15-411: Mutable Store

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Pointers and Arrays

We will see how static and dynamic semantics make it easy to introduce and specify advanced language features

- Static semantics of pointers
- Dynamic semantics of pointers
- Static semantics of arrays
- Dynamic semantics of arrays

Static semantics of pointers

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We add the following typing rules for expressions:

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- This type can be seen as a temporary placeholder
- When we constructed the type derivation we could replace any with a ‘concrete type’
- Another view is to say that `any*` has exactly one value: null

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Type Rules

Dereference and type instantiation

$$\frac{\Gamma \vdash e : any\,*}{\Gamma \vdash e : \tau *} \qquad \frac{\Gamma \vdash e : \tau * \quad \Gamma \not\vdash e : any\,*}{\Gamma \vdash *e : \tau}$$

Equality

$$\frac{\Gamma \vdash e_1 : \tau \quad \Gamma \vdash e_2 : \tau}{\Gamma \vdash e_1 \texttt{==} e_2 : \mathsf{bool}}$$

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Cannot dereference a Null pointer.

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Dynamic semantics of pointers

Configurations with Heap

- A value of a type $\tau *$ is an address that stores a value of type τ (or a special address 0)
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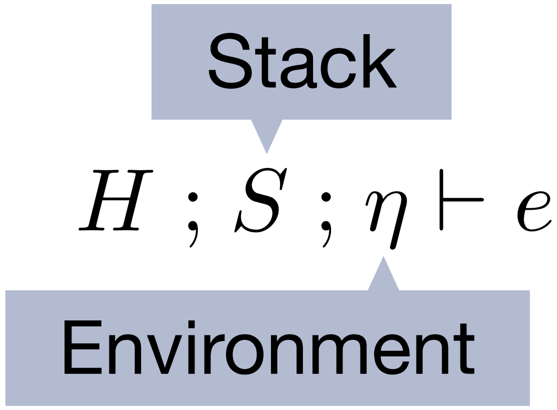

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Continuation

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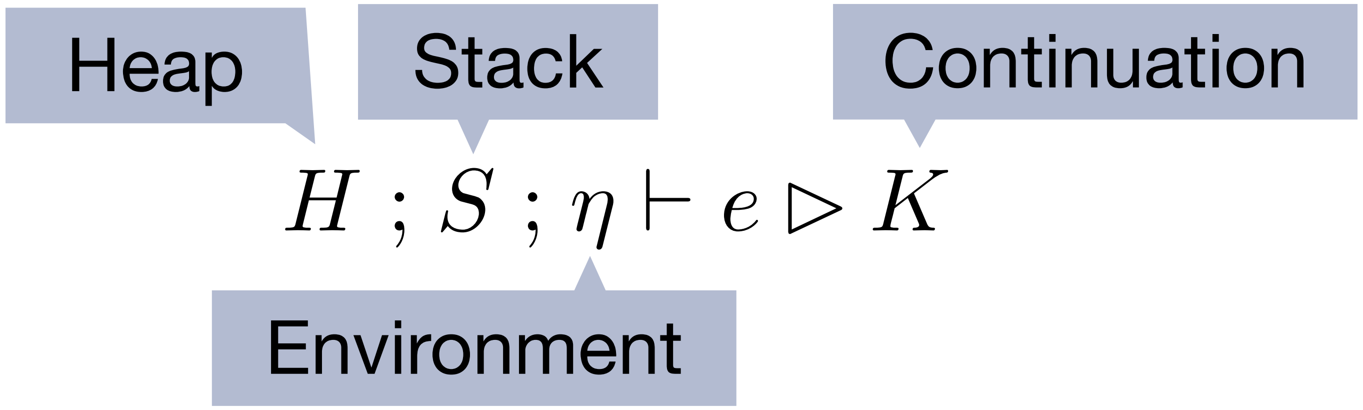

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Modeling the Heap

- Addresses are 64 bit words?
  Problem: We can run out of memory

- We don’t want to specify that programs fail due to memory errors (garbage collection, OS details, …)

- Approach: no out-of-memory errors at the high level
  -> **addresses are natural numbers**

$$H : (\mathbb{N} \cup \{\mathsf{next}\}) \to \mathsf{Val}$$

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We didn't model stack overflow.

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Previous runs are lifted:

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$$H \; ; \; S \; ; \; \eta \vdash \mathsf{alloc}(\tau) \rhd K \quad \longrightarrow \quad H[a \mapsto \mathsf{default}(\tau), \mathsf{next} \mapsto a + |\tau|] \; ; \; S \; ; \; \eta \vdash a \rhd K$$

$$a = H(\mathsf{next})$$

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Store a default value.

