description in English will always be ambiguous

➡ **Need precise ways of defining the meaning of programs**

Types of (Formal) Dynamic Semantics

- **Denotational Semantics:** Abstract and elegant.
  - Each part of a program is associated with a denotation (math. object)
  - For example: a procedure is associated with a mathematical function
- **Axiomatic Semantics:** Strongly related to program logic.
  - Gives meaning to phrases using logical axioms
  - The meaning is identical to the set of properties that can be proved
- **Operational Semantics:** Describes how programs are executed
  - Related to interpreters and abstract machines
  - Most popular and flexible form of semantics

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Dana Scott

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Tony Hoare

Gordon Plotkin

Operational Semantics

- **Many different styles**
  - Natural semantics (or big-step semantics or evaluation dynamics)
  - Structural operational semantics
  - Substructural operational semantics
  - Abstract machine (or small-step with continuation)
- **We will use an abstract machine**
  - Very general: can describe non-termination, concurrency, …
  - Low-level and elaborate

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How to pick the right dynamic semantics?

Evaluating Expressions

Continuations

Want to model a single evaluation step

$$e \to e'$$

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$$e \triangleright K$$

“Evaluate expression e and pass the result to K”

The continuation has a ‘hole’ for the result value of e.

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Use a continuation K:

A stack of partial computations.

$$e \rhd K$$

“Evaluate expression e and pass the result to K”

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Evaluation Rules: Addition

$$e_1 + e_2 \triangleright K \qquad \longrightarrow \qquad e_1 \triangleright (\_ + e_2 \ , K)$$

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First evaluate e1.

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Plug the result here.

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$$e_1 + e_2 \triangleright K \qquad \longrightarrow \qquad e_1 \triangleright (\_ + e_2 \ , \ K)$$

$$c_1 \triangleright (\_ + e_2 \ , \ K) \qquad \longrightarrow \qquad e_2 \triangleright (c_1 + \_ \ , \ K)$$

Evaluation Rules: Addition

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Plug the result here.

$$e_1 + e_2 \rhd K \quad \longrightarrow \quad e_1 \rhd (\_ + e_2 \ , \ K)$$

e is a constant.

$$c_1 \rhd (\_ + e_2 \ , \ K) \quad \longrightarrow \quad e_2 \rhd (c_1 + \_ \ , \ K)$$

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Continuation is an addition.

Evaluation Rules: Addition

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$$e_1 + e_2 \rhd K \quad\longrightarrow\quad e_1 \rhd (\_ + e_2\ ,\ K)$$

e is a constant.

Continue with evaluating e2.

$$c_1 \rhd (\_ + e_2\ ,\ K) \quad\longrightarrow\quad e_2 \rhd (c_1 + \_\ ,\ K)$$

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Evaluation Rules: Addition

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Two constants

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Continuation is an addition.

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Two constants

Actual addition.

Evaluation Rules: Binary Operations

Arithmetic operations are treated like addition

$$e_1 \oplus e_2 \rhd K \quad \longrightarrow \quad e_1 \rhd (\_ \oplus e_2 \ , K)$$

$$c_1 \rhd (\_ \oplus e_2 \ , K) \quad \longrightarrow \quad e_2 \rhd (c_1 \oplus \_ \ , K)$$

$$c_2 \rhd (c_1 \oplus \_ \ , K) \quad \longrightarrow \quad c \rhd K \qquad (c = c_1 \oplus c_2 \bmod 2^{32})$$

Arithmetic is modulo 2^{32} to match our x86 architecture

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Arithmetic is modulo 2^{32} to match our x86 architecture

What about effects?

Evaluation Rules: Binops with Effects

In case of an arithmetic exception: Abort the computation and report and error

$$
\begin{array}{llll}
e_1 \oslash e_2 \rhd K & \longrightarrow & e_1 \rhd (\_ \oslash e_2 \ , K) & \\
c_1 \rhd (\_ \oslash e_2 \ , K) & \longrightarrow & e_2 \rhd (c_1 \oslash \_ \ , K) & \\
c_2 \rhd (c_1 \oslash \_ \ , K) & \longrightarrow & c \rhd K & (c = c_1 \oslash c_2) \\
c_2 \rhd (c_1 \oslash \_ \ , K) & \longrightarrow & \mathsf{exception}(\mathsf{arith}) & (c_1 \oslash c_2 \text{ undefined})
\end{array}
$$

There is no rule for further evaluating an exception.

Example Evaluation

$$((4+5)*10)+2 \quad \triangleright \quad \cdot$$

Example Evaluation

$$
\begin{array}{llll}
 & ((4+5)*10)+2 & \triangleright & \cdot \\
\longrightarrow & (4+5)*10 & \triangleright & \_+2
\end{array}
$$

Example Evaluation

$$
\begin{array}{rlcl}
 & ((4+5)*10)+2 & \triangleright & \cdot \\
 & & & \\
\longrightarrow & (4+5)*10 & \triangleright & \_ + 2 \\
\longrightarrow & 4+5 & \triangleright & \_ * 10 \; , \; \_ + 2
\end{array}
$$

Example Evaluation

$$
\begin{array}{llll}
 & ((4+5)*10)+2 & \triangleright & \cdot \\
\\
\longrightarrow & (4+5)*10 & \triangleright & \_+2 \\
\longrightarrow & 4+5 & \triangleright & \_*10\ ,\ \_+2 \\
\longrightarrow & 4 & \triangleright & \_+5\ ,\ \_*10\ ,\ \_+2
\end{array}
$$

Example Evaluation

$$
\begin{array}{lllll}
 & ((4+5)*10)+2 & \rhd & \cdot \\
\longrightarrow & (4+5)*10 & \rhd & \_+2 \\
\longrightarrow & 4+5 & \rhd & \_*10\ ,\ \_+2 \\
\longrightarrow & 4 & \rhd & \_+5\ ,\ \_*10\ ,\ \_+2 \\
\longrightarrow & 5 & \rhd & 4+\_\ ,\ \_*10\ ,\ \_+2
\end{array}
$$

Example Evaluation

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\\
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\longrightarrow & 4+5 & \triangleright & \_*10\ ,\ \_+2 \\
\longrightarrow & 4 & \triangleright & \_+5\ ,\ \_*10\ ,\ \_+2 \\
\longrightarrow & 5 & \triangleright & 4+\_\ ,\ \_*10\ ,\ \_+2 \\
\longrightarrow & 9 & \triangleright & \_*10\ ,\ \_+2
\end{array}
$$

