16-782
Planning & Decision-making in Robotics

Search Algorithms:
Markov Property,
Dependent vs. Independent variables,
Dominance relationship

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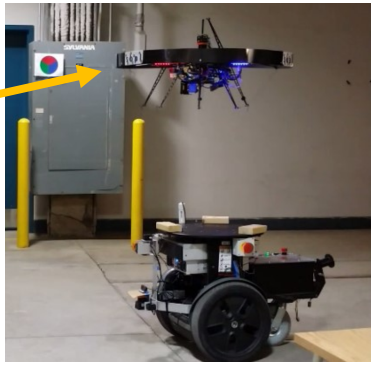
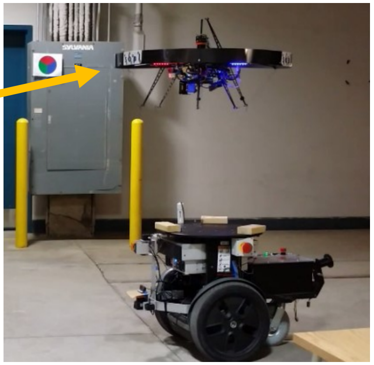
Consider Planning with Battery Constraint



- Suppose we are planning 2D *(x,y)* path for UAV
  - want a collision-free path to $s_{goal} = (x_{goal}, y_{goal})$
  - want to minimize some cost function associated with each transition (for example, minimize the risk of flying close to people)
  - subject to the trajectory being feasible given the UAV battery level L

What should be the variables defining each state (i.e., dimensions of the search)?

Consider Planning with Battery Constraint



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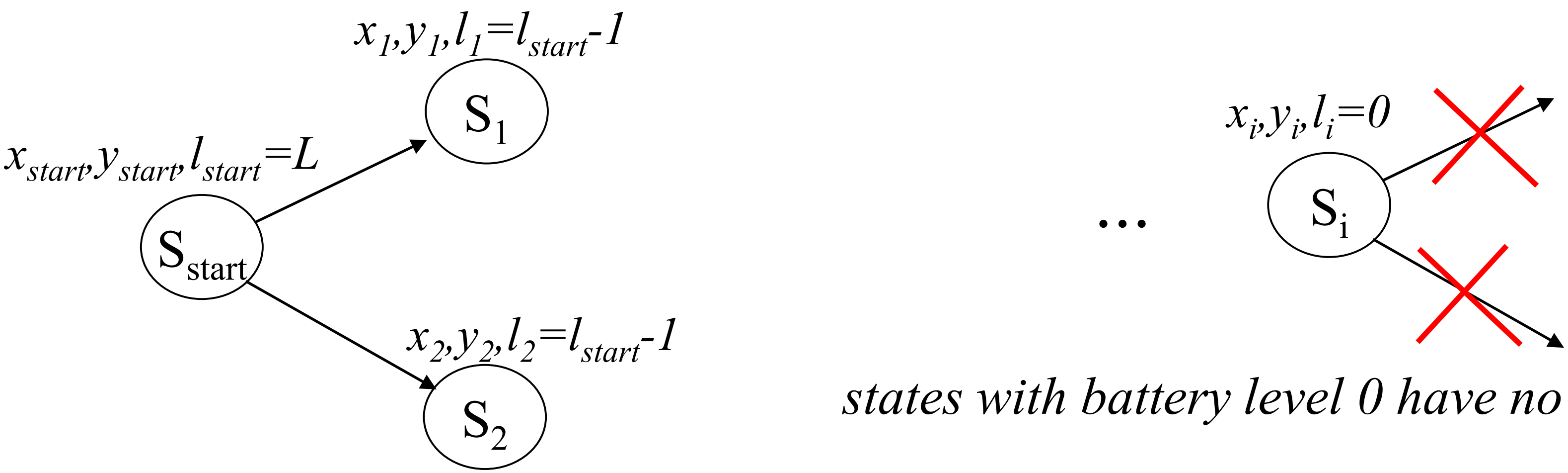
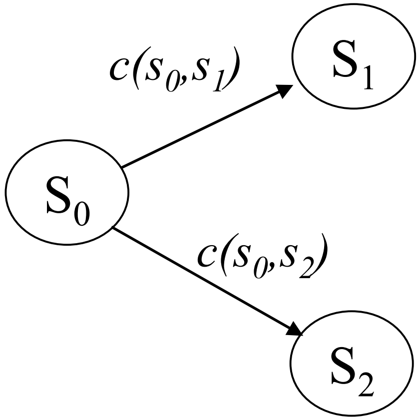
  - subject to the trajectory being feasible given the UAV battery level L
  - **Planning needs to be in (*x,y,l*), where *l* is the remaining battery level**



states with battery level 0 have no successors

Markov Property

- ***Cost and Set of Successors needs to depend <u>ONLY</u> on the current state (no dependence on the history of the path leading up to it!)***



for all states s: succ(s) = function of s
for all s' in succ(s): c(s,s')= function of s, s'

