10-301/601: Introduction to Machine Learning Lecture 10 – Regularization

Henry Chai & Matt Gormley

10/2/23

Front Matter

- Announcements:
  - HW4 released 9/29, due 10/9 at 11:59 PM

Recall: Logistic Regression

- Model:

$$p(y|\boldsymbol{x},\boldsymbol{\theta}) = \begin{cases} \sigma(\boldsymbol{\theta}^T\boldsymbol{x}) & \text{if } y = 1 \\ 1 - \sigma(\boldsymbol{\theta}^T\boldsymbol{x}) & \text{if } y = 0 \end{cases}$$

where $\sigma(z) = 1/1 + \exp(-z)$

- Derivatives

$$\frac{\partial J^{(i)}}{\partial \theta_m} = \frac{\partial}{\partial \theta_m}\left(-\log p\left(y^{(i)}\middle|\boldsymbol{x}^{(i)},\boldsymbol{\theta}\right)\right)$$

$$\vdots$$

$$= -\left(y^{(i)} - \sigma\left(\boldsymbol{\theta}^T\boldsymbol{x}^{(i)}\right)\right)\boldsymbol{x}_m^{(i)}$$

- Optimization: use GD or SGD; logistic regression does not permit a closed form solution

- Objective: minimize the negative conditional log-likelihood

$$J(\theta) = \frac{1}{N}\sum_{i=1}^{N} -\log p\left(y^{(i)}\middle|\boldsymbol{x}^{(i)},\boldsymbol{\theta}\right)$$

- Gradients

$$\nabla J^{(i)}(\boldsymbol{\theta}) = -\left(y^{(i)} - \sigma\left(\boldsymbol{\theta}^T\boldsymbol{x}^{(i)}\right)\right)\boldsymbol{x}^{(i)}$$

$$\nabla J(\boldsymbol{\theta}) = \frac{1}{N}\sum_{i=1}^{N}\nabla J^{(i)}$$

- Predictions

$$\hat{y} = \underset{y \in \{0,1\}}{\operatorname{argmax}}\ p\left(y\middle|\boldsymbol{x}',\hat{\boldsymbol{\theta}}\right)$$

