#### **Recitation 6: Dynamic Semantics Solutions**

## 3 March

Spring 2023

#### About Code Review Week

As you finish up Lab 3, you should spend some time on improving your codebase and making up for technical debt. If you used any terrible hacks to get register allocation or calling conventions to work, fix them. If you think any part of your code is a jumbled mess, refactor it. You'll want to have a solid base upon which to build Lab 4.

If you haven't already, you should sign up for a code review timeslot using the link on Piazza. Additionally, please also fill in this form with the commit hash or branch that you would like us to review. We will read your code beforehand and ask you questions about it during your team's code review meeting. While we will give you pointers on your style and structure, what we really want to assess how well you **understand** the code you and your partner have written. We will be using your git commit log to guide our understanding of who implemented what.

If there's any significant section of your compiler that your partner implemented and you did not read, you should read it. If you don't understand how part of your compiler works, you should ask your partner to explain it to you. That said, we don't expect you to remember every detail of your implementation—we just want to make sure that both team members are participating roughly equally and have a thorough understanding of the compiler's structure.

# **Recap: Dynamic Semantics for L2**

A configuration of an L2 program could be modeled as one of the two forms

- $\eta \vdash s \blacktriangleright K$  for *executing* statement s
- $\eta \vdash e \triangleright K$  for *evaluating* expression e

where  $\eta$  represents a map from variables to values and K represents the continuation (what to do next with the result of evaluating the current expression, or the next statement).

We're interested in the judgment  $c \rightarrow c'$ , indicating that a configuration c of the form above steps to a configuration c'. Here are a few rules for L2:

$\eta \vdash assign(x, e) \blacktriangleright K$	$\longrightarrow$	$\eta \vdash e \triangleright (assign(x,\_),K)$
$\eta \vdash v \triangleright (assign(x, \_), K)$	$\longrightarrow$	$\eta[x\mapsto v]\vdash nop\blacktriangleright K$
$\eta \vdash nop \blacktriangleright (s,K)$	$\longrightarrow$	$\eta \vdash s \blacktriangleright K$

We omit many rules-for a more complete set, refer to Lecture 13.

Let  $c_1$  be the initial configuration, and suppose  $c_i \to c_{i+1}$ . If  $c_n$  is a final configuration of the form  $\eta \vdash v \triangleright (\text{return}(\_), K)$ , then we say that  $c_1, c_2, \ldots c_n$  is the execution trace of  $c_1$ .

# Checkpoint 0

Draw the execution trace of configurations starting from:

 $\cdot \vdash \mathsf{seq}(\mathsf{assign}(x,3),\mathsf{return}(x+1)) \blacktriangleright \cdot$ 

#### Solution:

$$\begin{array}{cccc} \cdot \vdash \mathsf{seq}(\mathsf{assign}(x,3),\mathsf{return}(x)) \blacktriangleright \cdot & \longrightarrow & \cdot \vdash \mathsf{assign}(x,3) \blacktriangleright (\mathsf{return}(x), \cdot) \\ & \longrightarrow & \cdot \vdash 3 \triangleright (\mathsf{assign}(x,\_), (\mathsf{return}(x), \cdot)) \\ & \longrightarrow & [x \mapsto 3] \vdash \mathsf{nop} \blacktriangleright (\mathsf{return}(x), \cdot) \\ & \longrightarrow & [x \mapsto 3] \vdash \mathsf{return}(x) \blacktriangleright \cdot \\ & \longrightarrow & [x \mapsto 3] \vdash x \triangleright (\mathsf{return}(\_), \cdot) \\ & \longrightarrow & [x \mapsto 3] \vdash 3 \triangleright (\mathsf{return}(\_), \cdot) \end{array}$$

## **Dynamic Semantics for L3**

L3's dynamic semantics is slightly more interesting in that returning from a function call should restore state and control to the configuration prior to the call. We amend our configuration to hold a fourth element, the call stack S, which consists of tuples of the form  $\langle \eta, K \rangle$ . We reproduce the rules for single-argument functions below:

$$\begin{array}{cccc} S; \eta \vdash f(e) \triangleright K & \longrightarrow & S; \eta \vdash e \triangleright (f(\_), K) \\ S; \eta \vdash v \triangleright (f(\_), K) & \longrightarrow & (S, \langle \eta, K \rangle); [x \mapsto v] \vdash s_f \blacktriangleright \cdot \\ & supposing \ that \ f \ is \ defined \ as \ f(x)\{s_f;\} \\ (S, \langle \eta, K \rangle); \eta' \vdash v \triangleright (\text{return}(\_), K') & \longrightarrow & S; \eta \vdash v \triangleright K \end{array}$$

# **Checkpoint** 1

Draw the execution trace of the following program, starting execution at the beginning of main:

```
int f(int x) { return x; }
void g() { 4; }
int main() { int y = f(3); g(); return y; }
```