Markov Property

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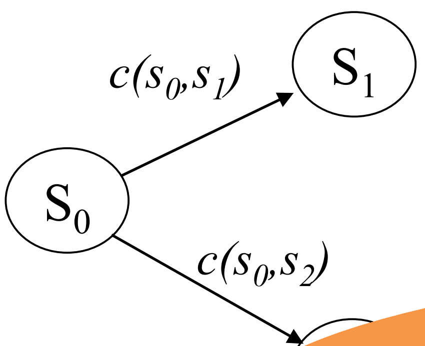


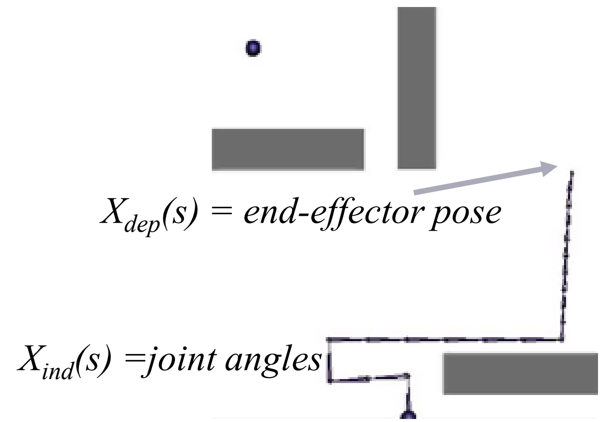
for all states s: succ(s) = function of s
for all s' in succ(s): c(s,s')= function of s, s'

Clearly true in an **explicit** (given) graph

Can be violated in **implicit** (dynamically generated) graphs, where *succ(s)* and *c(s,s')* are computed on-the-fly as a function of *s*,
when using dependent variables

Independent vs. Dependent Variables

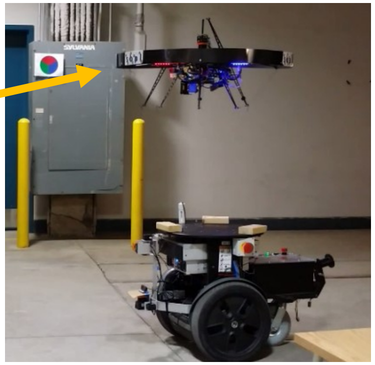
- *$X(s)$ – variables associated with s*
- *$X(s) = \{X_{ind}(s), X_{dep}(s)\}$*
- *$X_{ind}(s)$ – independent variables*
- *$X_{dep}(s)$ – dependent variables*



- ***Independent Variables*** are used to define state *s*
  - *two states s and s' are considered to be the same state if and only if $X_{ind}(s) = X_{ind}(s')$*

- ***Dependent Variables*** often used to help with computing cost or list of successor states
  - *if for all s, $X_{dep}(s) = f(X_{ind}(s))$ (that is, only depends on independent variables, then Markov Property holds true)*
  - **Often however, developers suggest to compute $X_{dep}(s)$ based on the path leading up to $X_{ind}(s)$**

Consider Planning with Battery Constraint



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  - subject to the trajectory being feasible given the UAV battery level L

  - **Consider $\mathbf{X_{ind}}$=(*x*,*y*), $\mathbf{X_{dep}}$=(*l*), where *l* is the remaining battery level**

Consider Planning with Battery Constraint

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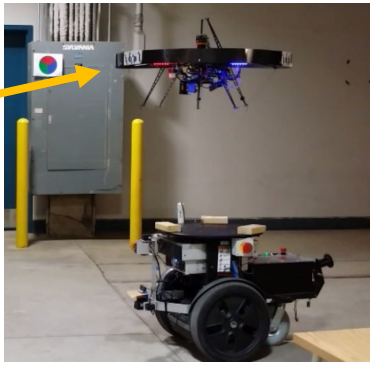