$$\vdots$$

$$= \text{“sign”}\left(\hat{\boldsymbol{\theta}}^T\boldsymbol{x}'\right)$$

Logistic Regression Decision Boundary

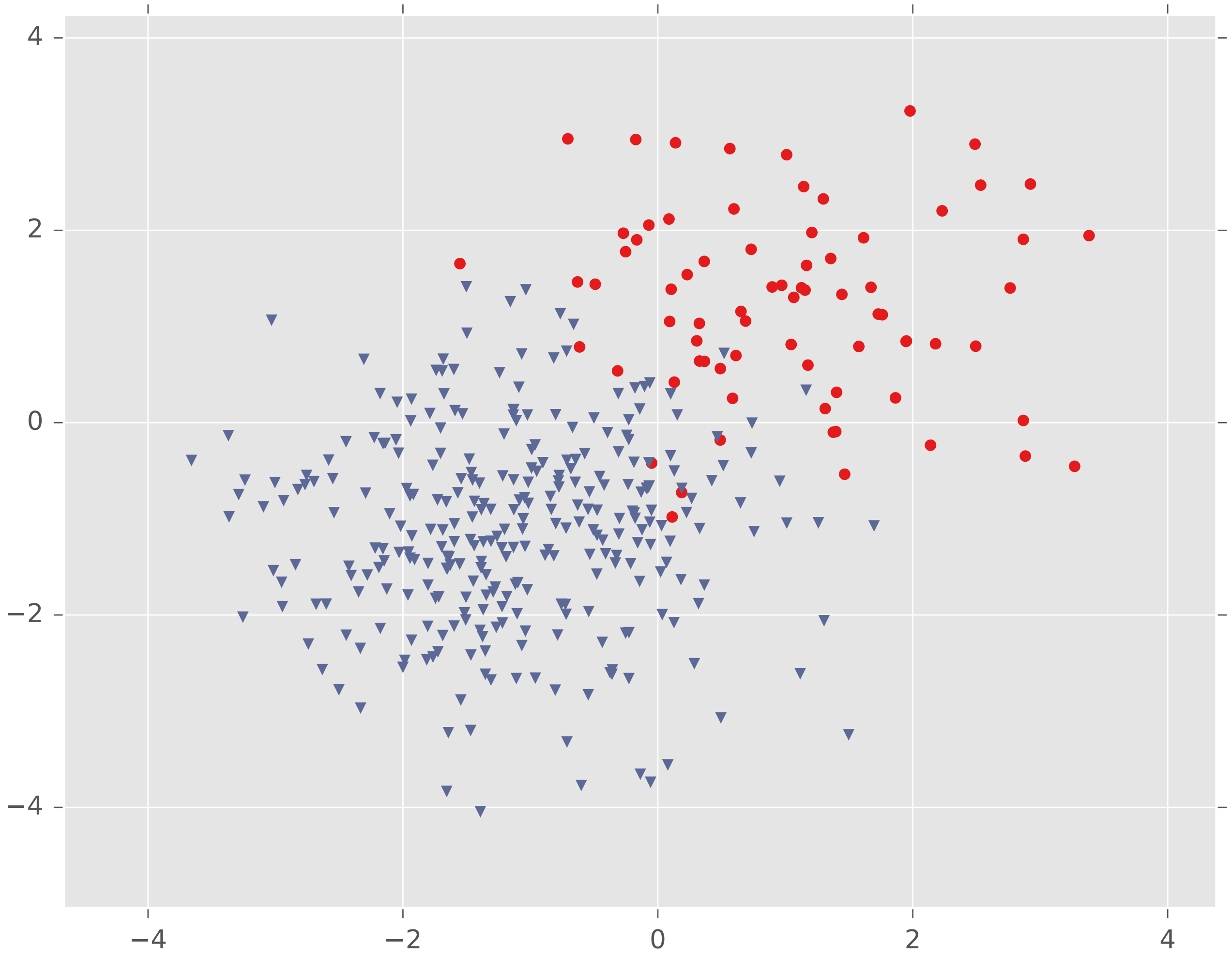


Figure courtesy of Matt Gormley

Logistic Regression Decision Boundary

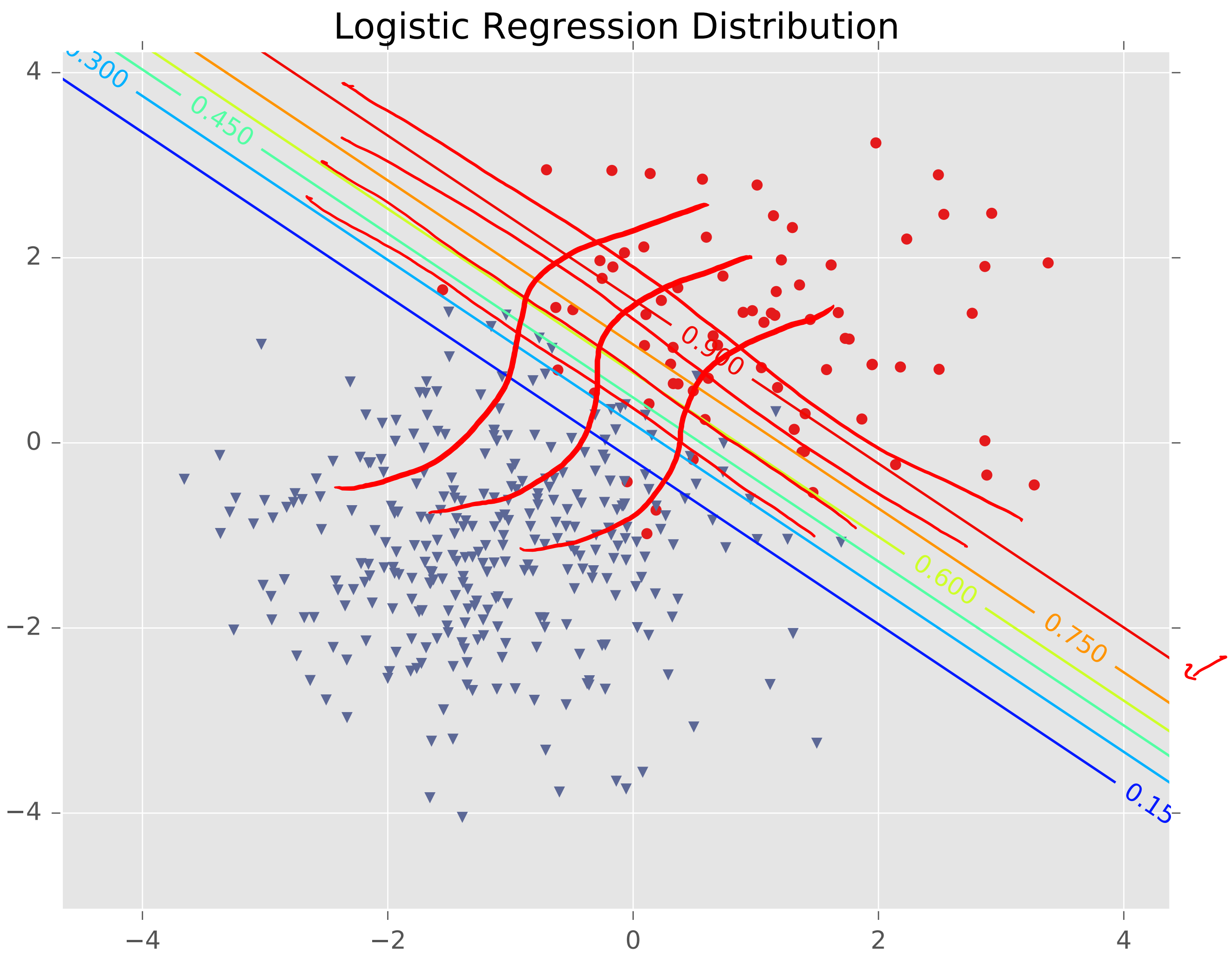


Figure courtesy of Matt Gormley

Logistic Regression Decision Boundary

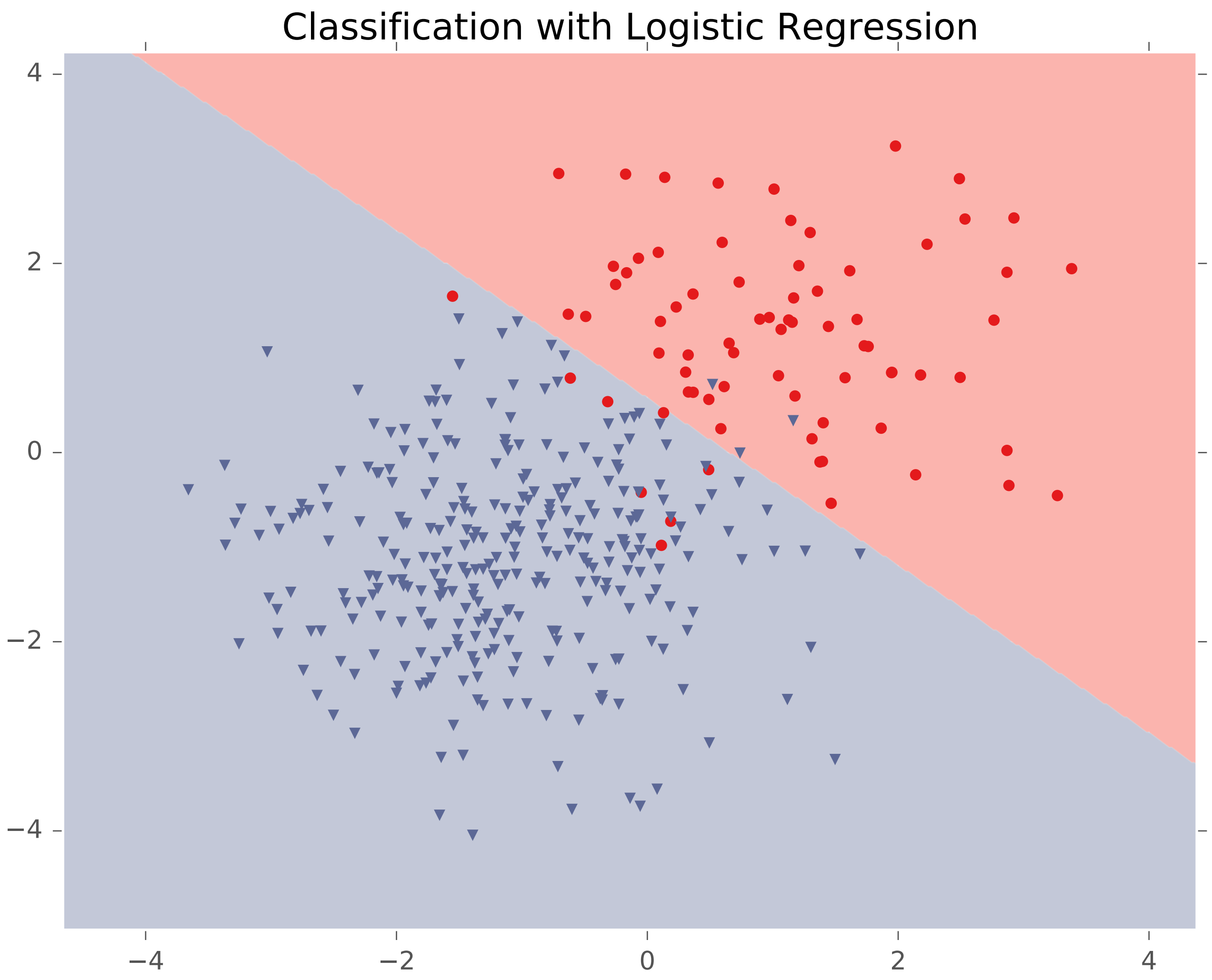


Figure courtesy of Matt Gormley

But is this the best that we could do, even if we knew p^*?