Example Evaluation

$$
\begin{array}{llll}
 & ((4+5)*10)+2 & \triangleright & \cdot \\
\\
\longrightarrow & (4+5)*10 & \triangleright & \_ + 2 \\
\longrightarrow & 4+5 & \triangleright & \_ * 10 \;,\; \_ + 2 \\
\longrightarrow & 4 & \triangleright & \_ + 5 \;,\; \_ * 10 \;,\; \_ + 2 \\
\longrightarrow & 5 & \triangleright & 4 + \_ \;,\; \_ * 10 \;,\; \_ + 2 \\
\longrightarrow & 9 & \triangleright & \_ * 10 \;,\; \_ + 2 \\
\longrightarrow & 10 & \triangleright & 9 * \_ \;,\; \_ + 2
\end{array}
$$

Example Evaluation

$$
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& ((4+5)*10)+2 & \rhd & \cdot \\
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\longrightarrow & 90 & \rhd & \_+2
\end{array}
$$

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\longrightarrow & 90 & \triangleright & \_+2 \\
\longrightarrow & 2 & \triangleright & 90+\_
\end{array}
$$

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\begin{array}{llll}
 & ((4+5)*10)+2 & \triangleright & \cdot \\
 & & & \\
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\longrightarrow & 90 & \triangleright & \_ + 2 \\
\longrightarrow & 2 & \triangleright & 90 + \_ \\
\longrightarrow & 92 & \triangleright & \cdot
\end{array}
$$

Evaluation Rules: End of and Evaluation

If we reach a constant and the empty continuation then we stop

$$c \rhd \cdot \quad \longrightarrow \quad \mathsf{value}(c)$$

Evaluation Rules: Boolean Expressions

$$e_1 \; \texttt{\&\&} \; e_2 \triangleright K \quad \longrightarrow \quad e_1 \triangleright (\_ \; \texttt{\&\&} \; e_2 \; , \; K)$$

$$\mathsf{false} \triangleright (\_ \; \texttt{\&\&} \; e_2 \; , \; K) \quad \longrightarrow \quad \mathsf{false} \triangleright K$$

$$\mathsf{true} \triangleright (\_ \; \texttt{\&\&} \; e_2 \; , \; K) \quad \longrightarrow \quad e_2 \triangleright K$$

true and *false* are also values

(We could also use 1 and 0 but distinguishing helps detect errors.)

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Notice the short-cutting.

$$\mathsf{true} \triangleright (\_ \texttt{ \&\& } e_2 \;,\; K) \quad \longrightarrow \quad e_2 \triangleright K$$

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$$\eta \vdash e \rhd K$$

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Integers or booleans.

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$$\eta \vdash e \triangleright K$$

Variables and Environments II

The rules we have seen so far just carry over

$$\eta \vdash e_1 \oplus e_2 \rhd K \quad\longrightarrow\quad \eta \vdash e_1 \rhd (\_ \oplus e_2\ ,\ K)$$

$$\eta \vdash c_1 \rhd (\_ \oplus e_2\ ,\ K) \quad\longrightarrow\quad \eta \vdash e_2 \rhd (c_1 \oplus \_\ ,\ K)$$

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$$\eta \vdash x \rhd K \quad\longrightarrow\quad \eta \vdash \eta(x) \rhd K$$

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The environment never changes when evaluating expressions

Executing Statements

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Executions of statements don’t pass values to the continuation

Statements have usually an effect on the environment

Machine configurations:

$$\eta \vdash s \blacktriangleright K$$

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$$\eta \vdash \mathsf{seq}(s_1, s_2) \blacktriangleright K \qquad \longrightarrow \qquad \eta \vdash s_1 \blacktriangleright (s_2\ ,\ K)$$

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No ops:

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A terminating execution ends with a nop.

Executing Statements II

Interaction with expressions is straightforward

Assignments:

$$\eta \vdash \mathsf{assign}(x, e) \blacktriangleright K \quad \longrightarrow \quad \eta \vdash e \rhd (\mathsf{assign}(x, \_)\ ,\ K)$$

$$\eta \vdash v \rhd (\mathsf{assign}(x, \_)\ ,\ K) \quad \longrightarrow$$

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Update the environment with new mapping.

Executing Statements III

Conditionals:

$$\eta \vdash \mathsf{if}(e, s_1, s_2) \blacktriangleright K \quad \longrightarrow \quad \eta \vdash e \rhd (\mathsf{if}(\_, s_1, s_2)\ ,\ K)$$

$$\eta \vdash \mathsf{true} \rhd (\mathsf{if}(\_, s_1, s_2), K) \quad \longrightarrow \quad \eta \vdash s_1 \blacktriangleright K$$

$$\eta \vdash \mathsf{false} \rhd (\mathsf{if}(\_, s_1, s_2), K) \quad \longrightarrow \quad \eta \vdash s_2 \blacktriangleright K$$

Executing Statements IV

Loops:

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Not that the following statements are equivalent:

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Non-termination:

$$s_0 \longrightarrow s_1 \longrightarrow s_2 \longrightarrow \cdots$$

We can make an infinite number of steps without reaching a final state

Executing Statements V

Assertions:

$$\eta \vdash \mathsf{assert}(e) \blacktriangleright K \quad \longrightarrow \quad \eta \vdash e \rhd (\mathsf{assert}(\_), K)$$

$$\eta \vdash \mathsf{true} \rhd (\mathsf{assert}(\_), K) \quad \longrightarrow \quad \eta \vdash \mathsf{nop} \blacktriangleright K$$

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$$\eta \vdash \mathsf{false} \rhd (\mathsf{assert}(\_), K) \quad \longrightarrow \quad \mathsf{exception}(\mathsf{abort})$$

Declarations:

$$\eta \vdash \mathsf{decl}(x, \tau, s) \blacktriangleright K \quad \longrightarrow \quad \eta[x \mapsto \mathsf{nothing}] \vdash s \blacktriangleright K$$

Executing Statements V

Assertions:

$$\eta \vdash \mathsf{assert}(e) \blacktriangleright K \quad \longrightarrow \quad \eta \vdash e \rhd (\mathsf{assert}(\_), K)$$

$$\eta \vdash \mathsf{true} \rhd (\mathsf{assert}(\_), K) \quad \longrightarrow \quad \eta \vdash \mathsf{nop} \blacktriangleright K$$

$$\eta \vdash \mathsf{false} \rhd (\mathsf{assert}(\_), K) \quad \longrightarrow \quad \mathsf{exception}(\mathsf{abort})$$

Declarations:

$$\eta \vdash \mathsf{decl}(x, \tau, s) \blacktriangleright K \quad \longrightarrow \quad \eta[x \mapsto \mathsf{nothing}] \vdash s \blacktriangleright K$$

If C0 had shadowing then we would have to be careful here.