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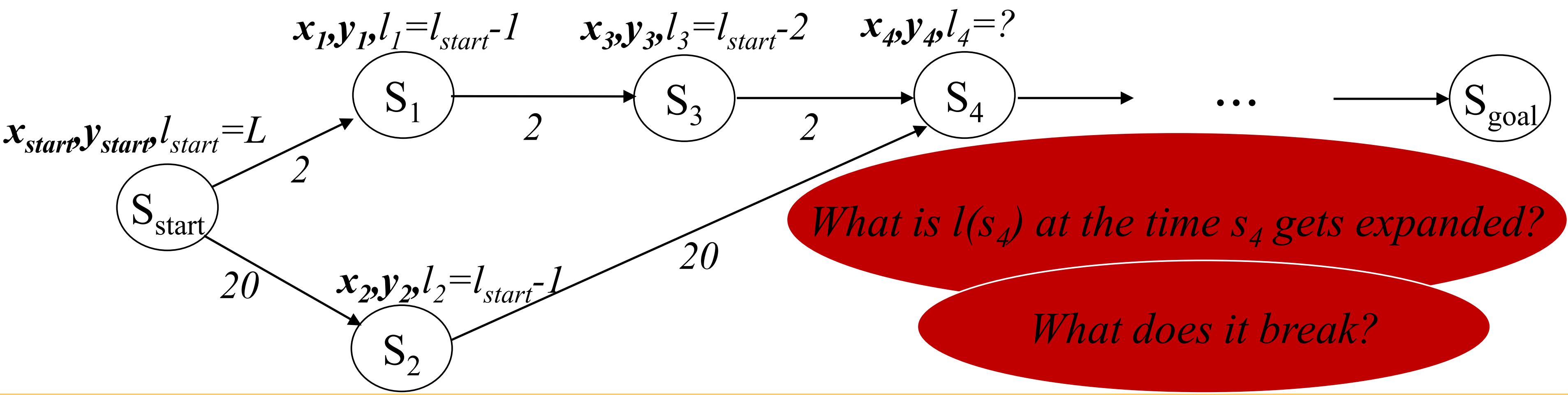
  - **Consider $X_{ind}=(x,y)$, $X_{dep}=(l)$, where l is the remaining battery level**



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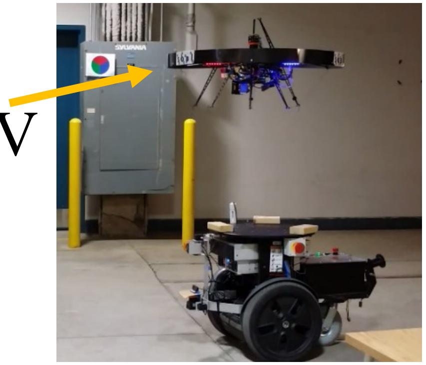

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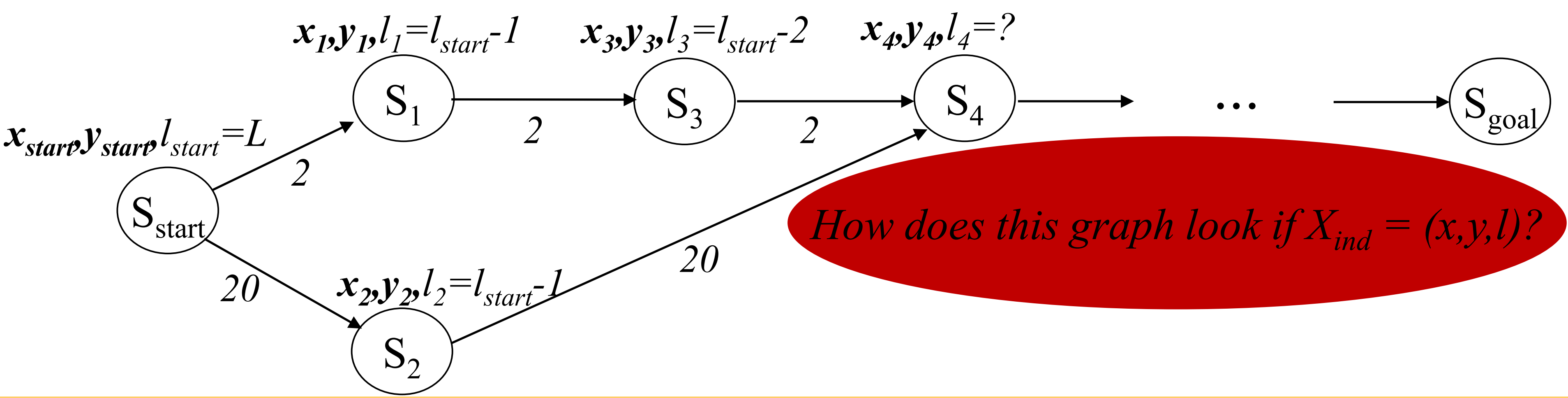


Consider Planning with Constraints on Rate of Turning

- Suppose we are planning 2D *(x,y)* path for a ground robot and constraining its heading to change by at most 45 degrees at each timestep based on the previous transition

  - **Consider $X_{ind}=(x,y)$, $X_{dep}=(\Theta)$, where Θ is robot's heading**

Example of incompleteness?

