15-411: Mutable Store

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Pointers and Arrays

We will see how static and dynamic semantics make it easy to introduce and specify advanced language features

- Static semantics of pointers
- Dynamic semantics of pointers
- Static semantics of arrays
- Dynamic semantics of arrays

Recap

Dynamic Semantics - Configurations

Call stack.

Continuation

$S\ ;\ \eta \vdash e \rhd K$ – Evaluating the expression e with the continuation K

$S\ ;\ \eta \vdash s \blacktriangleright K$ – Evaluating the statement s with the continuation K

Environment

$\mathsf{value}(c)$ – Final state, return a value

$\mathsf{exception}(E)$ – Final state, report an error

All ops.

Expressions	e	$::=$	$c \mid e_1 \odot e_2 \mid \mathsf{true} \mid \mathsf{false} \mid e_1 \;\&\&\; e_2 \mid x \mid f(e_1, e_2) \mid f(\,)$
Statements	s	$::=$	$\mathsf{nop} \mid \mathsf{seq}(s_1, s_2) \mid \mathsf{assign}(x, e) \mid \mathsf{decl}(x, \tau, s)$
		$\mid$	$\mathsf{if}(e, s_1, s_2) \mid \mathsf{while}(e, s) \mid \mathsf{return}(e) \mid \mathsf{assert}(e)$
Values	v	$::=$	$c \mid \mathsf{true} \mid \mathsf{false} \mid \mathsf{nothing}$
Environments	η	$::=$	$\cdot \mid \eta, x \mapsto c$
Stacks	S	$::=$	$\cdot \mid S\ ,\ \langle \eta, K \rangle$
Cont. frames	ϕ	$::=$	$\_ \odot e \mid c \odot \_ \mid \_ \;\&\&\; e \mid f(\_, e) \mid f(c, \_)$
		$\mid$	$s \mid \mathsf{assign}(x, \_) \mid \mathsf{if}(\_, s_1, s_2) \mid \mathsf{return}(\_) \mid \mathsf{assert}(\_)$
Continuations	K	$::=$	$\cdot \mid \phi\ ,\ K$
Exceptions	E	$::=$	$\mathsf{arith} \mid \mathsf{abort} \mid \mathsf{mem}$

Definitions

$$
\begin{array}{lll}
S\ ;\eta \vdash e_1 \odot e_2 \rhd K & \longrightarrow & S\ ;\eta \vdash e_1 \rhd (\_ \odot e_2\ , K) \\
S\ ;\eta \vdash c_1 \rhd (\_ \odot e_2\ , K) & \longrightarrow & S\ ;\eta \vdash e_2 \rhd (c_1 \odot \_\ , K) \\
S\ ;\eta \vdash c_2 \rhd (c_1 \odot \_\ , K) & \longrightarrow & S\ ;\eta \vdash c \rhd K \qquad (c = c_1 \odot c_2) \\
S\ ;\eta \vdash c_2 \rhd (c_1 \odot \_\ , K) & \longrightarrow & \mathsf{exception}(\mathsf{arith}) \qquad (c_1 \odot c_2 \text{ undefined}) \\
\\
S\ ;\eta \vdash e_1\ \texttt{\&\&}\ e_2 \rhd K & \longrightarrow & S\ ;\eta \vdash e_1 \rhd (\_\ \texttt{\&\&}\ e_2\ , K) \\
S\ ;\eta \vdash \mathsf{false} \rhd (\_\ \texttt{\&\&}\ e_2\ , K) & \longrightarrow & S\ ;\eta \vdash \mathsf{false} \rhd K \\
S\ ;\eta \vdash \mathsf{true} \rhd (\_\ \texttt{\&\&}\ e_2\ , K) & \longrightarrow & S\ ;\eta \vdash e_2 \rhd K \\
\\
S\ ;\eta \vdash x \rhd K & \longrightarrow & S\ ;\eta \vdash \eta(x) \rhd K
\end{array}
$$

