

Gödel's System T

15-814: Types and Programming Languages

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1 Syntax

Typ	$\tau ::=$	nat	number
		$\tau_1 \rightarrow \tau_2$	function
Exp	$e ::=$	x	variable
		z	zero
		$\mathbf{s}(e)$	successor
		$\mathbf{rec}\{e_0; x.y.e_1\}(e)$	recursion
		$\lambda(x : \tau) e$	abstraction
		$e_1(e_2)$	application

2 Statics

$$\frac{}{\Gamma, x : \tau \vdash x : \tau} T_{\text{var}} \quad \frac{}{\Gamma \vdash \mathbf{z} : \mathbf{nat}} T_{\mathbf{z}} \quad \frac{\Gamma \vdash e : \mathbf{nat}}{\Gamma \vdash \mathbf{s}(e) : \mathbf{nat}} T_{\mathbf{s}}$$
$$\frac{\Gamma \vdash e : \mathbf{nat} \quad \Gamma \vdash e_0 : \tau \quad \Gamma, x : \mathbf{nat}, y : \tau \vdash e_1 : \tau}{\Gamma \vdash \mathbf{rec}\{e_0; x.y.e_1\}(e) : \tau} T_{\text{rec}}$$
$$\frac{\Gamma, x : \tau_1 \vdash e_2 : \tau_2}{\Gamma \vdash \lambda(x : \tau_1) e_2 : \tau_1 \rightarrow \tau_2} T_{\text{abs}} \quad \frac{\Gamma \vdash e_1 : \tau_1 \rightarrow \tau_2 \quad \Gamma \vdash e_2 : \tau_1}{\Gamma \vdash e_1(e_2) : \tau_2} T_{\text{ap}}$$

3 Call-by-name dynamics

3.1 Values

$$\frac{}{\lambda(x : \tau) e \text{ val}} V_{\text{lam}} \quad \frac{}{\mathbf{z} \text{ val}} V_{\mathbf{z}} \quad \frac{e \text{ val}}{\mathbf{s}(e) \text{ val}} V_{\mathbf{s}}$$

3.2 Transitions

$$\frac{e \mapsto e'}{\mathbf{s}(e) \mapsto \mathbf{s}(e')} E_{\mathbf{s}} \quad \frac{e \mapsto e'}{\mathbf{rec}\{e_0; x.y.e_1\}(e) \mapsto \mathbf{rec}\{e_0; x.y.e_1\}(e')} E_{\text{rec1}}$$
$$\frac{}{\mathbf{rec}\{e_0; x.y.e_1\}(\mathbf{z}) \mapsto e_0} E_{\text{rec2}} \quad \frac{\mathbf{s}(e) \text{ val}}{\mathbf{rec}\{e_0; x.y.e_1\}(\mathbf{s}(e)) \mapsto [e, \mathbf{rec}\{e_0; x.y.e_1\}(e)/x, y]e_1} E_{\text{rec3}}$$

$$\frac{e_1 \mapsto e'_1}{e_1(e_2) \mapsto e'_1(e_2)} E_{\text{ap1}} \quad \frac{}{(\lambda(x : \tau) e)(e_2) \mapsto [e_2/x]e} E_{\text{ap2}}$$