Consider Planning with Continuous (x,y,Θ)

- Suppose we are planning 3D *(x,y,Θ)* path for a ground robot but we don't have motion primitives (lattice) that move the robot exactly between the centers of 3D cells

  - **Consider $X_{ind}=(x_{disc},y_{disc},\Theta_{disc})$, $X_{dep}=(x_{cont},y_{cont},\Theta_{cont})$, where X_{dep} keeps track of the continuous robot pose along its path** [Barraquand, J. & Latombe, '93]

Example of "incompleteness"?

Consider Planning in Dynamic Environments

- Suppose we are planning a path among moving obstacles

  - want a collision-free path to s_{goal}
  - want to minimize some cost function associated with each transition

  - **Consider X_{ind}=(*robot pose*), X_{dep}=(*t*), where *t* is time**

Example of incompleteness?

Consider Planning in Dynamic Environments

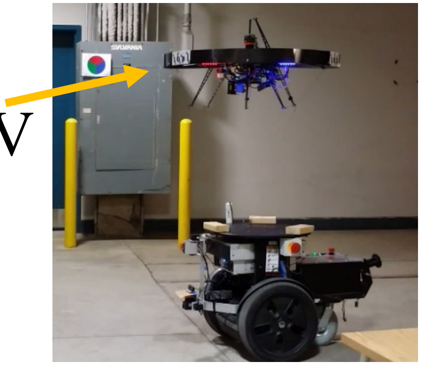
- Suppose we are planning a path among moving obstacles

  - want a collision-free path to s_{goal}
  - assume cost function is time

  - **Consider X_{ind}=(*robot pose*), X_{dep}=(*t*), where *t* is time**

Is it incomplete?

Back to Planning with Battery Constraint



- Suppose we are planning 2D *(x,y)* path for UAV
  - want a collision-free path to $s_{goal} = (x_{goal}, y_{goal})$
  - assume cost function is battery consumption
  - subject to the trajectory being feasible given the UAV battery level L
  - **Consider $X_{ind}=(x,y)$, $X_{dep}=(l)$, where l is the remaining battery level**

Is it incomplete?

Back to Planning with Battery Constraint

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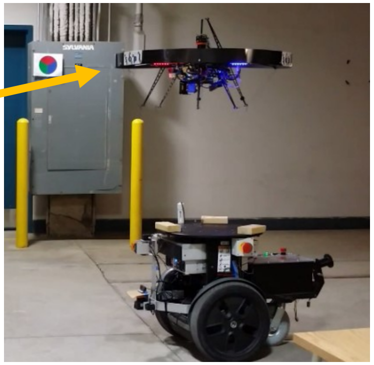

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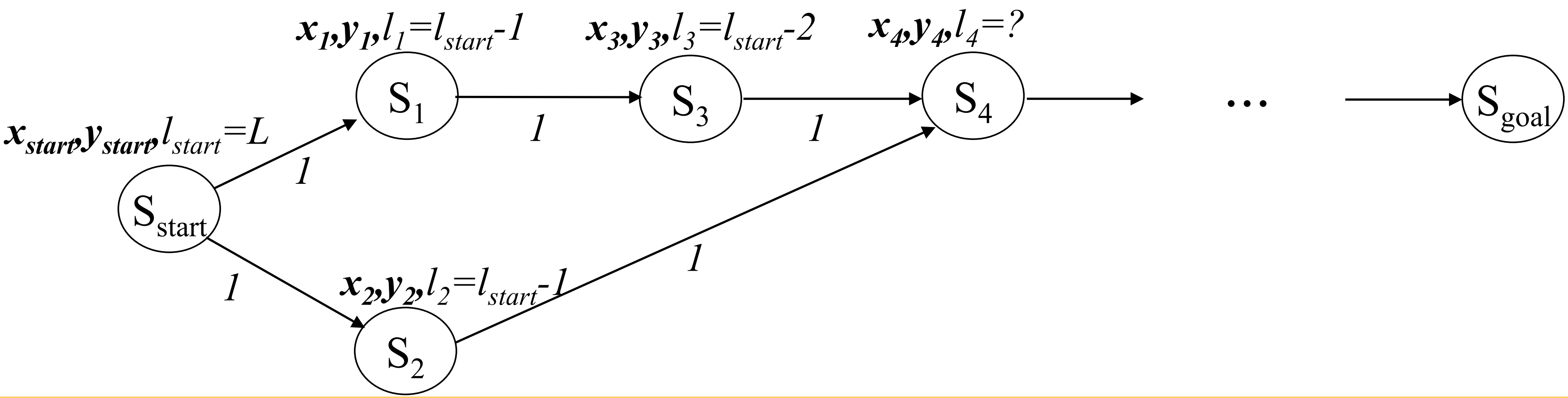