Executing Statements V

Assertions:

$$\eta \vdash \mathsf{assert}(e) \blacktriangleright K \quad \longrightarrow \quad \eta \vdash e \rhd (\mathsf{assert}(\_), K)$$

$$\eta \vdash \mathsf{true} \rhd (\mathsf{assert}(\_), K) \quad \longrightarrow \quad \eta \vdash \mathsf{nop} \blacktriangleright K$$

$$\eta \vdash \mathsf{false} \rhd (\mathsf{assert}(\_), K) \quad \longrightarrow \quad \mathsf{exception}(\mathsf{abort})$$

Declarations:

$$\eta \vdash \mathsf{decl}(x, \tau, s) \blacktriangleright K \quad \longrightarrow \quad \eta[x \mapsto \mathsf{nothing}] \vdash s \blacktriangleright K$$

If C0 had shadowing then we would have to be careful here.

Final states:

$$\mathsf{exception}(E), \qquad \mathsf{nop} \blacktriangleright \cdot$$

Example: Infinite Loop

$\mathsf{while}(x > 0, \mathsf{assign}(x, x+1)) \qquad \eta = [x \mapsto 1]$

$\longrightarrow$

$\longrightarrow$

$\longrightarrow$

$\longrightarrow$

$\longrightarrow$

$\longrightarrow$

$\longrightarrow$

$\longrightarrow$

$\longrightarrow$

$\longrightarrow$

$\longrightarrow$

$\longrightarrow$

$\longrightarrow$

$\longrightarrow$

$\longrightarrow$

Example: Infinite Loop

$\mathsf{while}(x > 0, \mathsf{assign}(x, x+1)) \qquad \eta = [x \mapsto 1]$

$$[x \mapsto 1] \vdash \mathsf{while}(x > 0, s) \quad \blacktriangleright \quad \cdot$$

$\longrightarrow$

$\longrightarrow$

$\longrightarrow$

$\longrightarrow$

$\longrightarrow$

$\longrightarrow$

$\longrightarrow$

$\longrightarrow$

$\longrightarrow$

$\longrightarrow$

$\longrightarrow$

$\longrightarrow$

$\longrightarrow$

$\longrightarrow$

$\longrightarrow$

Example: Infinite Loop

$\mathsf{while}(x > 0, \mathsf{assign}(x, x+1)) \qquad \eta = [x \mapsto 1]$

$$
\begin{array}{ll}
 & [x \mapsto 1] \vdash \mathsf{while}(x > 0, s) \quad \blacktriangleright \quad \cdot \\
\longrightarrow & [x \mapsto 1] \vdash \mathsf{if}(x > 0, \mathsf{seq}(s, \mathsf{while}(x > 0, s)), \mathsf{nop}) \quad \blacktriangleright \quad \cdot \\
\longrightarrow & \\
\longrightarrow & \\
\longrightarrow & \\
\longrightarrow & \\
\longrightarrow & \\
\longrightarrow & \\
\longrightarrow & \\
\longrightarrow & \\
\longrightarrow & \\
\longrightarrow & \\
\longrightarrow & \\
\longrightarrow & \\
\longrightarrow & \\
\longrightarrow & \\
\longrightarrow &
\end{array}
$$

Example: Infinite Loop

$\mathsf{while}(x > 0, \mathsf{assign}(x, x+1)) \qquad \eta = [x \mapsto 1]$

$$
\begin{array}{llll}
 & [x \mapsto 1] \vdash \mathsf{while}(x > 0, s) & \blacktriangleright & \cdot \\
\longrightarrow & [x \mapsto 1] \vdash \mathsf{if}(x{>}0, \mathsf{seq}(s, \mathsf{while}(x{>}0, s)), \mathsf{nop}) & \blacktriangleright & \cdot \\
\longrightarrow & [x \mapsto 1] \vdash x > 0 & \triangleright & \mathsf{if}(\_, \mathsf{seq}(s, \mathsf{while}(x > 0, s)), \mathsf{nop}) \\
\longrightarrow & & & \\
\longrightarrow & & & \\
\longrightarrow & & & \\
\longrightarrow & & & \\
\longrightarrow & & & \\
\longrightarrow & & & \\
\longrightarrow & & & \\
\longrightarrow & & & \\
\longrightarrow & & & \\
\longrightarrow & & & \\
\longrightarrow & & & \\
\longrightarrow & & & \\
\longrightarrow & & &
\end{array}
$$

Example: Infinite Loop

$\mathsf{while}(x > 0, \mathsf{assign}(x, x + 1)) \qquad \eta = [x \mapsto 1]$

$$
\begin{array}{lllll}
 & [x\mapsto 1] \vdash \mathsf{while}(x > 0, s) & \blacktriangleright & \cdot \\
\longrightarrow & [x\mapsto 1] \vdash \mathsf{if}(x>0, \mathsf{seq}(s, \mathsf{while}(x>0, s)), \mathsf{nop}) & \blacktriangleright & \cdot \\
\longrightarrow & [x\mapsto 1] \vdash x > 0 & \triangleright & \mathsf{if}(\_, \mathsf{seq}(s, \mathsf{while}(x > 0, s)), \mathsf{nop}) \\
\longrightarrow & [x\mapsto 1] \vdash x & \triangleright & \_ > 0; \mathsf{if}(\_, \mathsf{seq}(s, \mathsf{while}(x > 0, s)), \mathsf{nop}) \\
\longrightarrow & & & \\
\longrightarrow & & & \\
\longrightarrow & & & \\
\longrightarrow & & & \\
\longrightarrow & & & \\
\longrightarrow & & & \\
\longrightarrow & & & \\
\longrightarrow & & & \\
\longrightarrow & & & \\
\longrightarrow & & & \\
\longrightarrow & & & \\
\longrightarrow & & &
\end{array}
$$

Example: Infinite Loop

$\mathsf{while}(x \; > \; 0, \mathsf{assign}(x, x+1)) \qquad \eta \; = \; [x{\mapsto}1]$

$$
\begin{array}{lll}
 & [x{\mapsto}1] \vdash \mathsf{while}(x > 0, s) & \blacktriangleright \quad \cdot \\
\longrightarrow & [x{\mapsto}1] \vdash \mathsf{if}(x{>}0, \mathsf{seq}(s, \mathsf{while}(x{>}0, s)), \mathsf{nop}) & \blacktriangleright \quad \cdot \\
\longrightarrow & [x{\mapsto}1] \vdash x > 0 & \triangleright \quad \mathsf{if}(\_, \mathsf{seq}(s, \mathsf{while}(x > 0, s)), \mathsf{nop}) \\
\longrightarrow & [x{\mapsto}1] \vdash x & \triangleright \quad \_ > 0; \mathsf{if}(\_, \mathsf{seq}(s, \mathsf{while}(x > 0, s)), \mathsf{nop}) \\
\longrightarrow & [x{\mapsto}1] \vdash 1 & \triangleright \quad \_ > 0; \mathsf{if}(\_, \mathsf{seq}(s, \mathsf{while}(x > 0, s)), \mathsf{nop}) \\
\longrightarrow & & \\
\longrightarrow & & \\
\longrightarrow & & \\
\longrightarrow & & \\
\longrightarrow & & \\
\longrightarrow & & \\
\longrightarrow & & \\
\longrightarrow & & \\
\longrightarrow & & \\
\longrightarrow & & \\
\longrightarrow & &
\end{array}
$$