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Evaluation Rules: Allocation

$$H\ ;\ S\ ;\ \eta \vdash \mathsf{alloc}(\tau) \rhd K \qquad \longrightarrow \qquad \begin{array}{l} H[a \mapsto \mathsf{default}(\tau), \mathsf{next} \mapsto a + |\tau|]\ ;\ S\ ;\ \eta \vdash a \rhd K \\ a = H(\mathsf{next}) \end{array}$$

Default values

default(bool) = false

default(int) = 0

default($\tau*$) = null

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Type sizes (x86-64):

$$\begin{array}{lcl} |\mathsf{int}| & = & 4 \\ |\mathsf{bool}| & = & 4 \\ |\tau *| & = & 8 \\ |\tau[\,]| & = & 8 \end{array}$$

Evaluation Rules: Dereference

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$$H\ ;\ S\ ;\ \eta \vdash *e \triangleright K \qquad\qquad \longrightarrow \qquad\qquad H\ ;\ S\ ;\ \eta \vdash e \triangleright (*\_\ ,\ K)$$

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Implementing memory exceptions

- Use signal SIGUSR2 instead of SIGSEGV
- Better for debugging: better distinguishable from stack overflow and “accidental” memory errors

Assignments: Typing

Destinations (or l-values):

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Return type of current function.

Assignment: Evaluation Rules

Variables:

$$H\ ;S\ ;\eta \vdash \mathsf{assign}(x,e) \blacktriangleright K \qquad \longrightarrow \qquad H\ ;S\ ;\eta \vdash e \triangleright (\mathsf{assign}(x,\_)\ ,\ K)$$

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$$H \; ; S \; ; \eta \vdash c \triangleright (\mathsf{assign}(*a, \_) \; , K) \quad \longrightarrow \quad \mathsf{exception}(\mathsf{mem}) \qquad (a = 0)$$

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Arrays

Arrays: Typing

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There are no “null” arrays.

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There are no "null" arrays.

However, there are default arrays.

Type rules:

$$\frac{\Gamma \vdash e_1 : \tau[\,] \quad \Gamma \vdash e_2 : \mathsf{int}}{\Gamma \vdash e_1[e_2] : \tau}$$

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Array Evaluation: Access

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$$H\ ;\ S\ ;\ \eta \vdash i \rhd (a[\_]\ ,\ K) \qquad \longrightarrow \qquad H\ ;\ S\ ;\ \eta \vdash H(a + i|\tau|) \rhd K$$

$$a \neq 0, 0 \leq i < \mathsf{length}(a), a : \tau[\,]$$

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$$a \neq 0, 0 \leq i < \mathsf{length}(a), a : \tau[\,]$$

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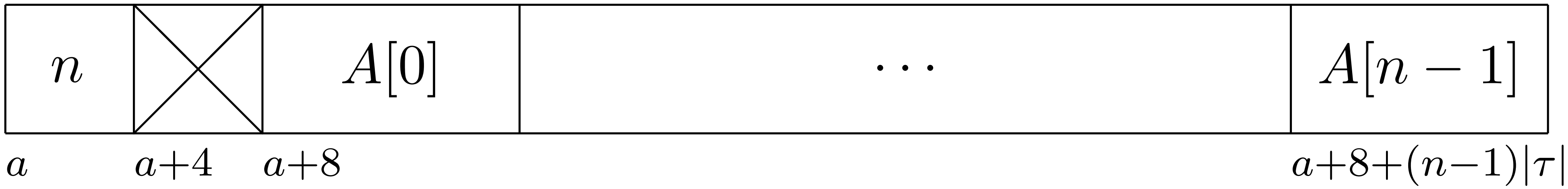
Lengths?

- Not known at compile time
- In `alloc_array(`τ`,e),` `e` can be an arbitrary expression