Transitions: Expressions

$$S\ ;\eta \vdash \mathsf{seq}(s_1, s_2) \blacktriangleright K \quad\longrightarrow\quad S\ ;\eta \vdash s_1 \blacktriangleright (s_2\ ,\ K)$$
$$S\ ;\eta \vdash \mathsf{nop} \blacktriangleright (s\ ,\ K) \quad\longrightarrow\quad S\ ;\eta \vdash s \blacktriangleright K$$
$$S\ ;\eta \vdash \mathsf{assign}(x, e) \blacktriangleright K \quad\longrightarrow\quad S\ ;\eta \vdash e \rhd (\mathsf{assign}(x, \_)\ ,\ K)$$
$$S\ ;\eta \vdash c \rhd (\mathsf{assign}(x, \_)\ ,\ K) \quad\longrightarrow\quad S\ ;\eta[x \mapsto c] \vdash \mathsf{nop} \blacktriangleright K$$

$$S\ ;\eta \vdash \mathsf{decl}(x, \tau, s) \blacktriangleright K \quad\longrightarrow\quad S\ ;\eta[x \mapsto \mathsf{nothing}] \vdash s \blacktriangleright K$$

$$S\ ;\eta \vdash \mathsf{assert}(e) \blacktriangleright K \quad\longrightarrow\quad S\ ;\eta \vdash e \rhd (\mathsf{assert}(\_)\ ,\ K)$$
$$S\ ;\eta \vdash \mathsf{true} \rhd (\mathsf{assert}(\_)\ ,\ K) \quad\longrightarrow\quad S\ ;\eta \vdash \mathsf{nop} \blacktriangleright K$$
$$S\ ;\eta \vdash \mathsf{false} \rhd (\mathsf{assert}(\_)\ ,\ K) \quad\longrightarrow\quad \mathsf{exception}(\mathsf{abort})$$

$$S\ ;\eta \vdash \mathsf{if}(e, s_1, s_2) \blacktriangleright K \quad\longrightarrow\quad S\ ;\eta \vdash e \rhd (\mathsf{if}(\_, s_1, s_2)\ ,\ K)$$
$$S\ ;\eta \vdash \mathsf{true} \rhd (\mathsf{if}(\_, s_1, s_2), K) \quad\longrightarrow\quad S\ ;\eta \vdash s_1 \blacktriangleright K$$
$$S\ ;\eta \vdash \mathsf{false} \rhd (\mathsf{if}(\_, s_1, s_2), K) \quad\longrightarrow\quad S\ ;\eta \vdash s_2 \blacktriangleright K$$

$$S\ ;\eta \vdash \mathsf{while}(e, s) \blacktriangleright K \quad\longrightarrow\quad S\ ;\eta \vdash \mathsf{if}(e, \mathsf{seq}(s, \mathsf{while}(e, s)), \mathsf{nop}) \blacktriangleright K$$

Transitions: Statements

$$S\ ;\eta \vdash f(e_1,e_2) \rhd K \quad\longrightarrow\quad S\ ;\eta \vdash e_1 \rhd (f(\_,e_2)\ ,\ K)$$

$$S\ ;\eta \vdash c_1 \rhd (f(\_,e_2)\ ,\ K) \quad\longrightarrow\quad S\ ;\eta \vdash e_2 \rhd (f(c_1,\_)\ ,\ K)$$

$$S\ ;\eta \vdash c_2 \rhd (f(c_1,\_)\ ,\ K) \quad\longrightarrow\quad (S\ ,\ \langle \eta, K\rangle)\ ;\ [x_1 \mapsto c_1, x_2 \mapsto c_2] \vdash s \blacktriangleright \cdot$$

(given that f is defined as $f(x_1,x_2)\{s\}$)

$$S\ ;\eta \vdash f(\,) \rhd K \quad\longrightarrow\quad (S\ ,\ \langle \eta, K\rangle)\ ;\ \cdot \vdash s \blacktriangleright \cdot$$