Example: Infinite Loop

$\mathsf{while}(x > 0, \mathsf{assign}(x, x+1)) \qquad \eta = [x{\mapsto}1]$

$$
\begin{array}{lll}
 & [x{\mapsto}1] \vdash \mathsf{while}(x > 0, s) & \blacktriangleright \quad \cdot \\
\longrightarrow & [x{\mapsto}1] \vdash \mathsf{if}(x{>}0, \mathsf{seq}(s, \mathsf{while}(x{>}0, s)), \mathsf{nop}) & \blacktriangleright \quad \cdot \\
\longrightarrow & [x{\mapsto}1] \vdash x > 0 & \triangleright \quad \mathsf{if}(\_, \mathsf{seq}(s, \mathsf{while}(x > 0, s)), \mathsf{nop}) \\
\longrightarrow & [x{\mapsto}1] \vdash x & \triangleright \quad \_ > 0; \mathsf{if}(\_, \mathsf{seq}(s, \mathsf{while}(x > 0, s)), \mathsf{nop}) \\
\longrightarrow & [x{\mapsto}1] \vdash 1 & \triangleright \quad \_ > 0; \mathsf{if}(\_, \mathsf{seq}(s, \mathsf{while}(x > 0, s)), \mathsf{nop}) \\
\longrightarrow & [x{\mapsto}1] \vdash 0 & \triangleright \quad 1 > \_; \mathsf{if}(\_, \mathsf{seq}(s, \mathsf{while}(x > 0, s)), \mathsf{nop}) \\
\longrightarrow & & \\
\longrightarrow & & \\
\longrightarrow & & \\
\longrightarrow & & \\
\longrightarrow & & \\
\longrightarrow & & \\
\longrightarrow & & \\
\longrightarrow & & \\
\longrightarrow & & \\
\longrightarrow & &
\end{array}
$$

Example: Infinite Loop

$\mathsf{while}(x > 0, \mathsf{assign}(x, x+1)) \qquad \eta = [x \mapsto 1]$

$$
\begin{array}{lllll}
 & [x\mapsto 1] \vdash \mathsf{while}(x > 0, s) & \blacktriangleright & \cdot \\
\longrightarrow & [x\mapsto 1] \vdash \mathsf{if}(x{>}0, \mathsf{seq}(s, \mathsf{while}(x{>}0, s)), \mathsf{nop}) & \blacktriangleright & \cdot \\
\longrightarrow & [x\mapsto 1] \vdash x > 0 & \rhd & \mathsf{if}(\_, \mathsf{seq}(s, \mathsf{while}(x > 0, s)), \mathsf{nop}) \\
\longrightarrow & [x\mapsto 1] \vdash x & \rhd & \_ > 0; \mathsf{if}(\_, \mathsf{seq}(s, \mathsf{while}(x > 0, s)), \mathsf{nop}) \\
\longrightarrow & [x\mapsto 1] \vdash 1 & \rhd & \_ > 0; \mathsf{if}(\_, \mathsf{seq}(s, \mathsf{while}(x > 0, s)), \mathsf{nop}) \\
\longrightarrow & [x\mapsto 1] \vdash 0 & \rhd & 1 > \_; \mathsf{if}(\_, \mathsf{seq}(s, \mathsf{while}(x > 0, s)), \mathsf{nop}) \\
\longrightarrow & [x\mapsto 1] \vdash \mathsf{true} & \rhd & \mathsf{if}(\_, \mathsf{seq}(s, \mathsf{while}(x > 0, s)), \mathsf{nop}) \\
\longrightarrow & & & \\
\longrightarrow & & & \\
\longrightarrow & & & \\
\longrightarrow & & & \\
\longrightarrow & & & \\
\longrightarrow & & & \\
\longrightarrow & & & \\
\longrightarrow & & & \\
\longrightarrow & & &
\end{array}
$$

Example: Infinite Loop

$\mathsf{while}(x > 0, \mathsf{assign}(x, x+1)) \qquad \eta = [x \mapsto 1]$

$$
\begin{array}{lll}
 & [x\mapsto 1] \vdash \mathsf{while}(x > 0, s) & \blacktriangleright \quad \cdot \\
\longrightarrow & [x\mapsto 1] \vdash \mathsf{if}(x{>}0, \mathsf{seq}(s, \mathsf{while}(x{>}0, s)), \mathsf{nop}) & \blacktriangleright \quad \cdot \\
\longrightarrow & [x\mapsto 1] \vdash x > 0 & \triangleright \quad \mathsf{if}(\_, \mathsf{seq}(s, \mathsf{while}(x > 0, s)), \mathsf{nop}) \\
\longrightarrow & [x\mapsto 1] \vdash x & \triangleright \quad \_ > 0; \mathsf{if}(\_, \mathsf{seq}(s, \mathsf{while}(x > 0, s)), \mathsf{nop}) \\
\longrightarrow & [x\mapsto 1] \vdash 1 & \triangleright \quad \_ > 0; \mathsf{if}(\_, \mathsf{seq}(s, \mathsf{while}(x > 0, s)), \mathsf{nop}) \\
\longrightarrow & [x\mapsto 1] \vdash 0 & \triangleright \quad 1 > \_; \mathsf{if}(\_, \mathsf{seq}(s, \mathsf{while}(x > 0, s)), \mathsf{nop}) \\
\longrightarrow & [x\mapsto 1] \vdash \mathsf{true} & \triangleright \quad \mathsf{if}(\_, \mathsf{seq}(s, \mathsf{while}(x > 0, s)), \mathsf{nop}) \\
\longrightarrow & [x\mapsto 1] \vdash \mathsf{seq}(s, \mathsf{while}(x > 0, s)) & \blacktriangleright \quad \cdot \\
\longrightarrow & & \\
\longrightarrow & & \\
\longrightarrow & & \\
\longrightarrow & & \\
\longrightarrow & & \\
\longrightarrow & & \\
\longrightarrow & & \\
\longrightarrow & &
\end{array}
$$