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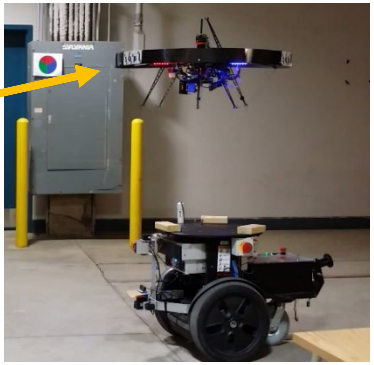

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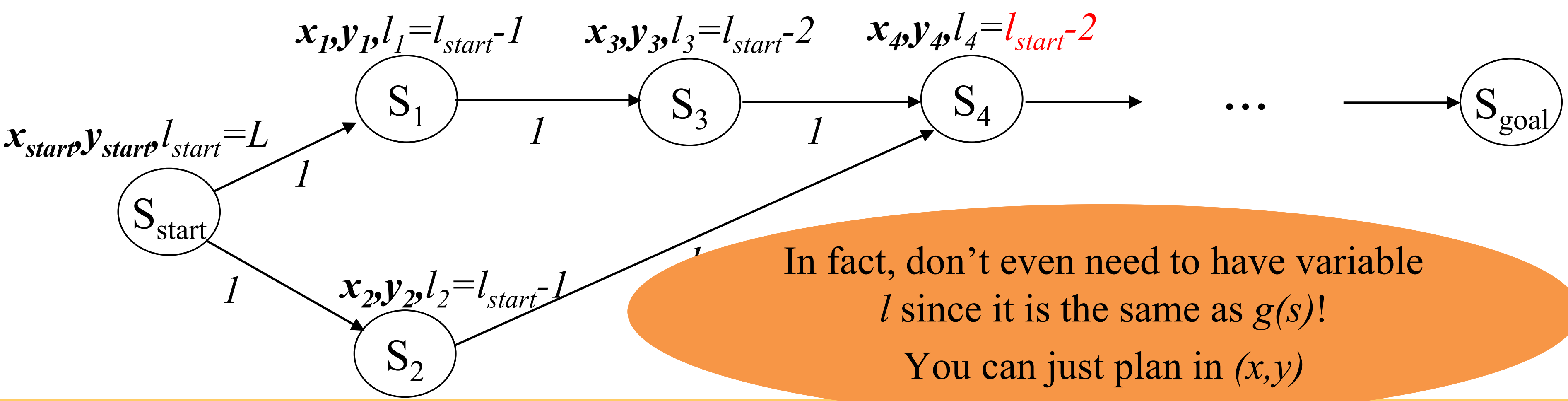
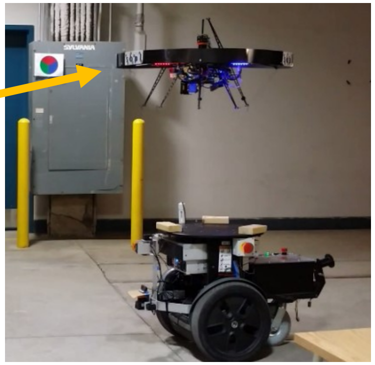


In fact, don't even need to have variable *l* since it is the same as *g(s)*!
You can just plan in *(x,y)*