(given that f is defined as $f(\,)\{s\}$)

$$S\ ;\eta \vdash \mathsf{return}(e) \blacktriangleright K \quad\longrightarrow\quad S\ ;\eta \vdash e \rhd (\mathsf{return}(\_)\ ,\ K)$$

$$(S\ ,\ \langle \eta', K'\rangle)\ ;\eta \vdash v \rhd (\mathsf{return}(\_)\ ,\ K) \quad\longrightarrow\quad S\ ;\eta' \vdash v \rhd K'$$

$$\cdot\ ;\eta \vdash c \rhd (\mathsf{return}(\_)\ ,\ K) \quad\longrightarrow\quad \mathsf{value}(c)$$

Special case: returning void

$$S\ ,\ \langle \eta', K'\rangle\ ;\eta \vdash \mathsf{nop} \blacktriangleright \cdot \quad\longrightarrow\quad S\ ;\eta' \vdash \mathsf{nothing} \rhd K'$$

Expressions as statements:

$$S\ ;\eta \vdash e \blacktriangleright K \quad\longrightarrow\quad S\ ;\eta \vdash e \rhd (\mathsf{discard}, K)$$

$$S\ ;\eta \vdash v \rhd (\mathsf{discard}, K) \quad\longrightarrow\quad S\ ;\eta \vdash \mathsf{nop} \blacktriangleright K$$

Transitions: Functions

Static semantics of pointers

Static Semantics of Pointers

Extend types with pointer types:

$$\tau ::= \mathsf{int} \mid \mathsf{bool} \mid \tau*$$

Extend expressions with allocations, dereference, and null pointers:

$$e ::= \ldots \mid \mathsf{alloc}(\tau) \mid *e \mid \mathsf{null}$$

We add the following typing rules for expressions:

We cannot synthesize this type.

$$\frac{}{\Gamma \vdash \mathsf{alloc}(\tau) : \tau*} \qquad \frac{\Gamma \vdash e : \tau*}{\Gamma \vdash *e : \tau} \qquad \frac{}{\Gamma \vdash \mathsf{null} : \tau*}$$

How to Type null?

Idea: Use an indefinite (polymorphic) type `any*` for synthesis

$$\frac{}{\Gamma \vdash \textsf{null} : any*}$$

- This type can be seen as a temporary placeholder
- When we constructed the type derivation we could replace any with a ‘concrete type’
- Another view is to say that `any*` has exactly one value: null

Example: Pointer Equality

We can compare two pointers using `p==q` if the have the same type

- If p and q both have definite type $\tau *$ then `p==q` is well-typed
- If p has definite type $\tau_1 *$
  and q has definite type $\tau_2 *$ for different types τ_1 and τ_2
  then `p==q` is rejected
- If `p` has definite type $\tau *$
  and `q` has type `any*`
  then `p==q` is well typed because we can compare every pointer to null
- If both `p` and `q` have type `any*` then `p==q` is well-typed

Type Rules

Dereference and type instantiation

Cannot dereference a Null pointer.

$$\frac{\Gamma \vdash e : any*}{\Gamma \vdash e : \tau*}$$

$$\frac{\Gamma \vdash e : \tau* \quad \Gamma \nvdash e : any*}{\Gamma \vdash *e : \tau}$$

Equality

$$\frac{\Gamma \vdash e_1 : \tau \quad \Gamma \vdash e_2 : \tau}{\Gamma \vdash e_1 \texttt{==} e_2 : \mathsf{bool}}$$

Dynamic semantics of pointers

Configurations with Heap

- A value of a type τ^* is an address that stores a value of type τ (or a special address 0)
- Allocations return fresh (unused) addresses
- Dereferencing retrieves a stored value

➡ Need **heap** that maps addresses to values

Heap | Stack | Continuation

$$H \; ; S \; ; \eta \vdash e \triangleright K$$

Environment

Modeling the Heap

- Addresses are 64 bit words?
  Problem: We can run out of memory

We didn't model stack overflow.