$ \begin{array}{cccc} & \cdot \vdash \operatorname{call}(f,3) \triangleright (\operatorname{assign}(y,\_),(\operatorname{seq}(\operatorname{call}(g),\operatorname{return}(y)),\cdot)) \\ & \cdot \vdash 3 \triangleright (\operatorname{call}(f,\_),(\operatorname{assign}(y,\_),(\operatorname{seq}(\operatorname{call}(g),\operatorname{return}(y)),\cdot))) \\ & \langle (\cdot,(\operatorname{assign}(y,\_),(\operatorname{seq}(\operatorname{call}(g),\operatorname{return}(y)),\cdot))) \rangle; [x \mapsto 3] \vdash \operatorname{return}(x) \blacktriangleright \cdot \\ & \rightarrow & \langle (\cdot,(\operatorname{assign}(y,\_),(\operatorname{seq}(\operatorname{call}(g),\operatorname{return}(y)),\cdot))) \rangle; [x \mapsto 3] \vdash x \triangleright (\operatorname{return}(\_),\cdot) \\ & \rightarrow & \langle (\cdot,(\operatorname{assign}(y,\_),(\operatorname{seq}(\operatorname{call}(g),\operatorname{return}(y)),\cdot))) \rangle; [x \mapsto 3] \vdash x \triangleright (\operatorname{return}(\_),\cdot) \\ & \rightarrow & \langle (\cdot,(\operatorname{assign}(y,\_),(\operatorname{seq}(\operatorname{call}(g),\operatorname{return}(y)),\cdot))) \rangle; [x \mapsto 3] \vdash 3 \triangleright (\operatorname{return}(\_),\cdot) \\ & \rightarrow & \langle (\cdot,(\operatorname{assign}(y,\_),(\operatorname{seq}(\operatorname{call}(g),\operatorname{return}(y)),\cdot))) \\ & \rightarrow & (y \mapsto 3] \vdash \operatorname{reall}(g),\operatorname{return}(y)), \bullet \end{pmatrix} \\ & & (y \mapsto 3] \vdash \operatorname{call}(g) \triangleright (\operatorname{return}(y),\cdot) \\ & \rightarrow & (y \mapsto 3] \vdash \operatorname{call}(g) \triangleright (\operatorname{discard},\operatorname{return}(y),\cdot) \\ & \rightarrow & \langle ([y \mapsto 3],(\operatorname{discard},\operatorname{return}(y),\cdot)) \rangle; \cdot \vdash \operatorname{seq}(4,\operatorname{return}(\operatorname{nothing}),\cdot) \\ & & \langle ([y \mapsto 3],(\operatorname{discard},\operatorname{return}(y),\cdot)) \rangle; \cdot \vdash \operatorname{d} \triangleright (\operatorname{return}(\operatorname{nothing}),\cdot) \\ & & \langle (([y \mapsto 3],(\operatorname{discard},\operatorname{return}(y),\cdot)) \rangle; \cdot \vdash \operatorname{nothing} \triangleright (\operatorname{return}(\operatorname{nothing}),\cdot) \\ & & \langle (([y \mapsto 3],(\operatorname{discard},\operatorname{return}(y),\cdot)) \rangle; \cdot \vdash \operatorname{nothing} \triangleright (\operatorname{return}(\operatorname{nothing}),\cdot) \\ & & \langle (([y \mapsto 3],(\operatorname{discard},\operatorname{return}(y),\cdot)) \rangle; \cdot \vdash \operatorname{nothing} \triangleright (\operatorname{return}(y),\cdot) \\ & \rightarrow & (y \mapsto 3] \vdash \operatorname{nothing} \triangleright (\operatorname{return}(y),\cdot) \\ & \rightarrow & (y \mapsto 3] \vdash \operatorname{nothing} \triangleright (\operatorname{return}(y),\cdot) \\ & \rightarrow & (y \mapsto 3] \vdash \operatorname{return}(y) \blacktriangleright \cdot \\ & & (y \mapsto 3] \vdash \operatorname{return}(y) \vdash \cdot) \\ & \rightarrow & (y \mapsto 3] \vdash \operatorname{return}(y) \vdash \cdot \\ & \rightarrow & (y \mapsto 3] \vdash \operatorname{return}(y) \vdash \cdot \\ & \rightarrow & (y \mapsto 3] \vdash \operatorname{return}(y),\cdot) \\ & \rightarrow & (y \mapsto 3] \vdash \operatorname{return}(y) \vdash \cdot \\ & \rightarrow & (y \mapsto 3] \vdash \operatorname{return}(y) \vdash \cdot \\ & \rightarrow & (y \mapsto 3] \vdash \operatorname{return}(y),\cdot) \\ & \rightarrow & (y \mapsto 3] \vdash \operatorname{return}(y) \vdash \cdot \\ & \rightarrow & (y \mapsto 3] \vdash \operatorname{return}(y) \vdash \cdot \\ & \rightarrow & (y \mapsto 3] \vdash \operatorname{return}(y),\cdot) \\ & \rightarrow & (y \mapsto 3] \vdash \operatorname{return}(y) \vdash \cdot \\ & \rightarrow & (y \mapsto 3] \vdash \operatorname{return}(y) \vdash \cdot \\ & \rightarrow & (y \mapsto 3] \vdash \operatorname{return}(y) \vdash \cdot \\ & \rightarrow & (y \mapsto 3] \vdash \operatorname{return}(y) \vdash \cdot \\ & \rightarrow & (y \mapsto 3] \vdash \operatorname{return}(y) \vdash \cdot \\ & \rightarrow & (y \mapsto (y \vdash y),\cdot) \\ & \rightarrow & (y \mapsto 3] \vdash \operatorname{return}(y) \vdash \cdot \\ & \rightarrow & (y \mapsto 3] \vdash \operatorname{return}(y) \to \cdot \\ & \rightarrow & (y \mapsto 3] \vdash \operatorname{return}(y) \to \cdot \\ & \rightarrow & (y \mapsto 3] \vdash \operatorname{return}(y) \to \cdot \\ &$	$\longrightarrow$	$\cdot \vdash \operatorname{assign}(y, \operatorname{call}(f, 3)) \blacktriangleright (\operatorname{seq}(\operatorname{call}(g), \operatorname{return}(y)), \cdot)$
$ \begin{array}{cccc} & & \cdot \vdash \exists \triangleright (\operatorname{call}(f, \_), (\operatorname{assign}(y, \_), (\operatorname{seq}(\operatorname{call}(g), \operatorname{return}(y)), \cdot))) \\ & & \langle (\cdot, (\operatorname{assign}(y, \_), (\operatorname{seq}(\operatorname{call}(g), \operatorname{return}(y)), \cdot))) \rangle; [x \mapsto \exists] \vdash \operatorname{return}(x) \blacktriangleright \cdot \\ & & & \langle (\cdot, (\operatorname{assign}(y, \_), (\operatorname{seq}(\operatorname{call}(g), \operatorname{return}(y)), \cdot))) \rangle; [x \mapsto \exists] \vdash x \triangleright (\operatorname{return}(\_), \cdot) \\ & & & \langle (\cdot, (\operatorname{assign}(y, \_), (\operatorname{seq}(\operatorname{call}(g), \operatorname{return}(y)), \cdot))) \rangle; [x \mapsto \exists] \vdash \exists \triangleright (\operatorname{return}(\_), \cdot) \\ & & & \cdot \vdash \exists \triangleright (\operatorname{assign}(y, \_), (\operatorname{seq}(\operatorname{call}(g), \operatorname{return}(y)), \cdot)) \\ & & & & (\cdot, (\operatorname{assign}(y, \_), (\operatorname{seq}(\operatorname{call}(g), \operatorname{return}(y)), \cdot)) \\ & & & & \cdot \vdash \exists \triangleright (\operatorname{assign}(y, \_), (\operatorname{seq}(\operatorname{call}(g), \operatorname{return}(y)), \cdot)) \\ & & & & (y \mapsto \exists] \vdash \operatorname{nop} \blacktriangleright (\operatorname{seq}(\operatorname{call}(g), \operatorname{return}(y)), \cdot) \\ & & & & (y \mapsto \exists] \vdash \operatorname{call}(g) \triangleright (\operatorname{return}(y), \cdot) \\ & & & & ([y \mapsto \exists], (\operatorname{discard}, \operatorname{return}(y), \cdot))); \cdot \vdash \operatorname{seq}(4, \operatorname{return}(\operatorname{nothing}), \cdot) \\ & & & & (([y \mapsto \exists], (\operatorname{discard}, \operatorname{return}(y), \cdot))); \cdot \vdash \operatorname{ab} (\operatorname{discard}, \operatorname{return}(\operatorname{nothing}), \cdot) \\ & & & & (([y \mapsto \exists], (\operatorname{discard}, \operatorname{return}(y), \cdot))); \cdot \vdash \operatorname{nop} \triangleright (\operatorname{return}(\operatorname{nothing}), \cdot) \\ & & & & (([y \mapsto \exists], (\operatorname{discard}, \operatorname{return}(y), \cdot))); \cdot \vdash \operatorname{nothing} \triangleright (\operatorname{return}(y), \cdot) \\ & & & & (([y \mapsto \exists], (\operatorname{discard}, \operatorname{return}(y), \cdot))); \cdot \vdash \operatorname{nothing} \triangleright (\operatorname{return}(y), \cdot) \\ & & & & (([y \mapsto \exists], (\operatorname{discard}, \operatorname{return}(y), \cdot))); \cdot \vdash \operatorname{nothing} \triangleright (\operatorname{return}(y), \cdot) \\ & & & & (([y \mapsto \exists], (\operatorname{discard}, \operatorname{return}(y), \cdot))); \cdot \vdash \operatorname{nothing} \triangleright (\operatorname{return}(y), \cdot) \\ & & & & & ([y \mapsto \exists] \vdash \operatorname{nop} \blacktriangleright (\operatorname{return}(y), \cdot) \\ & & & & ([y \mapsto \exists] \vdash \operatorname{nop} \triangleright (\operatorname{return}(y), \cdot) \\ & & & & & ([y \mapsto \exists] \vdash \operatorname{nop} \triangleright (\operatorname{return}(y), \cdot) \\ & & & & & ([y \mapsto \exists] \vdash \operatorname{return}(y) \triangleright \cdot \\ & & & & & ([y \mapsto \exists] \vdash \operatorname{return}(y), \cdot) \\ & & & & & & ([y \mapsto \exists] \vdash \operatorname{return}(y), \cdot) \\ & & & & & & ([y \mapsto \exists] \vdash \operatorname{return}(y) \triangleright \cdot \\ & & & & & & ([y \mapsto \exists] \vdash \operatorname{return}(y), \cdot) \\ & & & & & & & ([y \mapsto \exists] \vdash \operatorname{return}(y), \cdot) \\ & & & & & & & & ([y \mapsto \exists] \vdash \operatorname{return}(y) \vdash \cdot \\ & & & & & & & & & & & & & & & & & &$	$\longrightarrow$	$\cdot \vdash call(f,3) \triangleright (assign(y, -), (seq(call(g), return(y)), \cdot))$
$ \begin{array}{cccc} & & \langle(\cdot,(\operatorname{assign}(y,\_),(\operatorname{seq}(\operatorname{call}(g),\operatorname{return}(y)),\cdot)))\rangle;[x\mapsto 3]\vdash\operatorname{return}(x)\blacktriangleright \cdot \\ & & \langle(\cdot,(\operatorname{assign}(y,\_),(\operatorname{seq}(\operatorname{call}(g),\operatorname{return}(y)),\cdot)))\rangle;[x\mapsto 3]\vdash x \triangleright (\operatorname{return}(\_),\cdot) \\ & & & \langle(\cdot,(\operatorname{assign}(y,\_),(\operatorname{seq}(\operatorname{call}(g),\operatorname{return}(y)),\cdot)))\rangle;[x\mapsto 3]\vdash 3 \triangleright (\operatorname{return}(\_),\cdot) \\ & & & \cdot \vdash 3 \triangleright (\operatorname{assign}(y,\_),(\operatorname{seq}(\operatorname{call}(g),\operatorname{return}(y)),\cdot)) \\ & & & \cdot\vdash 3 \triangleright (\operatorname{assign}(y,\_),(\operatorname{seq}(\operatorname{call}(g),\operatorname{return}(y)),\cdot)) \\ & & & & [y\mapsto 3]\vdash \operatorname{nop}\blacktriangleright (\operatorname{seq}(\operatorname{call}(g),\operatorname{return}(y))) \blacktriangleright \cdot \\ & & & & [y\mapsto 3]\vdash \operatorname{call}(g)\triangleright (\operatorname{return}(y),\cdot) \\ & & & & & [y\mapsto 3]\vdash\operatorname{call}(g)\triangleright (\operatorname{return}(y),\cdot) \\ & & & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & \\ & & & & & \\ & & & & & & \\ & & & & & \\ & & & & & \\ & & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & \\ & & & & & & \\ & & & & & & & \\ & & & & & & & \\ & & & & & & & \\ & & & & & & \\ & & & & & & & \\ & & & & & & & \\ &$	$\longrightarrow$	$\cdot \vdash 3 \triangleright (call(f, \cdot), (assign(y, \cdot), (seq(call(g), return(y)), \cdot)))$
$ \begin{array}{cccc} & \longrightarrow & \langle(\cdot,(\operatorname{assign}(y,\_),(\operatorname{seq}(\operatorname{call}(g),\operatorname{return}(y)),\cdot)))\rangle;[x\mapsto 3]\vdash 3\triangleright(\operatorname{return}(\_),\cdot) \\ & \to & \cdot\vdash 3\triangleright(\operatorname{assign}(y,\_),(\operatorname{seq}(\operatorname{call}(g),\operatorname{return}(y)),\cdot)) \\ & \to & [y\mapsto 3]\vdash \operatorname{nop}\blacktriangleright(\operatorname{seq}(\operatorname{call}(g),\operatorname{return}(y)))\blacktriangleright \\ & \to & [y\mapsto 3]\vdash \operatorname{call}(g)\triangleright(\operatorname{return}(y))\blacktriangleright \\ & \to & [y\mapsto 3]\vdash \operatorname{call}(g)\triangleright(\operatorname{return}(y),\cdot) \\ & \to & (([y\mapsto 3],(\operatorname{discard},\operatorname{return}(y),\cdot)));\cdot\vdash \operatorname{seq}(4,\operatorname{return}(\operatorname{nothing}))\blacktriangleright \\ & \to & \langle([y\mapsto 3],(\operatorname{discard},\operatorname{return}(y),\cdot)));\cdot\vdash 4\vdash(\operatorname{return}(\operatorname{nothing}),\cdot) \\ & \to & \langle([y\mapsto 3],(\operatorname{discard},\operatorname{return}(y),\cdot)));\cdot\vdash 4\vdash(\operatorname{return}(\operatorname{nothing}),\cdot) \\ & \to & \langle([y\mapsto 3],(\operatorname{discard},\operatorname{return}(y),\cdot)));\cdot\vdash \operatorname{nop}\vdash(\operatorname{return}(\operatorname{nothing}),\cdot) \\ & \to & \langle([y\mapsto 3],(\operatorname{discard},\operatorname{return}(y),\cdot)));\cdot\vdash\operatorname{nothing}\triangleright(\operatorname{return}(\operatorname{nothing}),\cdot) \\ & \to & \langle([y\mapsto 3],(\operatorname{discard},\operatorname{return}(y),\cdot)));\cdot\vdash\operatorname{nothing}\triangleright(\operatorname{return}(\ldots),\cdot) \\ & \to & \langle([y\mapsto 3],(\operatorname{discard},\operatorname{return}(y),\cdot)));\cdot\vdash\operatorname{nothing}\triangleright(\operatorname{return}(y),\cdot) \\ & \to & \langle([y\mapsto 3],(\operatorname{discard},\operatorname{return}(y),\cdot)));\cdot\vdash\operatorname{nothing}\triangleright(\operatorname{return}(y),\cdot) \\ & \to & [y\mapsto 3]\vdash\operatorname{nop}\vdash(\operatorname{return}(y),\cdot) \\ & \to & [y\mapsto 3]\vdash\operatorname{return}(y)\leftarrow \cdot \\ & [y\mapsto 3]\vdash\operatorname{return}(y)\leftarrow \cdot \\ & [y\mapsto 3]\vdash\operatorname{y}\triangleright(\operatorname{return}(\_),\cdot) \end{array}$	$\longrightarrow$	$\langle (\cdot, (assign(y, \_), (seq(call(g), return(y)), \cdot))) \rangle; [x \mapsto 3] \vdash return(x) \triangleright \cdot$
$ \begin{array}{cccc} & & & \cdot \vdash 3 \triangleright (\operatorname{assign}(y, \_), (\operatorname{seq}(\operatorname{call}(g), \operatorname{return}(y)), \cdot)) \\ & & & & & & & & & & & & & & & & & &$	$\longrightarrow$	$\langle (\cdot, (assign(y, \_), (seq(call(g), return(y)), \cdot))) \rangle; [x \mapsto 3] \vdash x \triangleright (return(\_), \cdot)$
$ \begin{array}{c} & \qquad $	$\longrightarrow$	$\langle (\cdot, (assign(y, \_), (seq(call(g), return(y)), \cdot))) \rangle; [x \mapsto 3] \vdash 3 \triangleright (return(\_), \cdot) \rangle \rangle \rangle \rangle \rangle \rangle = \langle (return(y), \cdot) \rangle \langle (return(y), \cdot) \rangle \rangle \rangle \rangle \rangle \rangle \rangle \rangle \langle (return(y), \cdot) \rangle \langle (return(y), \cdot) \rangle $
$ \begin{array}{cccc} & & & & & & & & & & & & & & & & & $	$\longrightarrow$	$\cdot \vdash 3 \triangleright (assign(y, \_), (seq(call(g), return(y)), \cdot))$
$ \begin{array}{c} \longrightarrow \\ [y \mapsto 3] \vdash \operatorname{call}(g) \blacktriangleright (\operatorname{return}(y), \cdot) \\ (y \mapsto 3] \vdash \operatorname{call}(g) \triangleright (\operatorname{discard}, \operatorname{return}(y), \cdot) \\ ([y \mapsto 3], (\operatorname{discard}, \operatorname{return}(y), \cdot)) \rangle; \cdot \vdash \operatorname{seq}(4, \operatorname{return}(\operatorname{nothing})) \blacktriangleright \cdot \\ (([y \mapsto 3], (\operatorname{discard}, \operatorname{return}(y), \cdot)) \rangle; \cdot \vdash 4 \triangleright (\operatorname{return}(\operatorname{nothing}), \cdot) \\ (([y \mapsto 3], (\operatorname{discard}, \operatorname{return}(y), \cdot)) \rangle; \cdot \vdash 4 \triangleright (\operatorname{discard}, \operatorname{return}(\operatorname{nothing}), \cdot) \\ (([y \mapsto 3], (\operatorname{discard}, \operatorname{return}(y), \cdot)) \rangle; \cdot \vdash \operatorname{nop} \triangleright (\operatorname{return}(\operatorname{nothing}), \cdot) \\ (([y \mapsto 3], (\operatorname{discard}, \operatorname{return}(y), \cdot)) \rangle; \cdot \vdash \operatorname{nop} \vdash (\operatorname{return}(\operatorname{nothing}), \cdot) \\ (([y \mapsto 3], (\operatorname{discard}, \operatorname{return}(y), \cdot)) \rangle; \cdot \vdash \operatorname{nothing} \triangleright (\operatorname{return}(\ldots), \cdot) \\ (([y \mapsto 3], (\operatorname{discard}, \operatorname{return}(y), \cdot)) \rangle; \cdot \vdash \operatorname{nothing} \triangleright (\operatorname{return}(\ldots), \cdot) \\ ([y \mapsto 3], (\operatorname{discard}, \operatorname{return}(y), \cdot)) \rangle; \cdot \vdash \operatorname{nothing} \triangleright (\operatorname{return}(y), \cdot) \\ ([y \mapsto 3], (\operatorname{discard}, \operatorname{return}(y), \cdot)) \rangle; \cdot \vdash \operatorname{nothing} \triangleright (\operatorname{return}(y), \cdot) \\ (y \mapsto 3] \vdash \operatorname{nop} \triangleright (\operatorname{return}(y), \cdot) \\ (y \mapsto 3] \vdash \operatorname{return}(y) \vdash \cdot \end{array} $	$\longrightarrow$	$[y\mapsto 3]\vdash nop\blacktriangleright(seq(call(g),return(y)),\cdot)$
$ \begin{array}{c} \longrightarrow \\ [y \mapsto 3] \vdash \operatorname{call}(g) \triangleright (\operatorname{discard}, \operatorname{return}(y), \cdot) \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ $	$\longrightarrow$	$[y\mapsto 3]\vdash seq(call(g),return(y))\blacktriangleright\cdot$
$ \begin{array}{c} \longrightarrow \\ \langle ([y \mapsto 3], (\operatorname{discard}, \operatorname{return}(y), \cdot)) \rangle; \cdot \vdash \operatorname{seq}(4, \operatorname{return}(\operatorname{nothing})) \blacktriangleright \cdot \\ \langle ([y \mapsto 3], (\operatorname{discard}, \operatorname{return}(y), \cdot)) \rangle; \cdot \vdash 4 \blacktriangleright (\operatorname{return}(\operatorname{nothing}), \cdot) \\ \langle ([y \mapsto 3], (\operatorname{discard}, \operatorname{return}(y), \cdot)) \rangle; \cdot \vdash 4 \triangleright (\operatorname{discard}, \operatorname{return}(\operatorname{nothing}), \cdot) \\ \langle ([y \mapsto 3], (\operatorname{discard}, \operatorname{return}(y), \cdot)) \rangle; \cdot \vdash \operatorname{nop} \blacktriangleright (\operatorname{return}(\operatorname{nothing}), \cdot) \\ \langle ([y \mapsto 3], (\operatorname{discard}, \operatorname{return}(y), \cdot)) \rangle; \cdot \vdash \operatorname{nop} \vdash (\operatorname{return}(\operatorname{nothing}), \cdot) \\ \langle ([y \mapsto 3], (\operatorname{discard}, \operatorname{return}(y), \cdot)) \rangle; \cdot \vdash \operatorname{nothing} \triangleright (\operatorname{return}(-), \cdot) \\ \langle ([y \mapsto 3], (\operatorname{discard}, \operatorname{return}(y), \cdot)) \rangle; \cdot \vdash \operatorname{nothing} \triangleright (\operatorname{return}(-), \cdot) \\ (y \mapsto 3] \vdash \operatorname{nothing} \triangleright (\operatorname{discard}, \operatorname{return}(y), \cdot) \\ \to \\ [y \mapsto 3] \vdash \operatorname{nop} \triangleright (\operatorname{return}(y), \cdot) \\ \to \\ (y \mapsto 3] \vdash \operatorname{return}(y) \triangleright \cdot \\ [y \mapsto 3] \vdash y \triangleright (\operatorname{return}(-), \cdot) \end{array} $	$\longrightarrow$	$[y\mapsto 3]\vdash call(g)\blacktriangleright(return(y),\cdot)$