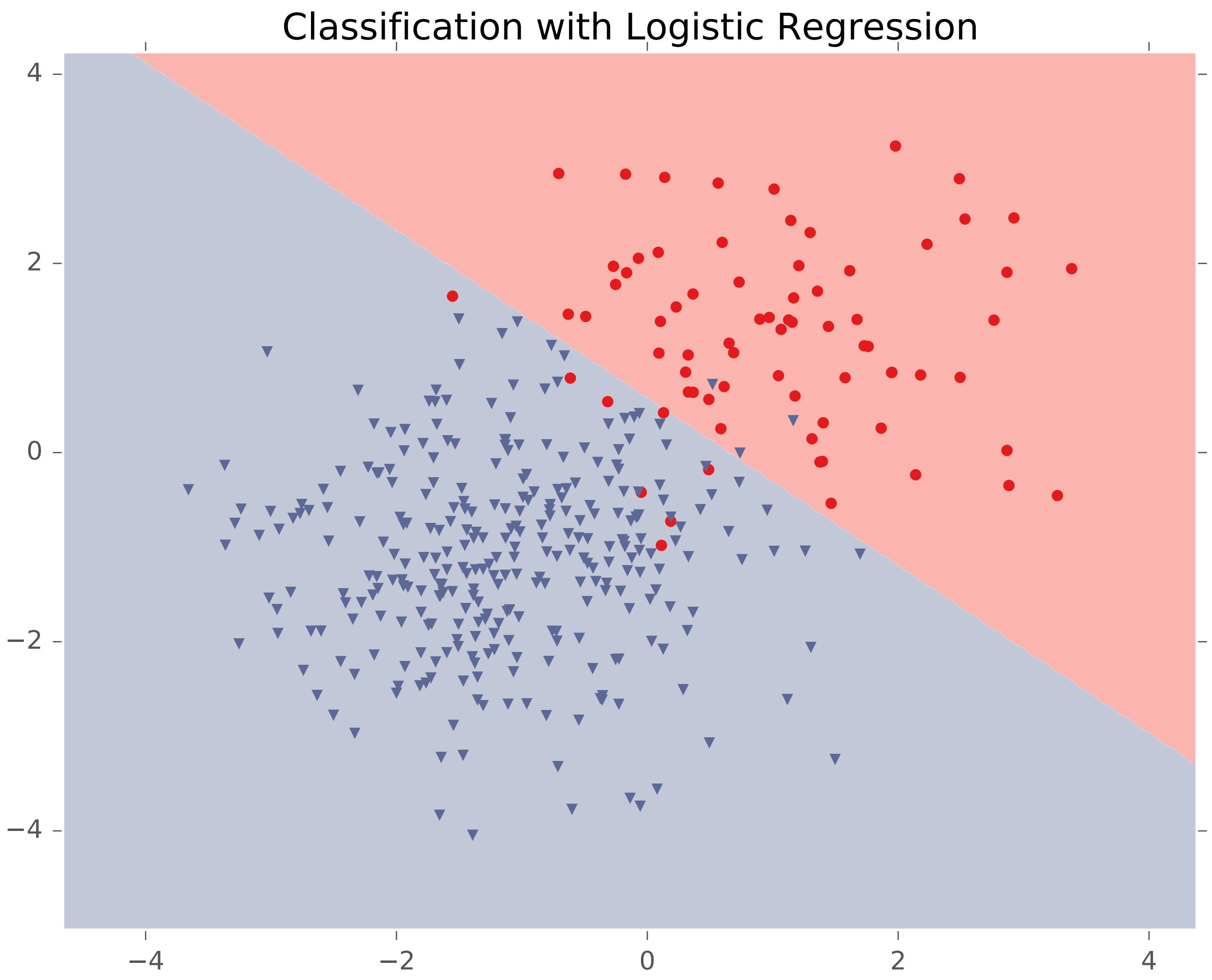


Figure courtesy of Matt Gormley

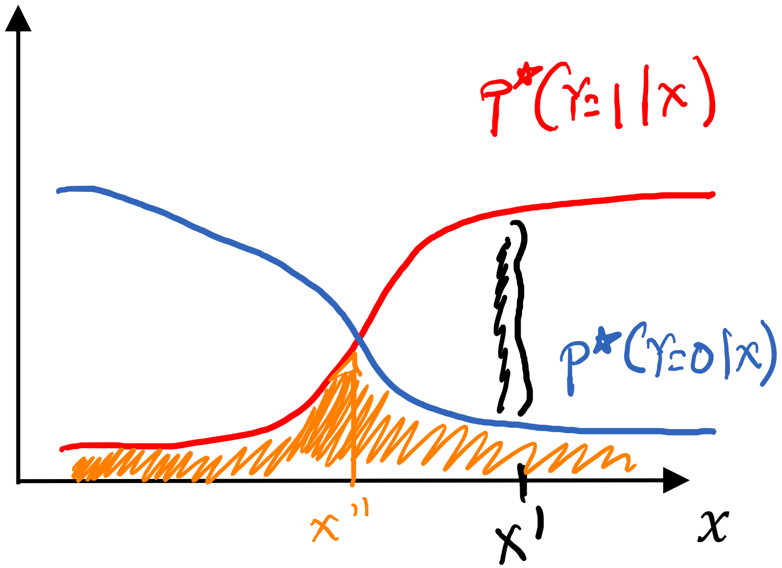
Bayes Optimal Classifier

- Suppose you knew $p^*(Y = 1|\boldsymbol{x})$ for all $\boldsymbol{x}$ and wanted to minimize the 0-1 loss

$$\ell(\hat{y}, y) = \mathbb{1}(\hat{y} \neq y)$$

- Then the optimal classifier in this setting, called the *Bayes optimal classifier*, is

$$\hat{y} = \begin{cases} 1 \text{ if } p^*(Y = 1|\boldsymbol{x}) \geq 0.5 \\ 0 \text{ otherwise} \end{cases}$$



- The *reducible error* of a classifier is the expected loss that could be eliminated if we knew p^*
- The *irreducible error* of a classifier is the expected loss even if we knew p^*

Stochastic Gradient Descent (SGD) for Logistic Regression

- Input: training dataset $\mathcal{D} = \{(\boldsymbol{x}^{(i)}, y^{(i)})\}_{i=1}^{N}$ and step size γ

1. Initialize $\boldsymbol{\theta}^{(0)}$ to all zeros and set $t = 0$
2. While TERMINATION CRITERION is not satisfied
   a. For $i \in \text{shuffle}(\{1, \ldots, N\})$
      i. Compute the pointwise gradient:

$$\nabla J^{(i)}(\boldsymbol{\theta}^{(t)}) = -\left(y^{(i)} - \sigma(\boldsymbol{\theta}^{T}\boldsymbol{x}^{(i)})\right)\boldsymbol{x}^{(i)}$$

      ii. Update $\boldsymbol{\theta}$: $\boldsymbol{\theta}^{(t+1)} \leftarrow \boldsymbol{\theta}^{(t)} - \gamma \nabla J^{(i)}(\boldsymbol{\theta}^{(t)})$
      iii. Increment t: $t \leftarrow t + 1$