Back to Planning with Battery Constraint



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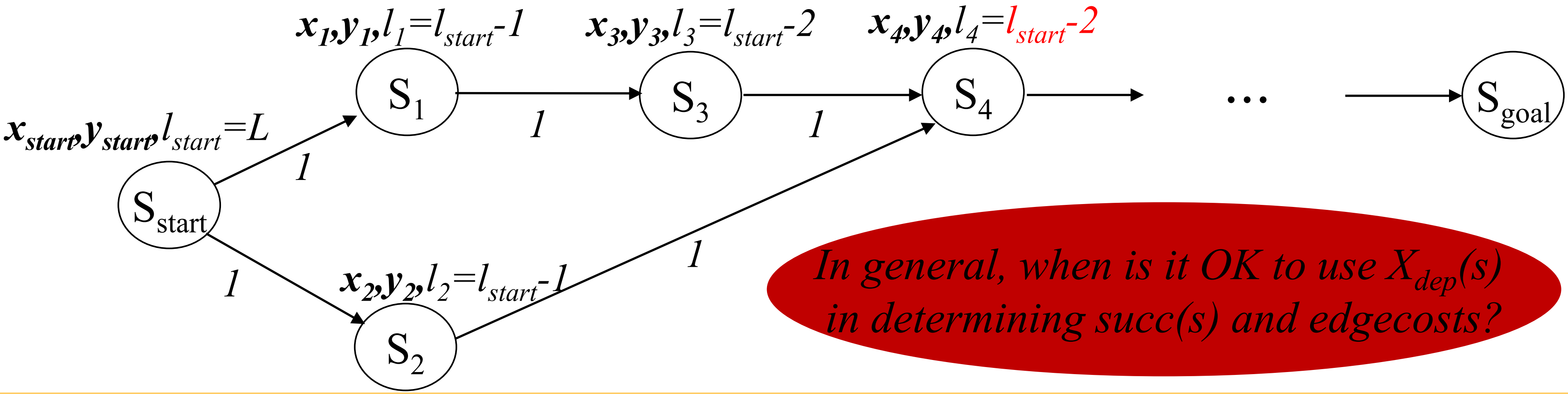
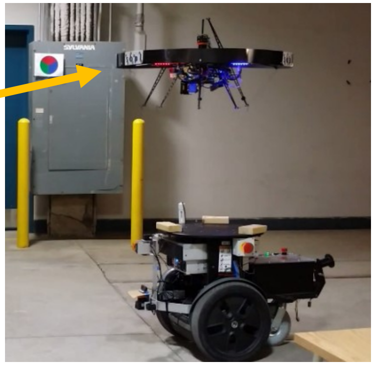


In general, when is it OK to use $X_{dep}(s)$ in determining succ(s) and edgecosts?

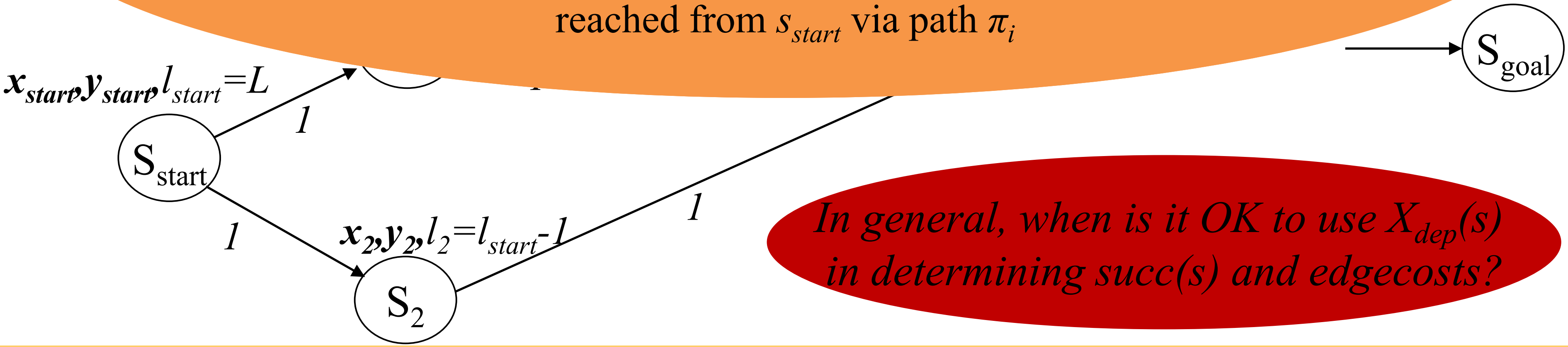
Back to Planning with Battery Constraint

- Suppose we are planning 2D *(x,y)* path for UAV



  - want a collision-free path to $s_{goal} = (x_{goal}, y_{goal})$
  - <u>assume cost function is battery consumption</u>
  - subject to the tra… the UAV battery level L

Whenever you can guarantee that for any state s:
if we have two paths $\pi_1(s_{start}, s)$ and $\pi_2(s_{start}, s)$ s.t. $c(\pi_1) \geq c(\pi_2)$,
then it implies that $c_1(s,s') \geq c_2(s,s')$,
where $c_i(s,s')$ – cost of a least-cost path from s to s' <u>after</u> s is reached from s_{start} via path π_i



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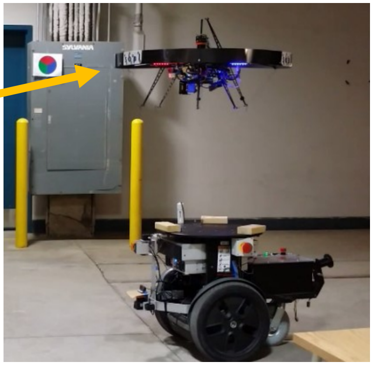