$ \begin{array}{c} \longrightarrow \\ \langle ([y \mapsto 3], (\operatorname{discard}, \operatorname{return}(y), \cdot)) \rangle; \cdot \vdash 4 \blacktriangleright (\operatorname{return}(\operatorname{nothing}), \cdot) \\ \langle ([y \mapsto 3], (\operatorname{discard}, \operatorname{return}(y), \cdot)) \rangle; \cdot \vdash 4 \triangleright (\operatorname{discard}, \operatorname{return}(\operatorname{nothing}), \cdot) \\ \langle ([y \mapsto 3], (\operatorname{discard}, \operatorname{return}(y), \cdot)) \rangle; \cdot \vdash \operatorname{nop} \blacktriangleright (\operatorname{return}(\operatorname{nothing}), \cdot) \\ \langle ([y \mapsto 3], (\operatorname{discard}, \operatorname{return}(y), \cdot)) \rangle; \cdot \vdash \operatorname{return}(\operatorname{nothing}) \blacktriangleright \cdot \\ \langle ([y \mapsto 3], (\operatorname{discard}, \operatorname{return}(y), \cdot)) \rangle; \cdot \vdash \operatorname{nothing} \triangleright (\operatorname{return}(\_), \cdot) \\ \langle ([y \mapsto 3], (\operatorname{discard}, \operatorname{return}(y), \cdot)) \rangle; \cdot \vdash \operatorname{nothing} \triangleright (\operatorname{return}(\_), \cdot) \\ \downarrow & \qquad \qquad$	$\longrightarrow$	$[y\mapsto 3]\vdash call(g) \triangleright (discard,return(y),\cdot)$
$ \begin{array}{c} \longrightarrow \\ \langle ([y \mapsto 3], (\operatorname{discard}, \operatorname{return}(y), \cdot)) \rangle; \cdot \vdash 4 \triangleright (\operatorname{discard}, \operatorname{return}(\operatorname{nothing}), \cdot) \\ \langle ([y \mapsto 3], (\operatorname{discard}, \operatorname{return}(y), \cdot)) \rangle; \cdot \vdash \operatorname{nop} \blacktriangleright (\operatorname{return}(\operatorname{nothing}), \cdot) \\ \rangle \\ \rightarrow \\ \langle ([y \mapsto 3], (\operatorname{discard}, \operatorname{return}(y), \cdot)) \rangle; \cdot \vdash \operatorname{return}(\operatorname{nothing}) \blacktriangleright \cdot \\ \langle ([y \mapsto 3], (\operatorname{discard}, \operatorname{return}(y), \cdot)) \rangle; \cdot \vdash \operatorname{nothing} \triangleright (\operatorname{return}(\_), \cdot) \\ \rangle \\ \rightarrow \\ ([y \mapsto 3], (\operatorname{discard}, \operatorname{return}(y), \cdot)) \rangle; \cdot \vdash \operatorname{nothing} \triangleright (\operatorname{return}(\_), \cdot) \\ ([y \mapsto 3] \vdash \operatorname{nothing} \triangleright (\operatorname{discard}, \operatorname{return}(y), \cdot) \\ \rightarrow \\ [y \mapsto 3] \vdash \operatorname{nop} \blacktriangleright (\operatorname{return}(y), \cdot) \\ \rightarrow \\ (y \mapsto 3] \vdash \operatorname{return}(y) \blacktriangleright \cdot \\ (y \mapsto 3] \vdash y \triangleright (\operatorname{return}(\_), \cdot) \end{array} $	$\longrightarrow$	$\langle ([y \mapsto 3], (discard, return(y), \cdot)) \rangle; \cdot \vdash seq(4, return(nothing)) \blacktriangleright \cdot$
$ \begin{array}{c} \longrightarrow \\ \langle ([y \mapsto 3], (\operatorname{discard}, \operatorname{return}(y), \cdot)) \rangle; \cdot \vdash \operatorname{nop} \blacktriangleright (\operatorname{return}(\operatorname{nothing}), \cdot) \\ \langle ([y \mapsto 3], (\operatorname{discard}, \operatorname{return}(y), \cdot)) \rangle; \cdot \vdash \operatorname{return}(\operatorname{nothing}) \blacktriangleright \cdot \\ \langle ([y \mapsto 3], (\operatorname{discard}, \operatorname{return}(y), \cdot)) \rangle; \cdot \vdash \operatorname{nothing} \triangleright (\operatorname{return}(\_), \cdot) \\ \rangle \\ \rightarrow \\ (([y \mapsto 3], (\operatorname{discard}, \operatorname{return}(y), \cdot)) \rangle; \cdot \vdash \operatorname{nothing} \triangleright (\operatorname{return}(\_), \cdot) \\ ([y \mapsto 3] \vdash \operatorname{nothing} \triangleright (\operatorname{discard}, \operatorname{return}(y), \cdot) \\ \rangle \\ \rightarrow \\ ([y \mapsto 3] \vdash \operatorname{nop} \blacktriangleright (\operatorname{return}(y), \cdot) \\ ([y \mapsto 3] \vdash \operatorname{return}(y) \blacktriangleright \cdot \\ ([y \mapsto 3] \vdash y \triangleright (\operatorname{return}(\_), \cdot) \\ \end{array} $	$\longrightarrow$	$\langle ([y \mapsto 3], (discard, return(y), \cdot)) \rangle; \cdot \vdash 4 \blacktriangleright (return(nothing), \cdot)$
