Lecture Notes on PCF

15-814: Types and Programming Languages
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1 Call-By-Value PCF

In the following, we present the syntax as semantics of *call-by-value* PCF or eager PCF.

Syntax

e	::=	x	variable
		$\mathtt{z}$	zero
		$\mathtt{s}(e)$	successor
		$\mathtt{ifz}\{e_0; x.e_1\}(e)$	conditional
		$\mathtt{fun}\{\tau_1; \tau_2\}(f.x.e)$	recursive function
		$\mathtt{ap}(e_1; e_2)$	function application

Static Semantics

$$\frac{}{\Gamma, x : \tau \vdash x : \tau}\ T_{\text{var}} \qquad \frac{}{\Gamma \vdash \mathtt{z} : \mathtt{nat}}\ T_{\mathtt{z}} \qquad \frac{\Gamma \vdash e : \mathtt{nat}}{\Gamma \vdash \mathtt{s}(e) : \mathtt{nat}}\ T_{\mathtt{s}}$$

$$\frac{\Gamma \vdash e : \mathtt{nat} \quad \Gamma \vdash e_0 : \tau \quad \Gamma, x : \mathtt{nat} \vdash e_1 : \tau}{\Gamma \vdash \mathtt{ifz}\{e_0; x.e_1\}(e) : \tau}\ T_{\text{ifz}}$$

$$\frac{\Gamma, f : \tau_1 \to \tau_2, x : \tau_1 \vdash e : \tau_2}{\Gamma \vdash \mathtt{fun}\{\tau_1; \tau_2\}(f.x.e) : \tau_1 \to \tau_2}\ T_{\text{fun}} \qquad \frac{\Gamma \vdash e_1 : \tau \to \tau' \quad \Gamma \vdash e_2 : \tau}{\Gamma \vdash \mathtt{ap}(e_1; e_2) : \tau'}\ T_{\text{ap}}$$

Values The values of call-by-value PCF are defined as follows. This definition does not change in the different dynamic semantics we consider.

$$\frac{}{\mathtt{z}\ \mathsf{val}}\ V_{\text{zero}} \qquad \frac{e\ \mathsf{val}}{\mathtt{s}(e)\ \mathsf{val}}\ V_{\text{succ}} \qquad \frac{}{\mathtt{fun}\{\tau_1;\tau_2\}(f.x.e)\ \mathsf{val}}\ V_{\text{fun}}$$

Dynamic Semantics The structural operational semantics is inductively defined as follows.

$$\frac{e \longmapsto e'}{\mathtt{ifz}\{e_0;x.e_1\}(e) \longmapsto \mathtt{ifz}\{e_0;x.e_1\}(e')}\ S_{\text{ifz1}} \qquad \frac{}{\mathtt{ifz}\{e_0;x.e_1\}(\mathtt{z}) \longmapsto e_0}\ S_{\text{ifz2}}$$

$$\frac{e\ \mathsf{val}}{\mathtt{ifz}\{e_0;x.e_1\}(\mathtt{s}(e)) \longmapsto [e/x]e_1}\ S_{\text{ifz3}} \qquad \frac{e_1 \longmapsto e_1'}{\mathtt{ap}(e_1;e_2) \longmapsto \mathtt{ap}(e_1';e_2)}\ S_{\text{ap1}}$$

$$\frac{e_1\ \mathsf{val} \quad e_2 \longmapsto e_2'}{\mathtt{ap}(e_1;e_2) \longmapsto \mathtt{ap}(e_1;e_2')}\ S_{\text{ap2}}$$

$$\frac{e_2\ \mathsf{val}}{\mathtt{ap}(\mathtt{fun}\{\tau_1;\tau_2\}(f.x.e);e_2) \longmapsto [\mathtt{fun}\{\tau_1;\tau_2\}(f.x.e),e_2/f,x]e}\ S_{\text{ap3}}$$