Example: Infinite Loop

$\text{while}(x > 0, \text{assign}(x, x+1)) \qquad \eta = [x \mapsto 1]$

$$
\begin{array}{llll}
 & [x \mapsto 1] \vdash \text{while}(x > 0, s) & \blacktriangleright & \cdot \\
\longrightarrow & [x \mapsto 1] \vdash \text{if}(x{>}0, \text{seq}(s, \text{while}(x{>}0, s)), \text{nop}) & \blacktriangleright & \cdot \\
\longrightarrow & [x \mapsto 1] \vdash x > 0 & \triangleright & \text{if}(\_, \text{seq}(s, \text{while}(x > 0, s)), \text{nop}) \\
\longrightarrow & [x \mapsto 1] \vdash x & \triangleright & \_ > 0; \text{if}(\_, \text{seq}(s, \text{while}(x > 0, s)), \text{nop}) \\
\longrightarrow & [x \mapsto 1] \vdash 1 & \triangleright & \_ > 0; \text{if}(\_, \text{seq}(s, \text{while}(x > 0, s)), \text{nop}) \\
\longrightarrow & [x \mapsto 1] \vdash 0 & \triangleright & 1 > \_; \text{if}(\_, \text{seq}(s, \text{while}(x > 0, s)), \text{nop}) \\
\longrightarrow & [x \mapsto 1] \vdash \text{true} & \triangleright & \text{if}(\_, \text{seq}(s, \text{while}(x > 0, s)), \text{nop}) \\
\longrightarrow & [x \mapsto 1] \vdash \text{seq}(s, \text{while}(x > 0, s)) & \blacktriangleright & \cdot \\
\longrightarrow & [x \mapsto 1] \vdash \text{assign}(x, x+1)) & \blacktriangleright & \text{while}(x > 0, \text{assign}(x, x+1)) \\
\longrightarrow & & & \\
\longrightarrow & & & \\
\longrightarrow & & & \\
\longrightarrow & & & \\
\longrightarrow & & & \\
\longrightarrow & & & \\
\longrightarrow & & &
\end{array}
$$

Example: Infinite Loop

$\mathsf{while}(x\ >\ 0, \mathsf{assign}(x, x+1)) \qquad \eta\ =\ [x{\mapsto}1]$

$$
\begin{array}{llcl}
 & [x{\mapsto}1] \vdash \mathsf{while}(x > 0, s) & \blacktriangleright & \cdot \\
\longrightarrow & [x{\mapsto}1] \vdash \mathsf{if}(x{>}0, \mathsf{seq}(s, \mathsf{while}(x{>}0, s)), \mathsf{nop}) & \blacktriangleright & \cdot \\
\longrightarrow & [x{\mapsto}1] \vdash x > 0 & \triangleright & \mathsf{if}(\_, \mathsf{seq}(s, \mathsf{while}(x > 0, s)), \mathsf{nop}) \\
\longrightarrow & [x{\mapsto}1] \vdash x & \triangleright & \_ > 0; \mathsf{if}(\_, \mathsf{seq}(s, \mathsf{while}(x > 0, s)), \mathsf{nop}) \\
\longrightarrow & [x{\mapsto}1] \vdash 1 & \triangleright & \_ > 0; \mathsf{if}(\_, \mathsf{seq}(s, \mathsf{while}(x > 0, s)), \mathsf{nop}) \\
\longrightarrow & [x{\mapsto}1] \vdash 0 & \triangleright & 1 > \_; \mathsf{if}(\_, \mathsf{seq}(s, \mathsf{while}(x > 0, s)), \mathsf{nop}) \\
\longrightarrow & [x{\mapsto}1] \vdash \mathsf{true} & \triangleright & \mathsf{if}(\_, \mathsf{seq}(s, \mathsf{while}(x > 0, s)), \mathsf{nop}) \\
\longrightarrow & [x{\mapsto}1] \vdash \mathsf{seq}(s, \mathsf{while}(x > 0, s)) & \blacktriangleright & \cdot \\
\longrightarrow & [x{\mapsto}1] \vdash \mathsf{assign}(x, x+1)) & \blacktriangleright & \mathsf{while}(x > 0, \mathsf{assign}(x, x+1)) \\
\longrightarrow & [x{\mapsto}1] \vdash x + 1 & \triangleright & \mathsf{assign}(x, \_)); \mathsf{while}(x > 0, s) \\
\longrightarrow & & & \\
\longrightarrow & & & \\
\longrightarrow & & & \\
\longrightarrow & & & \\
\longrightarrow & & & \\
\longrightarrow & & &
\end{array}
$$

Example: Infinite Loop

$\mathsf{while}(x > 0, \mathsf{assign}(x, x+1)) \qquad \eta = [x \mapsto 1]$

$$
\begin{array}{llll}
 & [x\mapsto 1] \vdash \mathsf{while}(x > 0, s) & \blacktriangleright & \cdot \\
\longrightarrow & [x\mapsto 1] \vdash \mathsf{if}(x{>}0, \mathsf{seq}(s, \mathsf{while}(x{>}0, s)), \mathsf{nop}) & \blacktriangleright & \cdot \\
\longrightarrow & [x\mapsto 1] \vdash x > 0 & \triangleright & \mathsf{if}(\_, \mathsf{seq}(s, \mathsf{while}(x > 0, s)), \mathsf{nop}) \\
\longrightarrow & [x\mapsto 1] \vdash x & \triangleright & \_ > 0; \mathsf{if}(\_, \mathsf{seq}(s, \mathsf{while}(x > 0, s)), \mathsf{nop}) \\
\longrightarrow & [x\mapsto 1] \vdash 1 & \triangleright & \_ > 0; \mathsf{if}(\_, \mathsf{seq}(s, \mathsf{while}(x > 0, s)), \mathsf{nop}) \\
\longrightarrow & [x\mapsto 1] \vdash 0 & \triangleright & 1 > \_; \mathsf{if}(\_, \mathsf{seq}(s, \mathsf{while}(x > 0, s)), \mathsf{nop}) \\
\longrightarrow & [x\mapsto 1] \vdash \mathsf{true} & \triangleright & \mathsf{if}(\_, \mathsf{seq}(s, \mathsf{while}(x > 0, s)), \mathsf{nop}) \\
\longrightarrow & [x\mapsto 1] \vdash \mathsf{seq}(s, \mathsf{while}(x > 0, s)) & \blacktriangleright & \cdot \\
\longrightarrow & [x\mapsto 1] \vdash \mathsf{assign}(x, x+1)) & \blacktriangleright & \mathsf{while}(x > 0, \mathsf{assign}(x, x+1)) \\
\longrightarrow & [x\mapsto 1] \vdash x + 1 & \triangleright & \mathsf{assign}(x, \_)); \mathsf{while}(x > 0, s) \\
\longrightarrow & [x\mapsto 1] \vdash x & \triangleright & \_ + 1; \mathsf{assign}(x, \_)); \mathsf{while}(x > 0, s) \\
\longrightarrow & & & \\
\longrightarrow & & & \\
\longrightarrow & & & \\
\longrightarrow & & & \\
\longrightarrow & & &
\end{array}
$$