➡Need to store array length

Storing Array Length

Alternative 1: Add length at the front, array address points to the start



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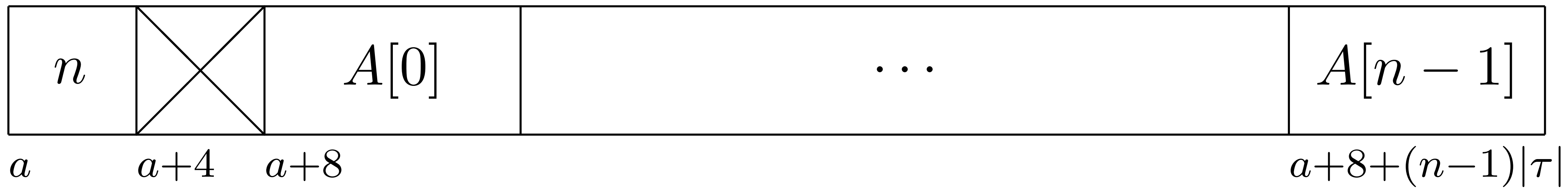
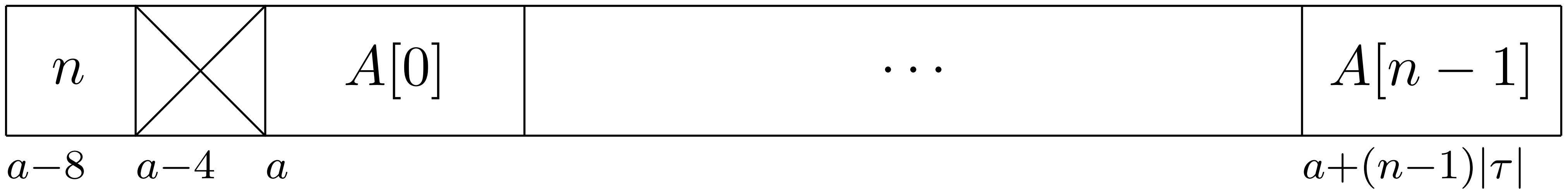


Alternative 2: Array address points to the first element



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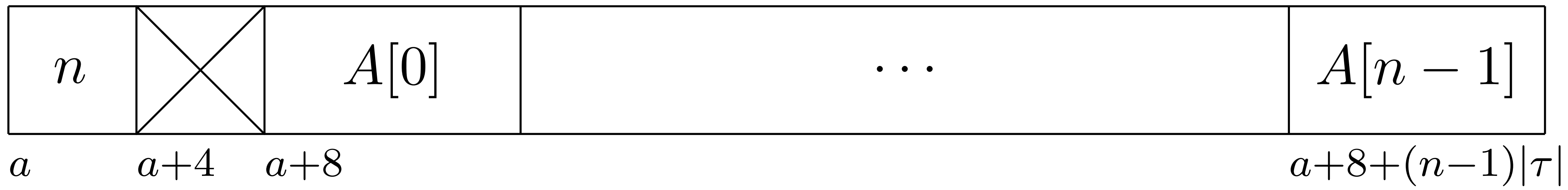

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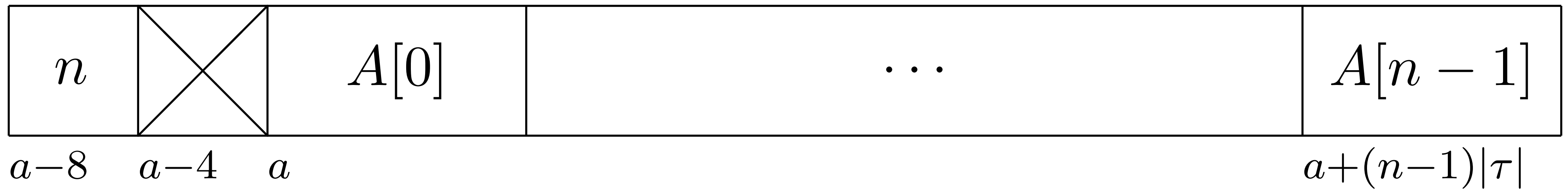

- Simplifies address arithmetic
- Allows to pass pointers to C (which wouldn't care about length info)

Updated Rules for Array Access

$$H\ ;S\ ;\eta \vdash e_1\{\tau\}[e_2] \rhd K \quad\longrightarrow\quad H\ ;S\ ;\eta \vdash e_1 \rhd (\_\{\tau\}[e_2]\ ,K)$$

$$H\ ;S\ ;\eta \vdash a \rhd (\_\{\tau\}[e_2]\ ,K) \quad\longrightarrow\quad H\ ;S\ ;\eta \vdash e_2 \rhd (a\{\tau\}[\_]\ ,K)$$

$$H\ ;S\ ;\eta \vdash i \rhd (a\{\tau\}[\_]\ ,K) \quad\longrightarrow\quad \begin{array}{l} H\ ;S\ ;\eta \vdash H(a+i|\tau|) \rhd K \\ a \neq 0, 0 \leq i < H(a-8) \end{array}$$

$$H\ ;S\ ;\eta \vdash i \rhd (a\{\tau\}[\_]\ ,K) \quad\longrightarrow\quad \begin{array}{l} \textsf{exception(mem)} \\ a = 0 \text{ or } i < 0 \text{ or } i \geq H(a-8) \end{array}$$