- We don't want to specify that programs fail due to memory errors (garbage collection, OS details, …)

- Approach: no out-of-memory errors at the high level
  -> **addresses are natural numbers**

$$H : (\mathbb{N} \cup \{\mathsf{next}\}) \rightarrow \mathsf{Val}$$

Special address that points to the next free address.

Evaluation Rules I

Heap is just passed through.

Previous runs are lifted:

$$H\ ;\ S\ ;\ \eta \vdash e_1 \odot e_2 \rhd K \quad \longrightarrow \quad H\ ;\ S\ ;\ \eta \vdash e_1 \rhd (\_ \odot e_2\ ,\ K)$$

New rules:

$$H\ ;\ S\ ;\ \eta \vdash \mathsf{null} \rhd K \quad \longrightarrow \quad H\ ;\ S\ ;\ \eta \vdash 0 \rhd K$$

Store a default value.

$$H\ ;\ S\ ;\ \eta \vdash \mathsf{alloc}(\tau) \rhd K \quad \longrightarrow \quad H[a \mapsto \mathsf{default}(\tau), \mathsf{next} \mapsto a + |\tau|]\ ;\ S\ ;\ \eta \vdash a \rhd K$$

$$a = H(\mathsf{next})$$

Evaluation Rules: Allocation

$H \; ; S \; ; \eta \vdash \mathsf{alloc}(\tau) \rhd K \qquad \longrightarrow \qquad H[a \mapsto \mathsf{default}(\tau), \mathsf{next} \mapsto a + |\tau|] \; ; S \; ; \eta \vdash a \rhd K$

$a = H(\mathsf{next})$

Default values

default(bool) = false

default(int) = 0

default(τ *) = null

➡ In the implementation you can initialize everything to 0

Type sizes (x86-64):

$$
\begin{aligned}
|\mathsf{int}| &= 4 \\
|\mathsf{bool}| &= 4 \\
|\tau *| &= 8 \\
|\tau[\,]| &= 8
\end{aligned}
$$

Evaluation Rules: Dereference

Dereferencing

$$H\ ;S\ ;\eta \vdash *e \rhd K \quad\longrightarrow\quad H\ ;S\ ;\eta \vdash e \rhd (*\_\ ,K)$$

$$H\ ;S\ ;\eta \vdash a \rhd (*\_\ ,K) \quad\longrightarrow\quad H\ ;S\ ;\eta \vdash H(a) \rhd K \qquad (a \neq 0)$$

$$H\ ;S\ ;\eta \vdash a \rhd (*\_\ ,K) \quad\longrightarrow\quad \mathsf{exception}(\mathsf{mem}) \qquad (a = 0)$$

Implementing memory exceptions

- Use signal SIGUSR2 instead of SIGSEGV
- Better for debugging: better distinguishable from stack overflow and “accidental” memory errors

Assignments: Typing

Destinations (or l-values):

$$d ::= x \mid *d$$

Arrays and structs will add more destinations.

Typing rule:

$$\frac{\Gamma \vdash d : \tau \quad \Gamma \vdash e : \tau}{\Gamma \vdash \mathsf{assign}(d, e) : [\tau']}$$

Return type of current function.