Example: Infinite Loop

$\mathsf{while}(x > 0, \mathsf{assign}(x, x+1)) \qquad \eta = [x \mapsto 1]$

$$
\begin{array}{lll}
 & [x\mapsto 1] \vdash \mathsf{while}(x > 0, s) & \blacktriangleright \quad \cdot \\
\longrightarrow & [x\mapsto 1] \vdash \mathsf{if}(x{>}0, \mathsf{seq}(s, \mathsf{while}(x{>}0, s)), \mathsf{nop}) & \blacktriangleright \quad \cdot \\
\longrightarrow & [x\mapsto 1] \vdash x > 0 & \triangleright \quad \mathsf{if}(\_, \mathsf{seq}(s, \mathsf{while}(x > 0, s)), \mathsf{nop}) \\
\longrightarrow & [x\mapsto 1] \vdash x & \triangleright \quad \_ > 0; \mathsf{if}(\_, \mathsf{seq}(s, \mathsf{while}(x > 0, s)), \mathsf{nop}) \\
\longrightarrow & [x\mapsto 1] \vdash 1 & \triangleright \quad \_ > 0; \mathsf{if}(\_, \mathsf{seq}(s, \mathsf{while}(x > 0, s)), \mathsf{nop}) \\
\longrightarrow & [x\mapsto 1] \vdash 0 & \triangleright \quad 1 > \_; \mathsf{if}(\_, \mathsf{seq}(s, \mathsf{while}(x > 0, s)), \mathsf{nop}) \\
\longrightarrow & [x\mapsto 1] \vdash \mathsf{true} & \triangleright \quad \mathsf{if}(\_, \mathsf{seq}(s, \mathsf{while}(x > 0, s)), \mathsf{nop}) \\
\longrightarrow & [x\mapsto 1] \vdash \mathsf{seq}(s, \mathsf{while}(x > 0, s)) & \blacktriangleright \quad \cdot \\
\longrightarrow & [x\mapsto 1] \vdash \mathsf{assign}(x, x+1)) & \blacktriangleright \quad \mathsf{while}(x > 0, \mathsf{assign}(x, x+1)) \\
\longrightarrow & [x\mapsto 1] \vdash x + 1 & \triangleright \quad \mathsf{assign}(x, \_)); \mathsf{while}(x > 0, s) \\
\longrightarrow & [x\mapsto 1] \vdash x & \triangleright \quad \_ + 1; \mathsf{assign}(x, \_)); \mathsf{while}(x > 0, s) \\
\longrightarrow & [x\mapsto 1] \vdash 1 & \triangleright \quad \_ + 1; \mathsf{assign}(x, \_)); \mathsf{while}(x > 0, s) \\
\longrightarrow & & \\
\longrightarrow & & \\
\longrightarrow & & \\
\longrightarrow & &
\end{array}
$$

Example: Infinite Loop

$\mathsf{while}(x > 0, \mathsf{assign}(x, x+1)) \qquad \eta = [x \mapsto 1]$

$$
\begin{array}{lll}
 & [x\mapsto 1] \vdash \mathsf{while}(x > 0, s) & \blacktriangleright \quad \cdot \\
\longrightarrow & [x\mapsto 1] \vdash \mathsf{if}(x{>}0, \mathsf{seq}(s, \mathsf{while}(x{>}0, s)), \mathsf{nop}) & \blacktriangleright \quad \cdot \\
\longrightarrow & [x\mapsto 1] \vdash x > 0 & \triangleright \quad \mathsf{if}(\_, \mathsf{seq}(s, \mathsf{while}(x > 0, s)), \mathsf{nop}) \\
\longrightarrow & [x\mapsto 1] \vdash x & \triangleright \quad \_ > 0; \mathsf{if}(\_, \mathsf{seq}(s, \mathsf{while}(x > 0, s)), \mathsf{nop}) \\
\longrightarrow & [x\mapsto 1] \vdash 1 & \triangleright \quad \_ > 0; \mathsf{if}(\_, \mathsf{seq}(s, \mathsf{while}(x > 0, s)), \mathsf{nop}) \\
\longrightarrow & [x\mapsto 1] \vdash 0 & \triangleright \quad 1 > \_; \mathsf{if}(\_, \mathsf{seq}(s, \mathsf{while}(x > 0, s)), \mathsf{nop}) \\
\longrightarrow & [x\mapsto 1] \vdash \mathsf{true} & \triangleright \quad \mathsf{if}(\_, \mathsf{seq}(s, \mathsf{while}(x > 0, s)), \mathsf{nop}) \\
\longrightarrow & [x\mapsto 1] \vdash \mathsf{seq}(s, \mathsf{while}(x > 0, s)) & \blacktriangleright \quad \cdot \\
\longrightarrow & [x\mapsto 1] \vdash \mathsf{assign}(x, x+1)) & \blacktriangleright \quad \mathsf{while}(x > 0, \mathsf{assign}(x, x+1)) \\
\longrightarrow & [x\mapsto 1] \vdash x + 1 & \triangleright \quad \mathsf{assign}(x, \_)); \mathsf{while}(x > 0, s) \\
\longrightarrow & [x\mapsto 1] \vdash x & \triangleright \quad \_ + 1; \mathsf{assign}(x, \_)); \mathsf{while}(x > 0, s) \\
\longrightarrow & [x\mapsto 1] \vdash 1 & \triangleright \quad \_ + 1; \mathsf{assign}(x, \_)); \mathsf{while}(x > 0, s) \\
\longrightarrow & [x\mapsto 1] \vdash 1 & \triangleright \quad 1 + \_; \mathsf{assign}(x, \_)); \mathsf{while}(x > 0, s) \\
\longrightarrow & & \\
\longrightarrow & & \\
\longrightarrow & &
\end{array}
$$