  - want a c
  - assume c
  - subject to the tra... the UAV battery level *L*

Assuming we are running optimal search (such as A).*

Whenever you can guarantee that for any state *s*:
if we have two paths $\pi_1(s_{start},s)$ and $\pi_2(s_{start},s)$ *s.t.* $c(\pi_1) \geq c(\pi_2)$,
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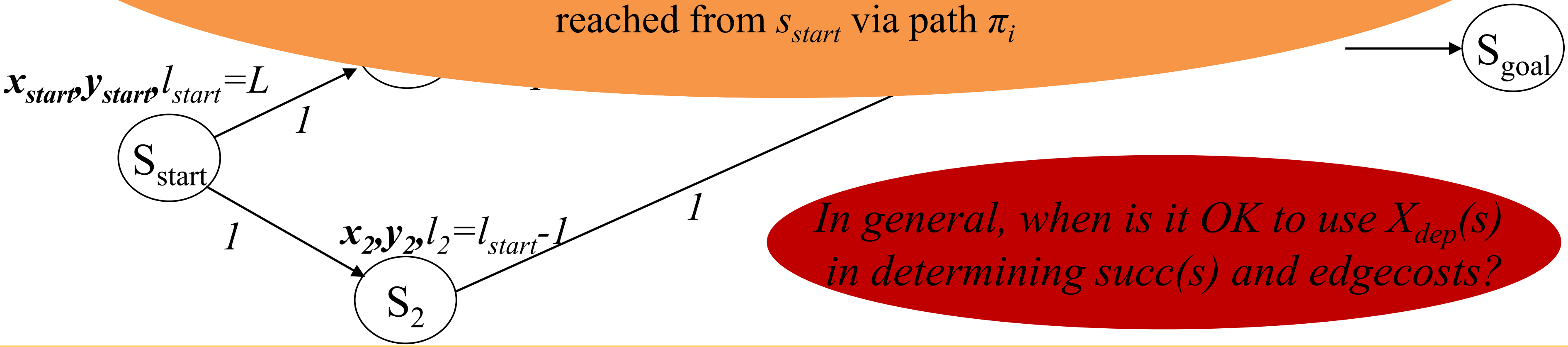

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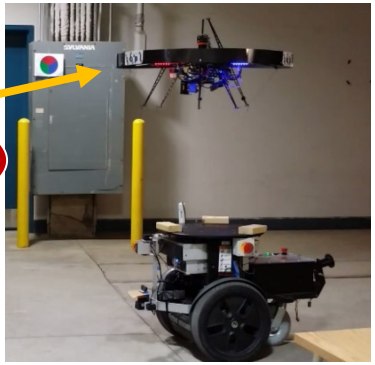


- Suppos

What happens if we are running suboptimal search such as weighted A?*

- want a
- assume
- subject to the tra... the UAV battery level L

Assuming we are running optimal search (such as A).*

Whenever you can guarantee that for any state s:

if we have two paths $\pi_1(s_{start},s)$ and $\pi_2(s_{start},s)$ s.t. $c(\pi_1) \geq c(\pi_2)$, then it implies that $c_1(s,s') \geq c_2(s,s')$,

where $c_i(s,s')$ – cost of a least-cost path from s to s' <u>after</u> s is reached from s_{start} via path π_i

$x_{start}, y_{start}, l_{start}=L$

S_{start}

1

1

$x_2, y_2, l_2=l_{start}-1$

S_2

1

S_{goal}

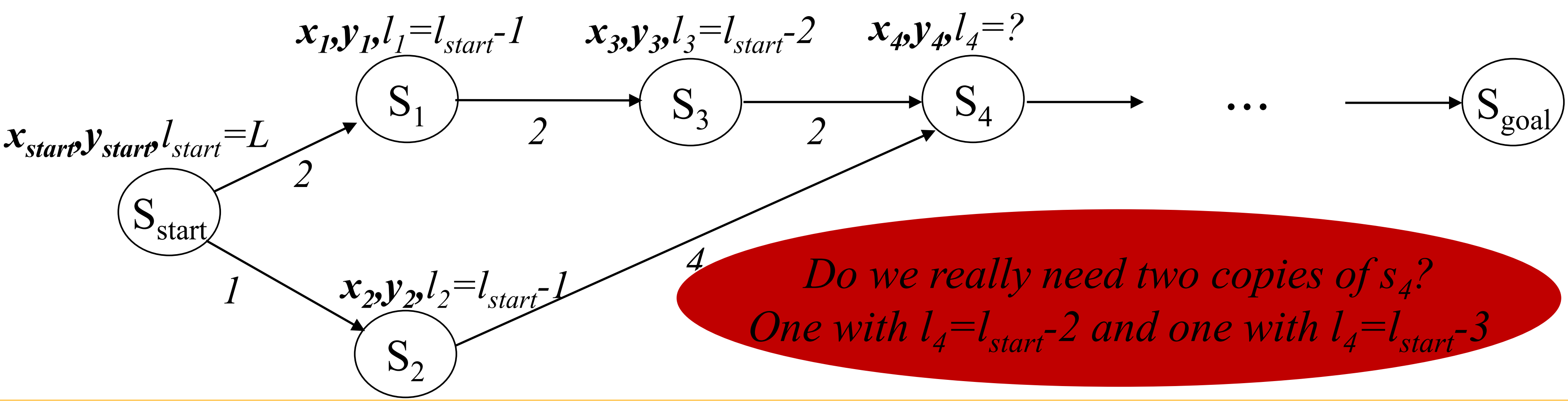
In general, when is it OK to use $X_{dep}(s)$ in determining succ(s) and edgecosts?