$ \begin{array}{c} \longrightarrow \\ \langle ([y \mapsto 3], (\operatorname{discard}, \operatorname{return}(y), \cdot)) \rangle; \cdot \vdash \operatorname{return}(\operatorname{nothing}) \blacktriangleright \cdot \\ \langle ([y \mapsto 3], (\operatorname{discard}, \operatorname{return}(y), \cdot)) \rangle; \cdot \vdash \operatorname{nothing} \triangleright (\operatorname{return}(\_), \cdot) \\ \\ \longrightarrow \\ [y \mapsto 3] \vdash \operatorname{nothing} \triangleright (\operatorname{discard}, \operatorname{return}(y), \cdot) \\ \\ \longrightarrow \\ [y \mapsto 3] \vdash \operatorname{nop} \blacktriangleright (\operatorname{return}(y), \cdot) \\ \\ \longrightarrow \\ [y \mapsto 3] \vdash \operatorname{return}(y) \blacktriangleright \cdot \\ \\ \\ \end{pmatrix} $	$\longrightarrow$	$\langle ([y \mapsto 3], (discard, return(y), \cdot)) \rangle; \cdot \vdash 4 \triangleright (discard, return(nothing), \cdot)$
$ \begin{array}{c} \longrightarrow \\ \langle ([y \mapsto 3], (\operatorname{discard}, \operatorname{return}(y), \cdot)) \rangle; \cdot \vdash \operatorname{nothing} \triangleright (\operatorname{return}(\_), \cdot) \\ \longrightarrow \\ & \qquad \qquad$	$\longrightarrow$	$\langle ([y \mapsto 3], (discard, return(y), \cdot)) \rangle; \cdot \vdash nop \blacktriangleright (return(nothing), \cdot)$
$\begin{array}{ccc} \longrightarrow & [y \mapsto 3] \vdash \operatorname{nothing} \triangleright (\operatorname{discard}, \operatorname{return}(y), \cdot) \\ \longrightarrow & [y \mapsto 3] \vdash \operatorname{nop} \blacktriangleright (\operatorname{return}(y), \cdot) \\ \longrightarrow & [y \mapsto 3] \vdash \operatorname{return}(y) \blacktriangleright \cdot \\ \longrightarrow & [y \mapsto 3] \vdash y \triangleright (\operatorname{return}(\_), \cdot) \end{array}$	$\longrightarrow$	$\langle ([y \mapsto 3], (discard, return(y), \cdot)) \rangle; \cdot \vdash return(nothing) \blacktriangleright \cdot$
$\begin{array}{c} \longrightarrow \\ \longrightarrow \\ \longrightarrow \\ \longrightarrow \\ \end{array} \begin{bmatrix} y \mapsto 3 \end{bmatrix} \vdash nop \blacktriangleright (return(y), \cdot) \\ [y \mapsto 3] \vdash return(y) \blacktriangleright \cdot \\ [y \mapsto 3] \vdash y \triangleright (return(\_), \cdot) \end{array}$	$\longrightarrow$	$\langle ([y \mapsto 3], (discard, return(y), \cdot)) \rangle; \cdot \vdash nothing \triangleright (return(\_), \cdot)$
$ \begin{array}{c} \longrightarrow \\ & [y \mapsto 3] \vdash return(y) \blacktriangleright \cdot \\ & \longrightarrow \\ & [y \mapsto 3] \vdash y \triangleright (return(\_), \cdot) \end{array} $	$\longrightarrow$	$[y\mapsto 3]\vdash nothing \triangleright (discard,return(y),\cdot)$
$\longrightarrow \qquad \qquad [y\mapsto 3]\vdash y\triangleright (return(\_),\cdot)$	$\longrightarrow$	$[y\mapsto 3]\vdash nop\blacktriangleright(return(y),\cdot)$
	$\longrightarrow$	$[y\mapsto 3]\vdash return(y)\blacktriangleright\cdot$
$\longrightarrow \qquad \qquad [y\mapsto 3]\vdash 3\triangleright (return(\_),\cdot)$	$\longrightarrow$	$[y\mapsto 3]\vdash y\triangleright (return(\_),\cdot)$
	$\longrightarrow$	$[y\mapsto 3]\vdash 3 \triangleright (return(\_), \cdot)$

 $\underline{\textbf{Solution:}} \ \cdot \vdash \mathsf{seq}(\mathsf{assign}(y,\mathsf{call}(f,3)),\mathsf{seq}(\mathsf{call}(g),\mathsf{return}(y))) \blacktriangleright \ \cdot$