- Output: $\boldsymbol{\theta}^{(t)}$

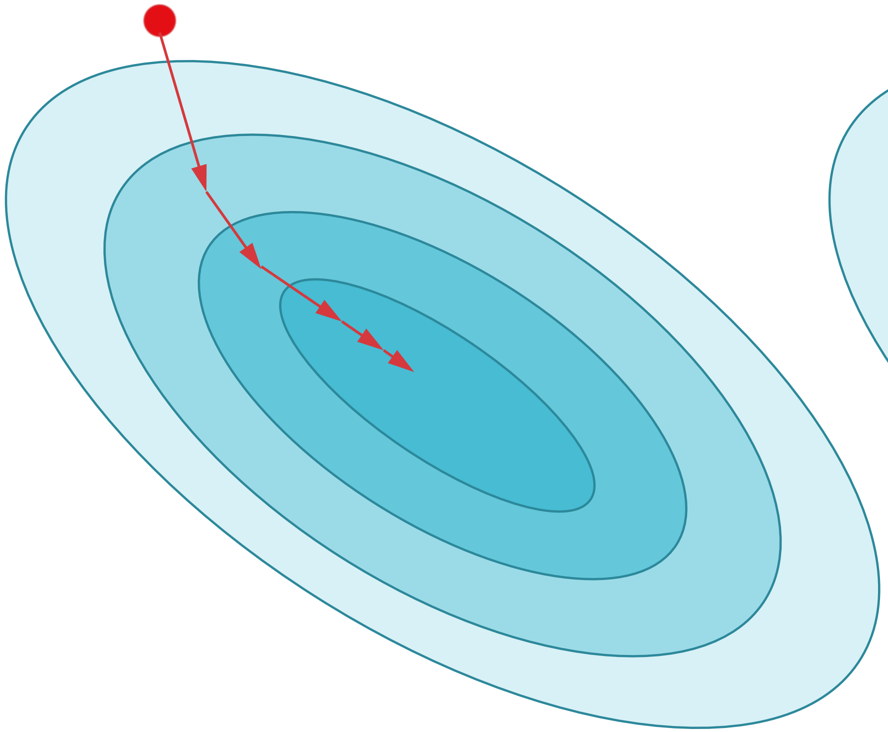
Stochastic Gradient Descent (SGD) for Logistic Regression

- Input: training dataset $\mathcal{D} = \{(\boldsymbol{x}^{(i)}, y^{(i)})\}_{i=1}^{N}$ and step size γ

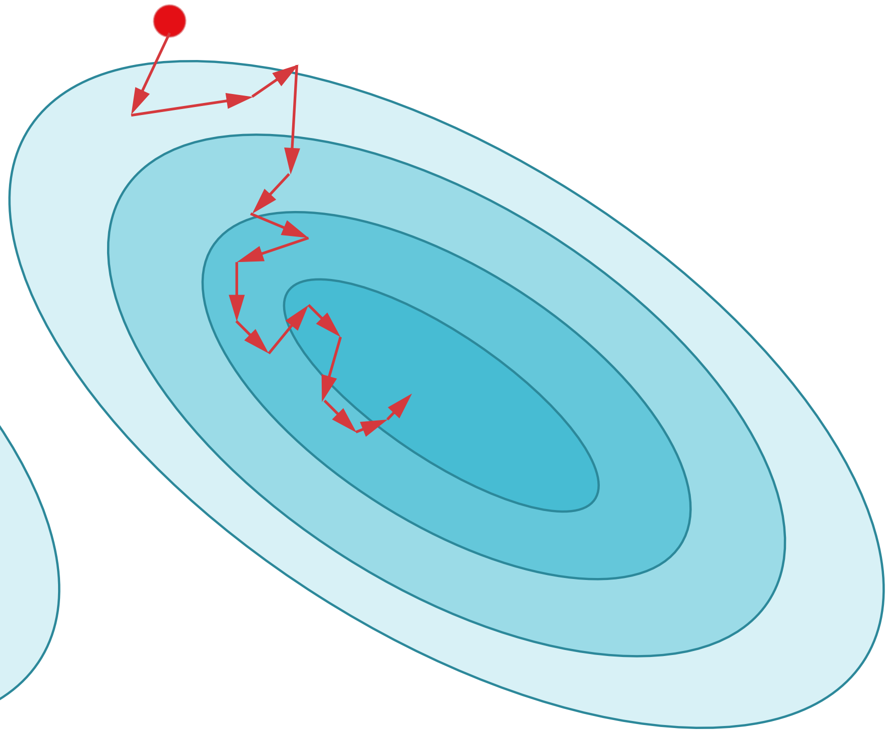
1. Initialize $\boldsymbol{\theta}^{(0)}$ to all zeros and set $t = 0$
2. While TERMINATION CRITERION is not satisfied
   a. For $i \in \text{shuffle}(\{1, \ldots, N\})$
      i. Compute the pointwise gradient:
      $$\nabla J^{(i)}(\boldsymbol{\theta}^{(t)}) = \left(P(Y = 1 | \boldsymbol{x}^{(i)}, \boldsymbol{\theta}^{(t)}) - y^{(i)}\right)\boldsymbol{x}^{(i)}$$
      ii. Update $\boldsymbol{\theta}$: $\boldsymbol{\theta}^{(t+1)} \leftarrow \boldsymbol{\theta}^{(t)} - \gamma \nabla J^{(i)}(\boldsymbol{\theta}^{(t)})$
      iii. Increment t: $t \leftarrow t + 1$