Example: Infinite Loop

$\mathsf{while}(x\ >\ 0, \mathsf{assign}(x, x + 1)) \qquad \eta\ =\ [x{\mapsto}1]$

$$
\begin{array}{llcl}
 & [x{\mapsto}1] \vdash \mathsf{while}(x > 0, s) & \blacktriangleright & \cdot \\
\longrightarrow & [x{\mapsto}1] \vdash \mathsf{if}(x{>}0, \mathsf{seq}(s, \mathsf{while}(x{>}0, s)), \mathsf{nop}) & \blacktriangleright & \cdot \\
\longrightarrow & [x{\mapsto}1] \vdash x > 0 & \triangleright & \mathsf{if}(\_, \mathsf{seq}(s, \mathsf{while}(x > 0, s)), \mathsf{nop}) \\
\longrightarrow & [x{\mapsto}1] \vdash x & \triangleright & \_ > 0; \mathsf{if}(\_, \mathsf{seq}(s, \mathsf{while}(x > 0, s)), \mathsf{nop}) \\
\longrightarrow & [x{\mapsto}1] \vdash 1 & \triangleright & \_ > 0; \mathsf{if}(\_, \mathsf{seq}(s, \mathsf{while}(x > 0, s)), \mathsf{nop}) \\
\longrightarrow & [x{\mapsto}1] \vdash 0 & \triangleright & 1 > \_; \mathsf{if}(\_, \mathsf{seq}(s, \mathsf{while}(x > 0, s)), \mathsf{nop}) \\
\longrightarrow & [x{\mapsto}1] \vdash \mathsf{true} & \triangleright & \mathsf{if}(\_, \mathsf{seq}(s, \mathsf{while}(x > 0, s)), \mathsf{nop}) \\
\longrightarrow & [x{\mapsto}1] \vdash \mathsf{seq}(s, \mathsf{while}(x > 0, s)) & \blacktriangleright & \cdot \\
\longrightarrow & [x{\mapsto}1] \vdash \mathsf{assign}(x, x + 1)) & \blacktriangleright & \mathsf{while}(x > 0, \mathsf{assign}(x, x + 1)) \\
\longrightarrow & [x{\mapsto}1] \vdash x + 1 & \triangleright & \mathsf{assign}(x, \_)); \mathsf{while}(x > 0, s) \\
\longrightarrow & [x{\mapsto}1] \vdash x & \triangleright & \_ + 1; \mathsf{assign}(x, \_)); \mathsf{while}(x > 0, s) \\
\longrightarrow & [x{\mapsto}1] \vdash 1 & \triangleright & \_ + 1; \mathsf{assign}(x, \_)); \mathsf{while}(x > 0, s) \\
\longrightarrow & [x{\mapsto}1] \vdash 1 & \triangleright & 1 + \_; \mathsf{assign}(x, \_)); \mathsf{while}(x > 0, s) \\
\longrightarrow & [x{\mapsto}1] \vdash 2 & \triangleright & \mathsf{assign}(x, \_)); \mathsf{while}(x > 0, s) \\
\longrightarrow & & & \\
\longrightarrow & & &
\end{array}
$$

Example: Infinite Loop

$\mathsf{while}(x > 0, \mathsf{assign}(x, x+1)) \qquad \eta = [x\mapsto 1]$

$$
\begin{array}{llll}
 & [x\mapsto 1] \vdash \mathsf{while}(x > 0, s) & \blacktriangleright & \cdot \\
\longrightarrow & [x\mapsto 1] \vdash \mathsf{if}(x>0, \mathsf{seq}(s, \mathsf{while}(x>0, s)), \mathsf{nop}) & \blacktriangleright & \cdot \\
\longrightarrow & [x\mapsto 1] \vdash x > 0 & \triangleright & \mathsf{if}(\_, \mathsf{seq}(s, \mathsf{while}(x > 0, s)), \mathsf{nop}) \\
\longrightarrow & [x\mapsto 1] \vdash x & \triangleright & \_ > 0; \mathsf{if}(\_, \mathsf{seq}(s, \mathsf{while}(x > 0, s)), \mathsf{nop}) \\
\longrightarrow & [x\mapsto 1] \vdash 1 & \triangleright & \_ > 0; \mathsf{if}(\_, \mathsf{seq}(s, \mathsf{while}(x > 0, s)), \mathsf{nop}) \\
\longrightarrow & [x\mapsto 1] \vdash 0 & \triangleright & 1 > \_; \mathsf{if}(\_, \mathsf{seq}(s, \mathsf{while}(x > 0, s)), \mathsf{nop}) \\
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\longrightarrow & & &
\end{array}
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Example: Infinite Loop

$\mathsf{while}(x > 0, \mathsf{assign}(x, x+1)) \qquad \eta = [x{\mapsto}1]$

$$
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 & [x{\mapsto}1] \vdash \mathsf{while}(x > 0, s) & \blacktriangleright \;\; \cdot \\
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Functions

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- Execute the body of the callee in a new environment that maps the formal parameters to the argument values
- Pass the return value to the environment of the caller

Call Stack

We need to keep track of continuations and environment in stack frames

Call stack:

$$S \quad ::= \quad \cdot \mid S \ , \langle \eta, K \rangle$$

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Configurations:

Evaluation $\quad S \; ; \eta \vdash e \triangleright K$

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Existing rules can be lifted to the new configurations by passing through the call stack

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We only show the special case of 0 and 2 arguments

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(given that f is defined as $f(\,)\{s\}$*)*

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Two arguments:

$$S\ ;\eta \vdash f(e_1, e_2) \rhd K \qquad \longrightarrow \qquad S\ ;\eta \vdash e_1 \rhd (f(\_, e_2)\ ,\ K)$$

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Rules for Returns

Need to restore continuation and environment and pass return value

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Special case: returning void

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Alternative: elaborate each function that returns void with return(nothing) statements.

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Statics, Dynamics, and Safety

Overview of Machine States (Configurations)

$S\,;\eta \vdash e \rhd K$ – Evaluating the expression e with the continuation K

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The language should be deterministic: there at most one transition per state

Progress

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then either $\mathcal{ST}_n$ is a final state or else $\mathcal{ST}_n$ is not-stuck because there exists a state $\mathcal{ST}'$ such that $\mathcal{ST}_n \longrightarrow \mathcal{ST}'$.

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15-312

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\begin{array}{llcl}
\text{Expressions} & e & ::= & c \mid e_1 \odot e_2 \mid \mathsf{true} \mid \mathsf{false} \mid e_1 \;\texttt{\&\&}\; e_2 \mid x \mid f(e_1, e_2) \mid f(\,) \\
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 & & \mid & \mathsf{if}(e, s_1, s_2) \mid \mathsf{while}(e, s) \mid \mathsf{return}(e) \mid \mathsf{assert}(e) \\
\text{Values} & v & ::= & c \mid \mathsf{true} \mid \mathsf{false} \mid \mathsf{nothing} \\
\text{Environments} & \eta & ::= & \cdot \mid \eta, x \mapsto c \\
\text{Stacks} & S & ::= & \cdot \mid S\ ,\ \langle \eta, K \rangle \\
\text{Cont. frames} & \phi & ::= & \_ \odot e \mid c \odot \_ \mid \_ \;\texttt{\&\&}\; e \mid f(\_, e) \mid f(c, \_) \\
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\text{Continuations} & K & ::= & \cdot \mid \phi\ ,\ K \\
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Summary I

All ops.