Assignment: Evaluation Rules

Variables:

$$H\ ;\ S\ ;\ \eta \vdash \textsf{assign}(x, e) \blacktriangleright K \quad \longrightarrow \quad H\ ;\ S\ ;\ \eta \vdash e \triangleright (\textsf{assign}(x, \_)\ ,\ K)$$

$$H\ ;\ S\ ;\ \eta \vdash c \triangleright (\textsf{assign}(x, \_)\ ,\ K) \quad \longrightarrow \quad H\ ;\ S\ ;\ \eta[x \mapsto c] \triangleright \textsf{nop} \blacktriangleright K$$

Memory destinations:

$$H\ ;\ S\ ;\ \eta \vdash \textsf{assign}(*d, e) \blacktriangleright K \quad \longrightarrow \quad H\ ;\ S\ ;\ \eta \vdash d \triangleright (\textsf{assign}(*\_, e)\ ,\ K)$$

$$H\ ;\ S\ ;\ \eta \vdash a \triangleright (\textsf{assign}(*\_, e)\ ,\ K) \quad \longrightarrow \quad H\ ;\ S\ ;\ \eta \vdash e \triangleright (\textsf{assign}(*a, \_)\ ,\ K)$$

$$H\ ;\ S\ ;\ \eta \vdash c \triangleright (\textsf{assign}(*a, \_)\ ,\ K) \quad \longrightarrow \quad H[a \mapsto c]\ ;\ S\ ;\ \eta \vdash \textsf{nop} \blacktriangleright K \qquad (a \neq 0)$$

$$H\ ;\ S\ ;\ \eta \vdash c \triangleright (\textsf{assign}(*a, \_)\ ,\ K) \quad \longrightarrow \quad \textsf{exception}(\textsf{mem}) \qquad (a = 0)$$

Examples

What happens if we evaluate the following statements?

```
int* p = NULL;
*p = 1/0;
```

Arithmetic exception.

```
int** p = NULL;
**p = 1/0;
```

Memory exception.

Arrays

Arrays: Typing

Types, expressions, destinations:

$$\tau \quad ::= \quad \ldots \mid \tau[\,]$$

$$e \quad ::= \quad \ldots \mid \mathsf{alloc\_array}(\tau, e) \mid e_1[e_2]$$

$$d \quad ::= \quad \ldots \mid d[e]$$

There are no "null" arrays.

However, there are default arrays.

Type rules:

$$\frac{\Gamma \vdash e_1 : \tau[\,] \quad \Gamma \vdash e_2 : \mathsf{int}}{\Gamma \vdash e_1[e_2] : \tau}$$

$$\frac{\Gamma \vdash e : \mathsf{int}}{\Gamma \vdash \mathsf{alloc\_array}(\tau, e) : \tau[\,]}$$

Array Evaluation: Access

$$H\ ;\ S\ ;\ \eta \vdash e_1[e_2] \rhd K \quad \longrightarrow \quad H\ ;\ S\ ;\ \eta \vdash e_1 \rhd (\_[e_2]\ ,\ K)$$

$$H\ ;\ S\ ;\ \eta \vdash a \rhd (\_[e_2]\ ,\ K) \quad \longrightarrow \quad H\ ;\ S\ ;\ \eta \vdash e_2 \rhd (a[\_]\ ,\ K)$$

Need types.

$$H\ ;\ S\ ;\ \eta \vdash i \rhd (a[\_]\ ,\ K) \quad \longrightarrow \quad H\ ;\ S\ ;\ \eta \vdash H(a + i|\tau|) \rhd K$$

$$a \neq 0, 0 \leq i < \mathsf{length}(a), a : \tau[\,]$$

Need array sizes.

$$H\ ;\ S\ ;\ \eta \vdash i \rhd (a[\_]\ ,\ K) \quad \longrightarrow \quad \mathsf{exception}(\mathsf{mem})$$

$$a = 0 \text{ or } i < 0 \text{ or } i \geq \mathsf{length}(a)$$

Arrays: Implementation

Types?