- Output: $\boldsymbol{\theta}^{(t)}$

Stochastic Gradient Descent vs. Gradient Descent

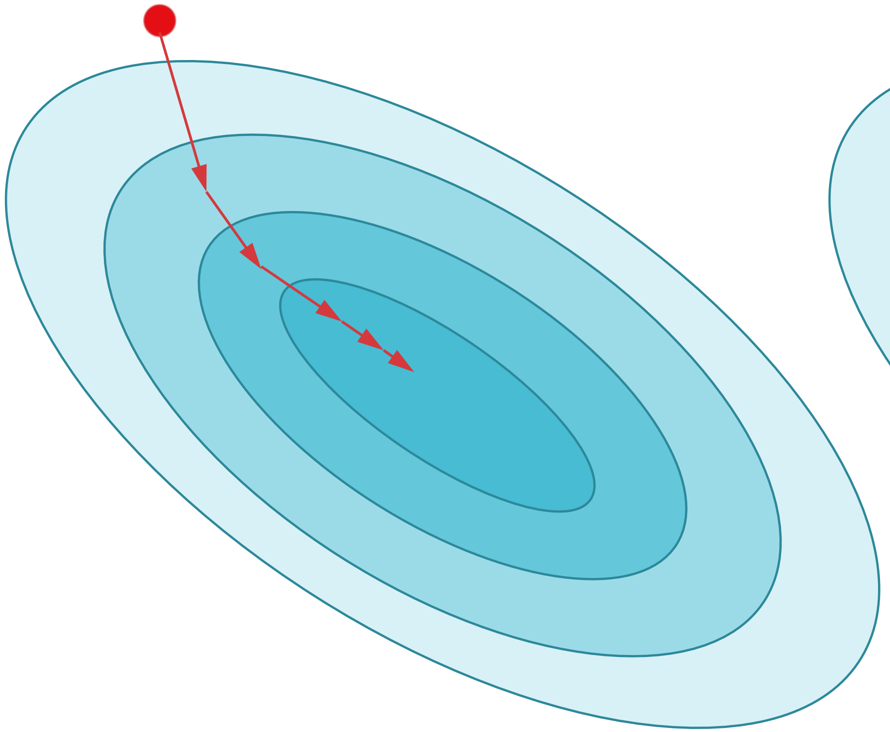


Gradient Descent

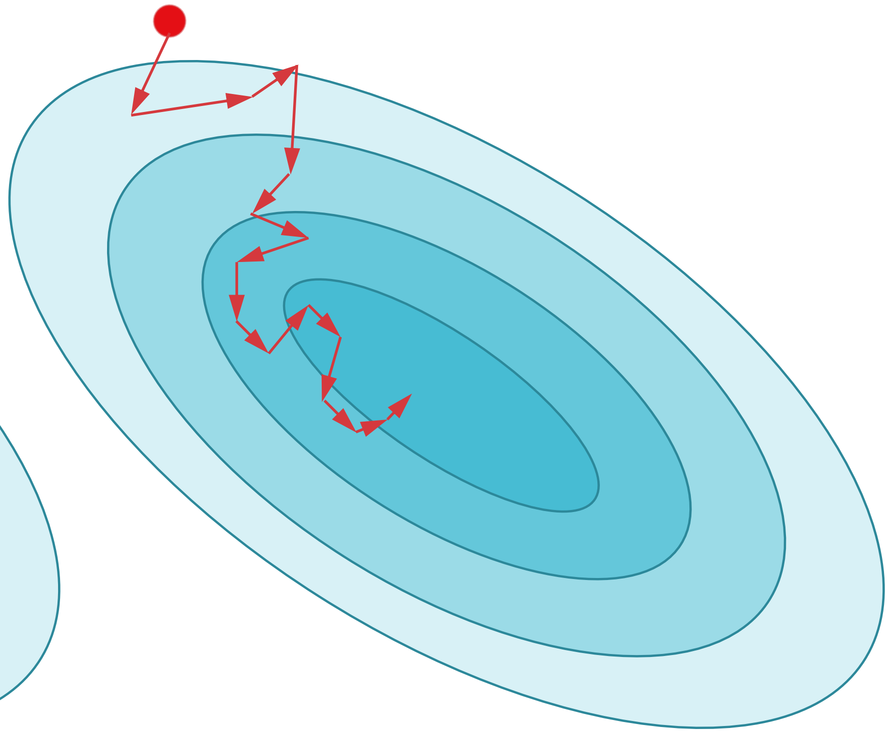


Stochastic Gradient Descent

Can we find some middle ground here?



Gradient Descent



Stochastic Gradient Descent

Mini-batch Stochastic Gradient Descent for Neural Networks

- Input: training dataset $\mathcal{D} = \{(\boldsymbol{x}^{(i)}, y^{(i)})\}_{i=1}^{N}$, step size γ, and batch size B

1. Initialize $\boldsymbol{\theta}^{(0)}$ to all zeros and set $t = 0$
2. While TERMINATION CRITERION is not satisfied
   a. Randomly sample B data points from $\mathcal{D}$, $\{(\boldsymbol{x}^{(b)}, y^{(b)})\}_{b=1}^{B}$
   b. Compute the gradient w.r.t. the sampled *batch*,

$$\nabla J^{(B)}(\boldsymbol{\theta}^{(t)}) = \frac{1}{B}\sum_{b=1}^{B}\left(P\left(Y = 1 | \boldsymbol{x}^{(b)}, \boldsymbol{\theta}^{(t)}\right) - y^{(b)}\right)\boldsymbol{x}^{(b)}$$

   c. Update $\boldsymbol{\theta}$: $\boldsymbol{\theta}^{(t+1)} \leftarrow \boldsymbol{\theta}^{(t)} - \gamma \nabla J^{(B)}(\boldsymbol{\theta}^{(t)})$
   d. Increment t: $t \leftarrow t + 1$