Expressions	e	$::=$	$c \mid e_1 \odot e_2 \mid \mathsf{true} \mid \mathsf{false} \mid e_1 \;\&\&\; e_2 \mid x \mid f(e_1, e_2) \mid f(\,)$
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Continuations	K	$::=$	$\cdot \mid \phi \;,\; K$
Exceptions	E	$::=$	$\mathsf{arith} \mid \mathsf{abort} \mid \mathsf{mem}$

Summary I

$$
\begin{array}{lcl}
S\ ;\eta \vdash e_1 \odot e_2 \rhd K & \longrightarrow & S\ ;\eta \vdash e_1 \rhd (\_ \odot e_2\ , K) \\
S\ ;\eta \vdash c_1 \rhd (\_ \odot e_2\ , K) & \longrightarrow & S\ ;\eta \vdash e_2 \rhd (c_1 \odot \_\ , K) \\
S\ ;\eta \vdash c_2 \rhd (c_1 \odot \_\ , K) & \longrightarrow & S\ ;\eta \vdash c \rhd K \qquad (c = c_1 \odot c_2) \\
S\ ;\eta \vdash c_2 \rhd (c_1 \odot \_\ , K) & \longrightarrow & \mathsf{exception}(\mathsf{arith}) \qquad (c_1 \odot c_2 \text{ undefined}) \\
\\
S\ ;\eta \vdash e_1\ \texttt{\&\&}\ e_2 \rhd K & \longrightarrow & S\ ;\eta \vdash e_1 \rhd (\_\ \texttt{\&\&}\ e_2\ , K) \\
S\ ;\eta \vdash \mathsf{false} \rhd (\_\ \texttt{\&\&}\ e_2\ , K) & \longrightarrow & S\ ;\eta \vdash \mathsf{false} \rhd K \\
S\ ;\eta \vdash \mathsf{true} \rhd (\_\ \texttt{\&\&}\ e_2\ , K) & \longrightarrow & S\ ;\eta \vdash e_2 \rhd K \\
\\
S\ ;\eta \vdash x \rhd K & \longrightarrow & S\ ;\eta \vdash \eta(x) \rhd K
\end{array}
$$

Summary: Expressions

$$
\begin{array}{lcl}
S\,;\eta \vdash \mathsf{seq}(s_1,s_2) \blacktriangleright K & \longrightarrow & S\,;\eta \vdash s_1 \blacktriangleright (s_2\ ,\ K) \\
S\,;\eta \vdash \mathsf{nop} \blacktriangleright (s\ ,\ K) & \longrightarrow & S\,;\eta \vdash s \blacktriangleright K \\
S\,;\eta \vdash \mathsf{assign}(x,e) \blacktriangleright K & \longrightarrow & S\,;\eta \vdash e \rhd (\mathsf{assign}(x,\_)\ ,\ K) \\
S\,;\eta \vdash c \rhd (\mathsf{assign}(x,\_)\ ,\ K) & \longrightarrow & S\,;\eta[x \mapsto c] \vdash \mathsf{nop} \blacktriangleright K \\
\\
S\,;\eta \vdash \mathsf{decl}(x,\tau,s) \blacktriangleright K & \longrightarrow & S\,;\eta[x \mapsto \mathsf{nothing}] \vdash s \blacktriangleright K \\
\\
S\,;\eta \vdash \mathsf{assert}(e) \blacktriangleright K & \longrightarrow & S\,;\eta \vdash e \rhd (\mathsf{assert}(\_)\ ,\ K) \\
S\,;\eta \vdash \mathsf{true} \rhd (\mathsf{assert}(\_)\ ,\ K) & \longrightarrow & S\,;\eta \vdash \mathsf{nop} \blacktriangleright K \\
S\,;\eta \vdash \mathsf{false} \rhd (\mathsf{assert}(\_)\ ,\ K) & \longrightarrow & \mathsf{exception}(\mathsf{abort}) \\
\\
S\,;\eta \vdash \mathsf{if}(e,s_1,s_2) \blacktriangleright K & \longrightarrow & S\,;\eta \vdash e \rhd (\mathsf{if}(\_,s_1,s_2)\ ,\ K) \\
S\,;\eta \vdash \mathsf{true} \rhd (\mathsf{if}(\_,s_1,s_2),K) & \longrightarrow & S\,;\eta \vdash s_1 \blacktriangleright K \\
S\,;\eta \vdash \mathsf{false} \rhd (\mathsf{if}(\_,s_1,s_2),K) & \longrightarrow & S\,;\eta \vdash s_2 \blacktriangleright K \\
\\
S\,;\eta \vdash \mathsf{while}(e,s) \blacktriangleright K & \longrightarrow & S\,;\eta \vdash \mathsf{if}(e,\mathsf{seq}(s,\mathsf{while}(e,s)),\mathsf{nop}) \blacktriangleright K
\end{array}
$$

Summary: Statements

$$
\begin{array}{lcl}
S\ ;\ \eta \vdash f(e_1, e_2) \rhd K & \longrightarrow & S\ ;\ \eta \vdash e_1 \rhd (f(\_, e_2)\ ,\ K) \\
S\ ;\ \eta \vdash c_1 \rhd (f(\_, e_2)\ ,\ K) & \longrightarrow & S\ ;\ \eta \vdash e_2 \rhd (f(c_1, \_)\ ,\ K) \\
S\ ;\ \eta \vdash c_2 \rhd (f(c_1, \_)\ ,\ K) & \longrightarrow & (S\ ,\ \langle \eta, K \rangle)\ ;\ [x_1 \mapsto c_1, x_2 \mapsto c_2] \vdash s \blacktriangleright \cdot \\
 & & \textit{(given that } f \textit{ is defined as } \ f(x_1, x_2)\{s\}\textit{)} \\
S\ ;\ \eta \vdash f(\,) \rhd K & \longrightarrow & (S\ ,\ \langle \eta, K \rangle)\ ;\ \cdot \vdash s \blacktriangleright \cdot \\
 & & \textit{(given that } f \textit{ is defined as } \ f(\,)\{s\}\textit{)} \\
S\ ;\ \eta \vdash \mathsf{return}(e) \blacktriangleright K & \longrightarrow & S\ ;\ \eta \vdash e \rhd (\mathsf{return}(\_)\ ,\ K) \\
(S\ ,\ \langle \eta', K' \rangle)\ ;\ \eta \vdash v \rhd (\mathsf{return}(\_)\ ,\ K) & \longrightarrow & S\ ;\ \eta' \vdash v \rhd K' \\
\cdot\ ;\ \eta \vdash c \rhd (\mathsf{return}(\_)\ ,\ K) & \longrightarrow & \mathsf{value}(c)
\end{array}
$$

Summary: Functions