- We know type τ[] of destination `e1` at compile time

➡ Just select the right constant when generating code

- In the dynamic semantics: assume $e_1[e_2]$ has been elaborated to

$$e_1\{\tau\}[e_2] \qquad \text{if} \qquad e_1 : \tau$$

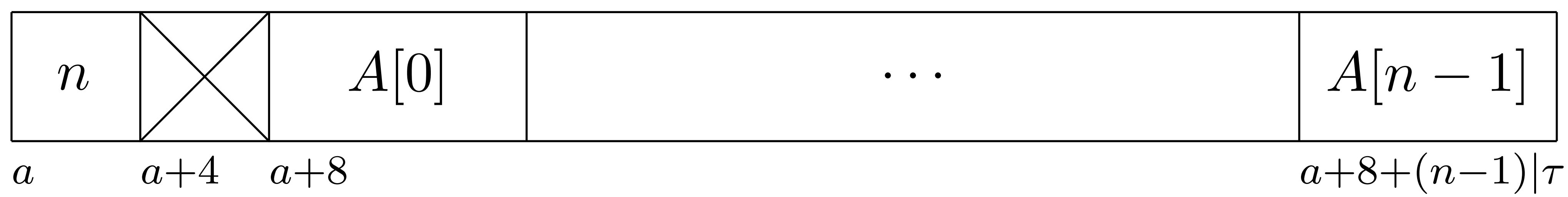
Lengths?

- Not known at compile time
- In `alloc_array(`τ`,e)`, `e` can be an arbitrary expression

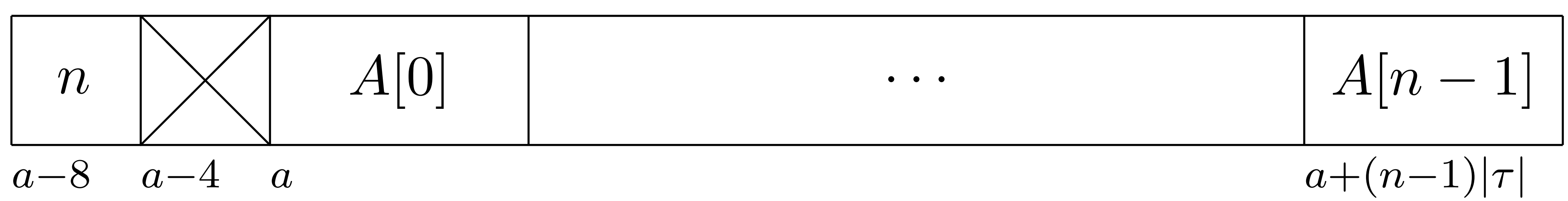
➡Need to store array length

Storing Array Length

Alternative 1: Add length at the front, array address points to the start



Alternative 2: Array address points to the first element



- Simplifies address arithmetic
- Allows to pass pointers to C (which wouldn’t care about length info)

Updated Rules for Array Access

$$H \; ; S \; ; \eta \vdash e_1\{\tau\}[e_2] \rhd K \quad \longrightarrow \quad H \; ; S \; ; \eta \vdash e_1 \rhd (\_\{\tau\}[e_2] \; , K)$$

$$H \; ; S \; ; \eta \vdash a \rhd (\_\{\tau\}[e_2] \; , K) \quad \longrightarrow \quad H \; ; S \; ; \eta \vdash e_2 \rhd (a\{\tau\}[\_] \; , K)$$

$$H \; ; S \; ; \eta \vdash i \rhd (a\{\tau\}[\_] \; , K) \quad \longrightarrow \quad H \; ; S \; ; \eta \vdash H(a + i|\tau|) \rhd K$$

$$a \neq 0, 0 \leq i < H(a - 8)$$

$$H \; ; S \; ; \eta \vdash i \rhd (a\{\tau\}[\_] \; , K) \quad \longrightarrow \quad \text{exception(mem)}$$

$$a = 0 \text{ or } i < 0 \text{ or } i \geq H(a - 8)$$

Array Access: Code Generation

The code pattern for $e_1\{\tau\}[e_2]$ and $|\tau| = k$ could be like this:

$$
\begin{array}{ll}
\mathsf{cogen}(e_1, a) & (a \text{ new}) \\
\mathsf{cogen}(e_2, i) & (i \text{ new}) \\
a_1 \leftarrow a - 8 & \\
t_2 \leftarrow M[a_1] & \\
\mathsf{if}\ (i < 0)\ \mathsf{goto\ error} & \\
\mathsf{if}\ (i \geq t_2)\ \mathsf{goto\ error} & \\
a_3 \leftarrow i * \$k & \\
a_4 \leftarrow a + a_3 & \\
t_5 \leftarrow M[a_4] &
\end{array}
$$