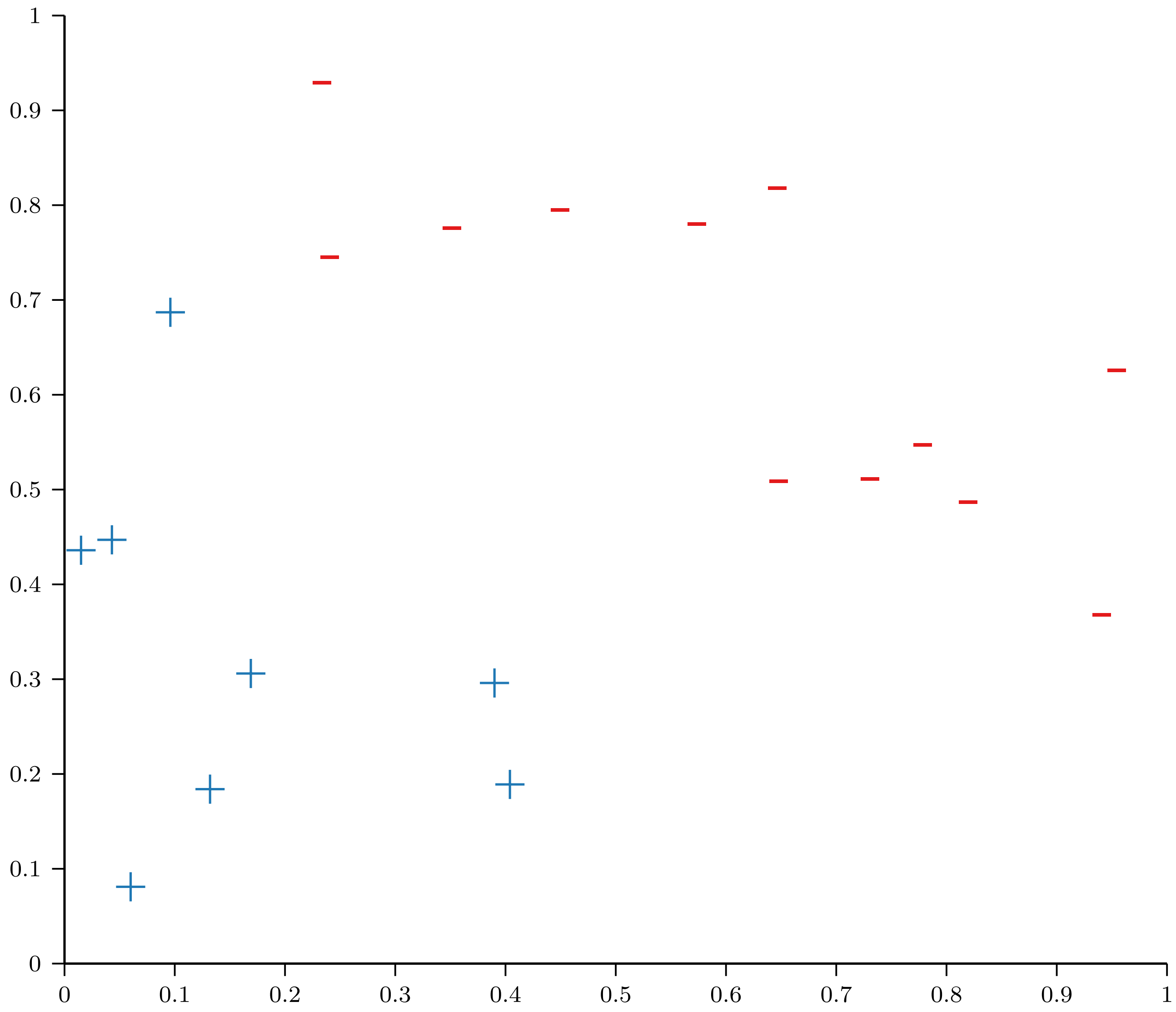
- Output: $\boldsymbol{\theta}^{(t)}$

Logistic Regression Learning Objectives

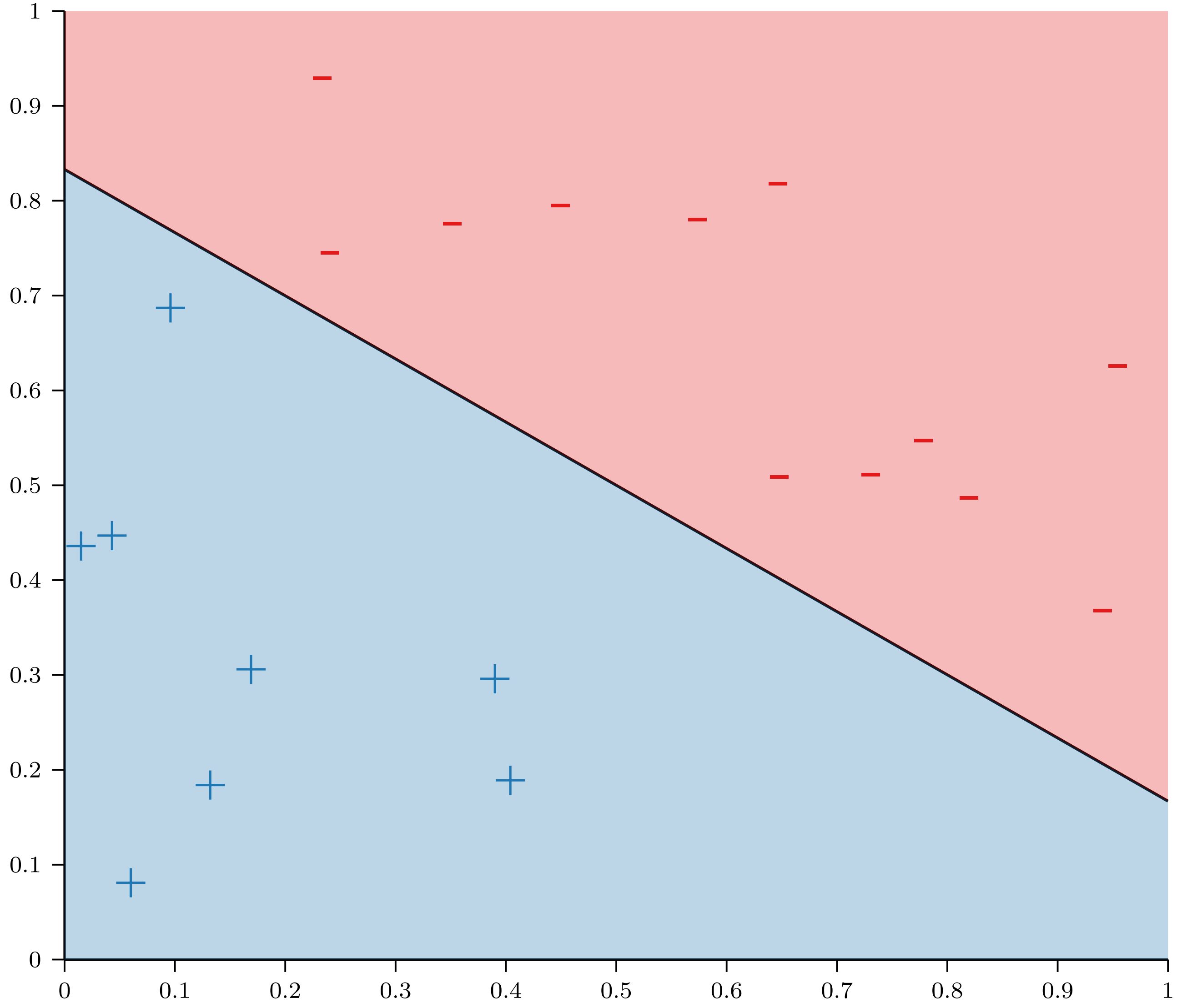
You should be able to…

- Apply the principle of maximum likelihood estimation (MLE) to learn the parameters of a probabilistic model
- Given a discriminative probabilistic model, derive the conditional log-likelihood, its gradient, and the corresponding Bayes Classifier
- Explain the practical reasons why we work with the log of the likelihood
- Implement logistic regression for binary classification
- Prove that the decision boundary of binary logistic regression is linear