Dominance Relationship

- Suppose we are planning 2D *(x,y)* path for UAV
  - want a collision-free path to $s_{goal} = (x_{goal}, y_{goal})$
  - want to minimize some cost function associated with each transition (for example, minimiz
  - subje

What are the general conditions for pruning "dominated" states?

  - **Consider** $\mathbf{X_{ind}=(x,y,l)}$



Do we really need two copies of s_4?
One with $l_4=l_{start}-2$ and one with $l_4=l_{start}-3$

Dominance Relationship

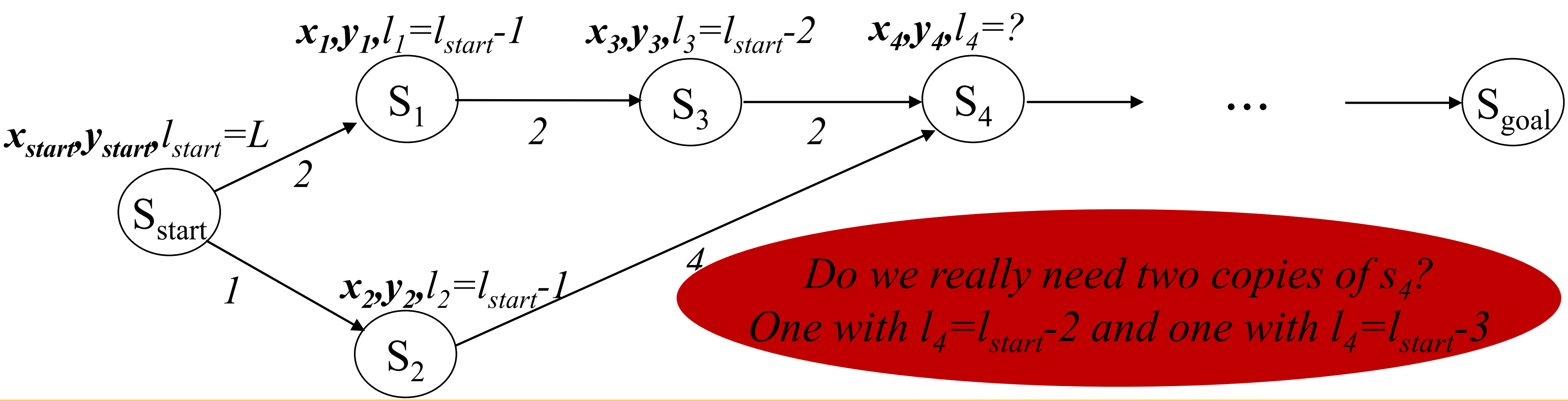
if *(g(s) ≤ g(s'))* and *s* **dominates** *s'*), then *s'* can be pruned by search

s dominates s' implies s cannot be part of a solution that is better than the solution from s'

- want to minimi… …with each transition (for example, minimize th…
- subje…

What are the general conditions for pruning "dominated" states?

- **Consider $X_{ind}=(x,y,l)$**



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A* Search with Dominance Check

Main function

$g(s_{start}) = 0$; all other g-values are infinite; $OPEN = \{s_{start}\}$;

ComputePath();

publish solution;

ComputePath function

while(s_{goal} is not expanded and $OPEN \neq 0$)

  remove s with the smallest $[f(s) = g(s)+h(s)]$ from *OPEN*;

  insert s into *CLOSED*;

  for every successor s' of s such that s' not in *CLOSED*

    if $g(s') > g(s) + c(s,s')$

      $g(s') = g(s) + c(s,s')$;

      if there exists state s'' such that $(g(s'') \leq g(s')$ AND s'' dominates $s')$

        continue; //skip inserting state s' into *OPEN*, i.e., prune

      insert s' into *OPEN*;

What You Should Know…

- Dependent vs. Independent variables.
- Definition of Markov Property
- The definition and the use of Dominance relationship