Array Access: Code Generation

The code pattern for $e_1\{\tau\}[e_2]$ and $|\tau| = k$ could be like this:

```
cogen$(e_1, a)$              ($a$ new)
cogen$(e_2, i)$              ($i$ new)
$a_1 \leftarrow a - 8$
$t_2 \leftarrow M[a_1]$
if $(i < 0)$ goto error
if $(i \geq t_2)$ goto error
$a_3 \leftarrow i * \$k$
$a_4 \leftarrow a + a_3$
$t_5 \leftarrow M[a_4]$
```

Array Evaluation: Allocation

Array Evaluation: Allocation

$$H\ ;\ S\ ;\ \eta \vdash \mathsf{alloc\_array}(\tau, e) \rhd K \qquad\qquad \longrightarrow \qquad\qquad H\ ;\ S\ ;\ \eta \vdash e \rhd (\mathsf{alloc\_array}(\tau, \_)\ ,\ K)$$

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$$H\ ;\ S\ ;\ \eta \vdash n \rhd (\mathsf{alloc\_array}(\tau, \_)\ ,\ K) \quad \longrightarrow \quad H'\ ;\ S\ ;\ \eta \vdash a' \rhd K \qquad (n \geq 0)$$

$$a = H(\mathsf{next}) \qquad a' = a + 8$$

$$H' = H[a' + 0|\tau| \mapsto \mathsf{default}(\tau), \ldots, a' + (n-1)|\tau| \mapsto \mathsf{default}(\tau), \mathsf{next} \mapsto a' + n|\tau|]$$

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$$H\ ;\ S\ ;\ \eta \vdash n \rhd (\mathsf{alloc\_array}(\tau, \_)\ ,\ K) \qquad \longrightarrow \qquad \mathsf{exception}(\mathsf{mem}) \qquad (n < 0)$$

Array Evaluation: Assignment

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$$H\ ;\ S\ ;\ \eta \vdash \mathsf{assign}(d\{\tau\}[e_2], e_3) \blacktriangleright K \qquad \longrightarrow \qquad H\ ;\ S\ ;\ \eta \vdash d \rhd (\mathsf{assign}(\_\{\tau\}[e_2], e_3)\ ,\ K)$$

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$$H\ ;\ S\ ;\ \eta \vdash a \rhd (\mathsf{assign}(\_\{\tau\}[e_2], e_3)\ ,\ K) \qquad \longrightarrow \qquad H\ ;\ S\ ;\ \eta \vdash e_2 \rhd (\mathsf{assign}(a\{\tau\}[\_], e_3)\ ,\ K)$$

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$$\begin{array}{lcr} H \; ; \; S \; ; \; \eta \vdash i \rhd (\mathsf{assign}(a\{\tau\}[\_], e_3) \; , \; K) & \longrightarrow & H \; ; \; S \; ; \; \eta \vdash e_3 \rhd (\mathsf{assign}(a + i|\tau|, \_) \; , \; K) \\ & & a \neq 0, 0 \leq i < \mathsf{length}(a) \end{array}$$

Array Evaluation: Assignment

length(a) = H(a-8)

$$H\ ;\ S\ ;\ \eta \vdash \mathsf{assign}(d\{\tau\}[e_2], e_3) \blacktriangleright K \qquad \longrightarrow \qquad H\ ;\ S\ ;\ \eta \vdash d \rhd (\mathsf{assign}(\_\{\tau\}[e_2], e_3)\ ,\ K)$$

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$$a \neq 0, 0 \leq i < \mathsf{length}(a)$$

$$H\ ;\ S\ ;\ \eta \vdash i \rhd (\mathsf{assign}(a\{\tau\}[\_], e_3)\ ,\ K) \quad\longrightarrow\quad \mathsf{exception}(\mathsf{mem})$$

$$a = 0 \text{ or } i < 0 \text{ or } i \geq \mathsf{length}(a)$$

$$H\ ;\ S\ ;\ \eta \vdash c \rhd (\mathsf{assign}(b, \_)\ ,\ K) \quad\longrightarrow\quad H[b \mapsto c]\ ;\ S\ ;\ \eta \vdash \mathsf{nop} \blacktriangleright K$$

Default Values of Array Type

We also need a default value for array types

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We also need a default value for array types

- We will just use 0 as the default value again
- It represents an array of length 0
- We can never legally access an array element in the default array
- Warning: Arrays can be compared with equality
- Make sure that alloc_array(t,0) returns a fresh address different from 0
- If arrays have address a=0 then you should not access M[a-8]

Compound Assignment Operators

- We translate x += e to x = x + e
- We cannot translate d1[e2] += e3 to d1[e2] = d1[e2] + e3

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Effects of e2 and d1 would be evaluated twice.