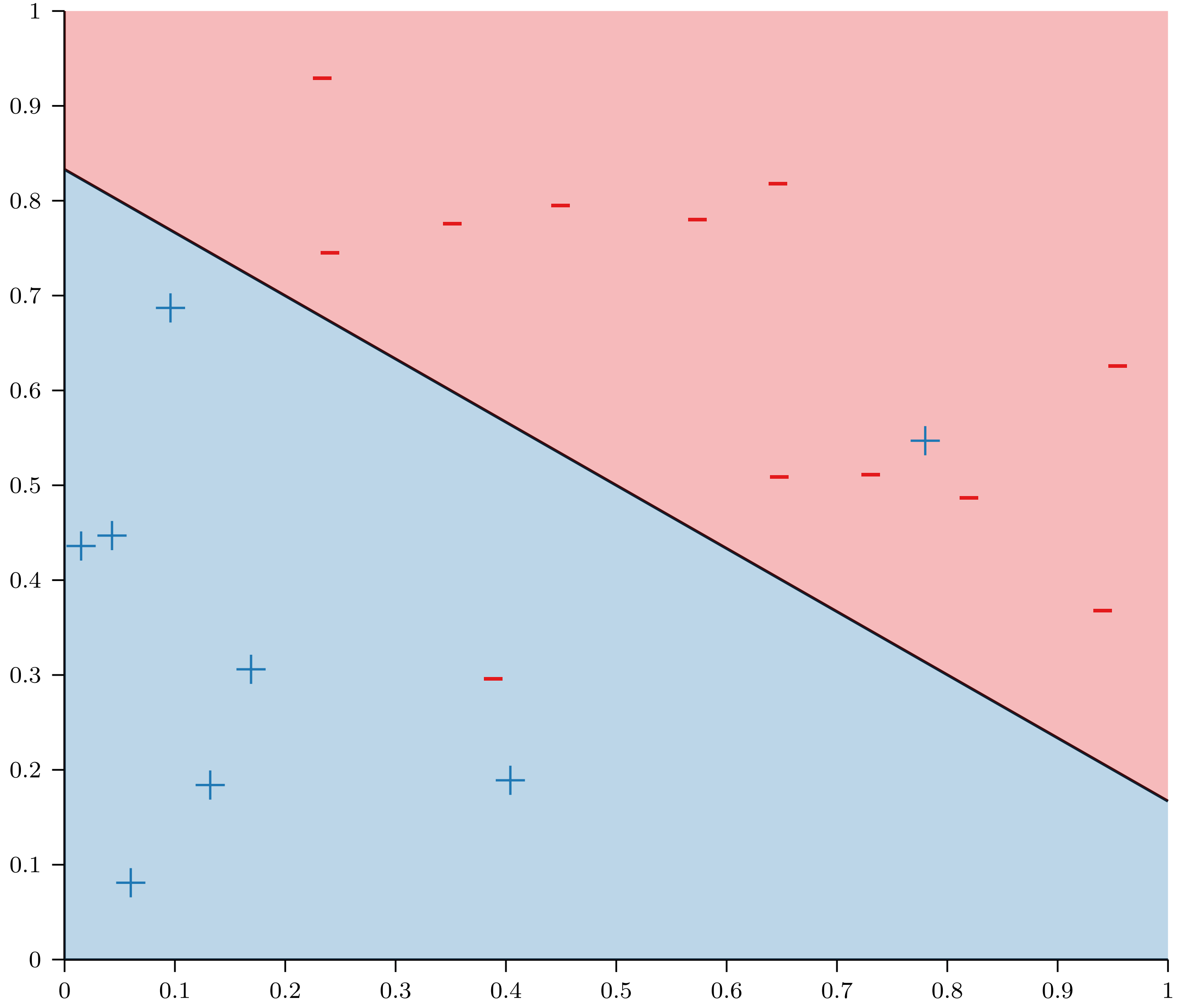
Linear Models



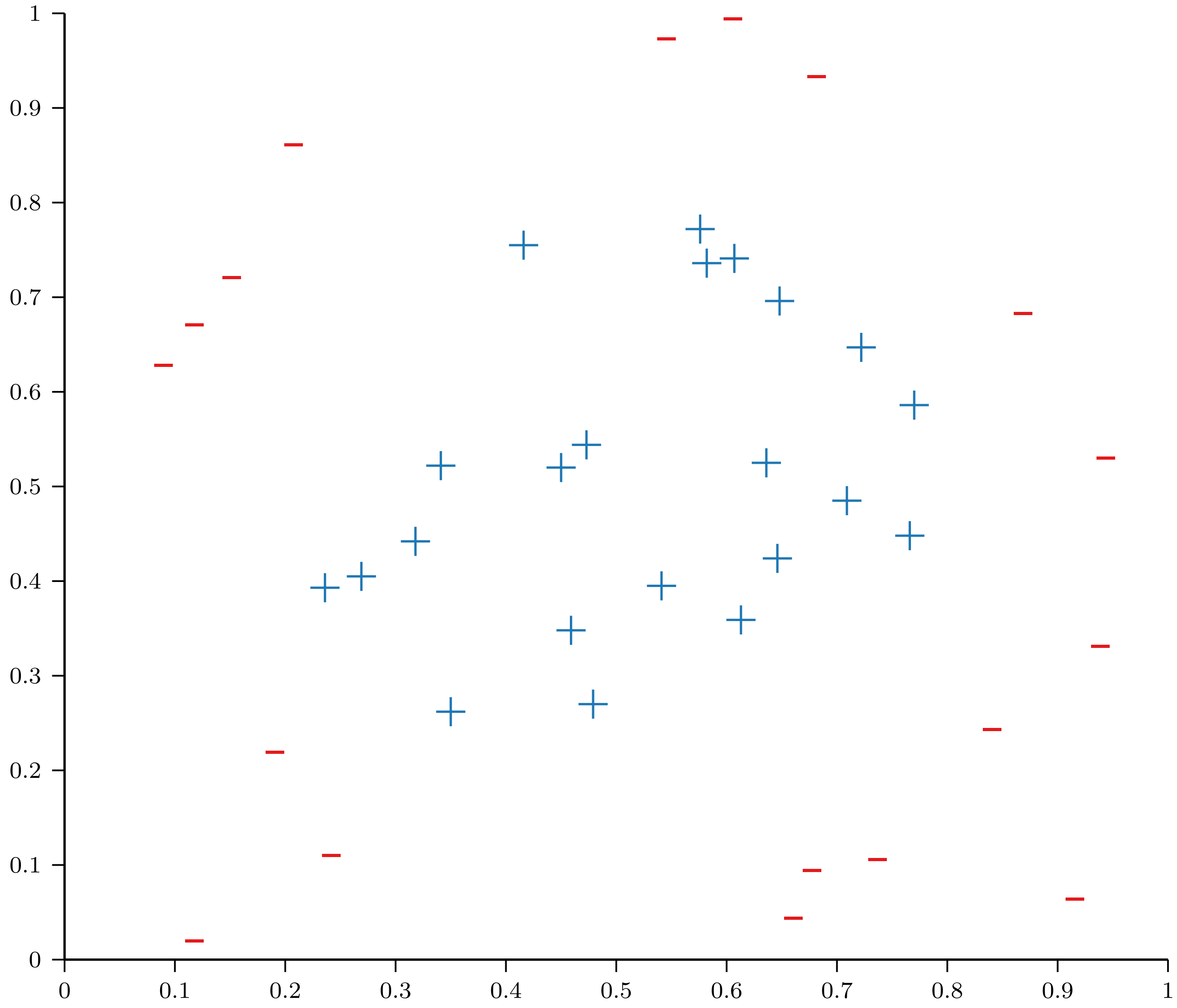
Linear Models