Array Evaluation: Allocation

$$H \; ; S \; ; \eta \vdash \mathsf{alloc\_array}(\tau, e) \rhd K \qquad \longrightarrow \qquad H \; ; S \; ; \eta \vdash e \rhd (\mathsf{alloc\_array}(\tau, \_) \; , K)$$

$$H \; ; S \; ; \eta \vdash n \rhd (\mathsf{alloc\_array}(\tau, \_) \; , K) \qquad \longrightarrow \qquad H' \; ; S \; ; \eta \vdash a' \rhd K \qquad (n \geq 0)$$

$$a = H(\mathsf{next}) \qquad a' = a + 8$$

$$H' = H[a \mapsto n, a' + 0|\tau| \mapsto \mathsf{default}(\tau), \ldots, a' + (n-1)|\tau| \mapsto \mathsf{default}(\tau), \mathsf{next} \mapsto a' + n|\tau|]$$

$$H \; ; S \; ; \eta \vdash n \rhd (\mathsf{alloc\_array}(\tau, \_) \; , K) \qquad \longrightarrow \qquad \mathsf{exception}(\mathsf{mem}) \qquad (n < 0)$$

Array Evaluation: Assignment

length(a) = H(a-8)

$$H \; ; S \; ; \eta \vdash \mathsf{assign}(d\{\tau\}[e_2], e_3) \blacktriangleright K \quad \longrightarrow \quad H \; ; S \; ; \eta \vdash d \rhd (\mathsf{assign}(\_\{\tau\}[e_2], e_3) \; , K)$$

$$H \; ; S \; ; \eta \vdash a \rhd (\mathsf{assign}(\_\{\tau\}[e_2], e_3) \; , K) \quad \longrightarrow \quad H \; ; S \; ; \eta \vdash e_2 \rhd (\mathsf{assign}(a\{\tau\}[\_], e_3) \; , K)$$

$$H \; ; S \; ; \eta \vdash i \rhd (\mathsf{assign}(a\{\tau\}[\_], e_3) \; , K) \quad \longrightarrow \quad H \; ; S \; ; \eta \vdash e_3 \rhd (\mathsf{assign}(a + i|\tau|, \_) \; , K)$$

$$a \neq 0, 0 \leq i < \mathsf{length}(a)$$

$$H \; ; S \; ; \eta \vdash i \rhd (\mathsf{assign}(a\{\tau\}[\_], e_3) \; , K) \quad \longrightarrow \quad \mathsf{exception}(\mathsf{mem})$$

$$a = 0 \text{ or } i < 0 \text{ or } i \geq \mathsf{length}(a)$$

$$H \; ; S \; ; \eta \vdash c \rhd (\mathsf{assign}(b, \_) \; , K) \quad \longrightarrow \quad H[b \mapsto c] \; ; S \; ; \eta \vdash \mathsf{nop} \blacktriangleright K$$

Default Values of Array Type

We also need a default value for array types

- We will just use 0 as the default value again
- It represents an array of length 0
- We can never legally access an array element in the default array
- Warning: Arrays can be compared with equality
- Make sure that alloc_array(t,0) returns a fresh address different from 0
- If arrays have address a=0 then you should not access M[a-8]

Compound Assignment Operators

- We translate x += e to x = x + e
- We cannot translate d1[e2] += e3 to d1[e2] = d1[e2] + e3

Effects of e2 and d1 would be evaluated twice.