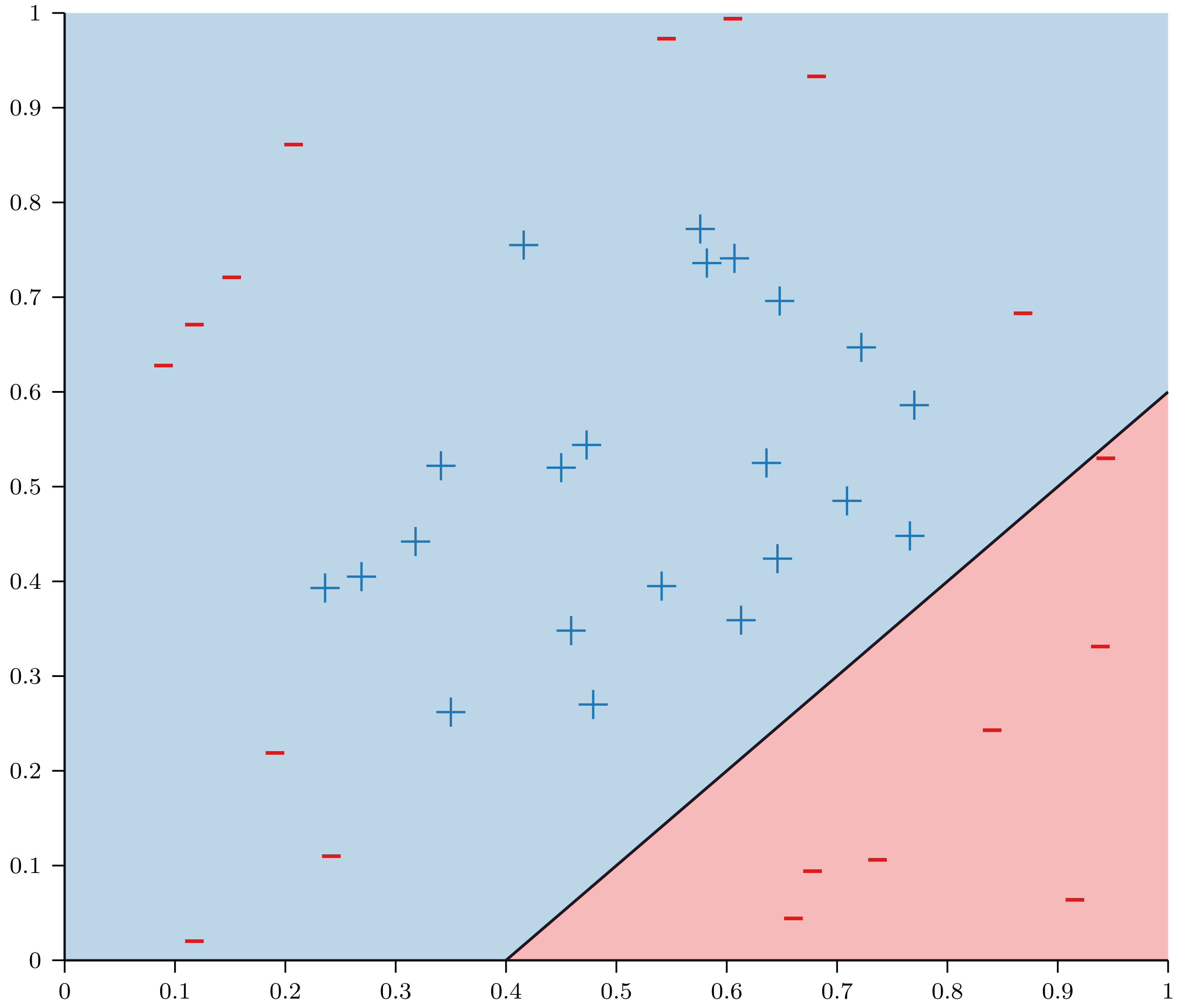
Linear Models



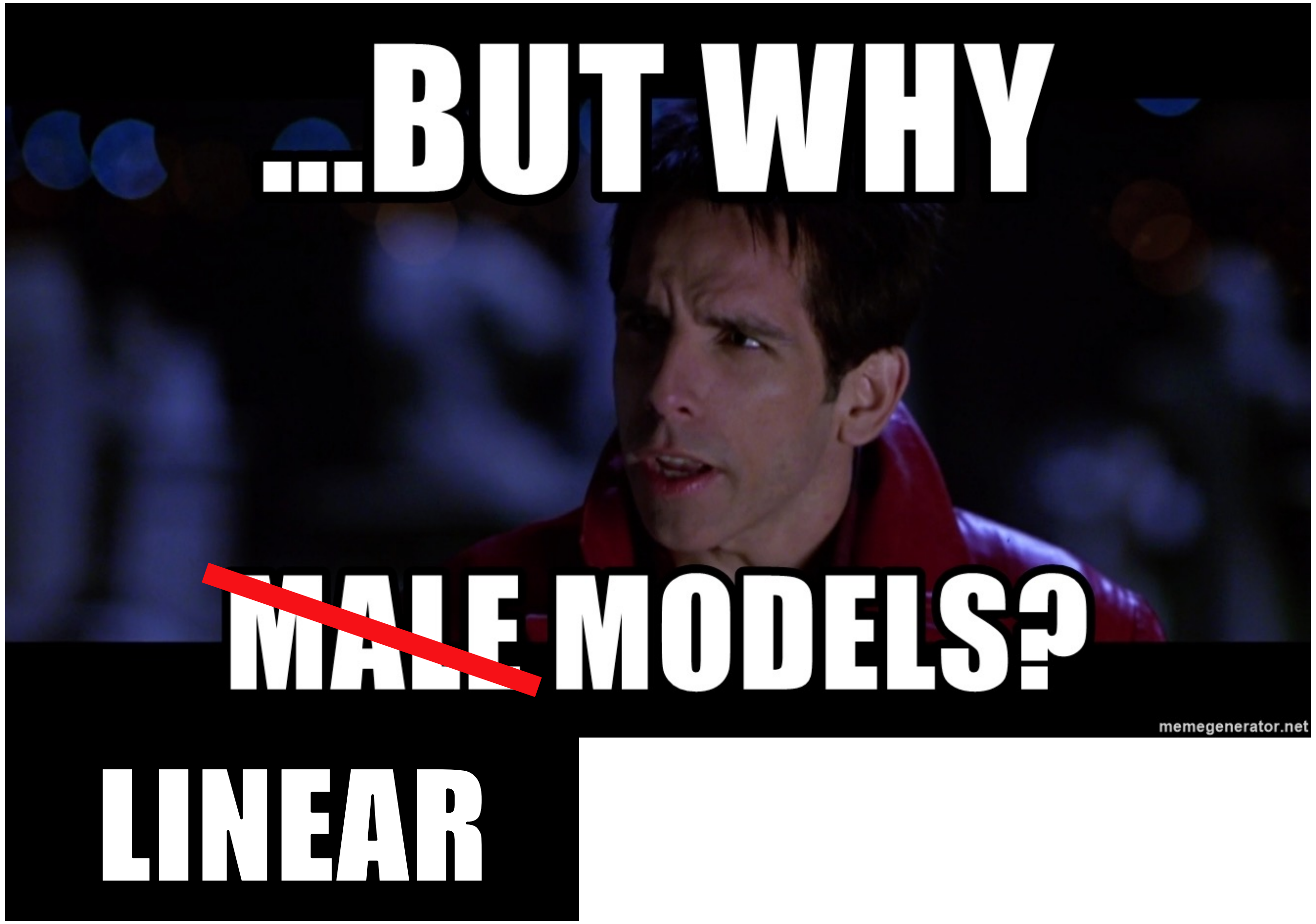
Linear Models?



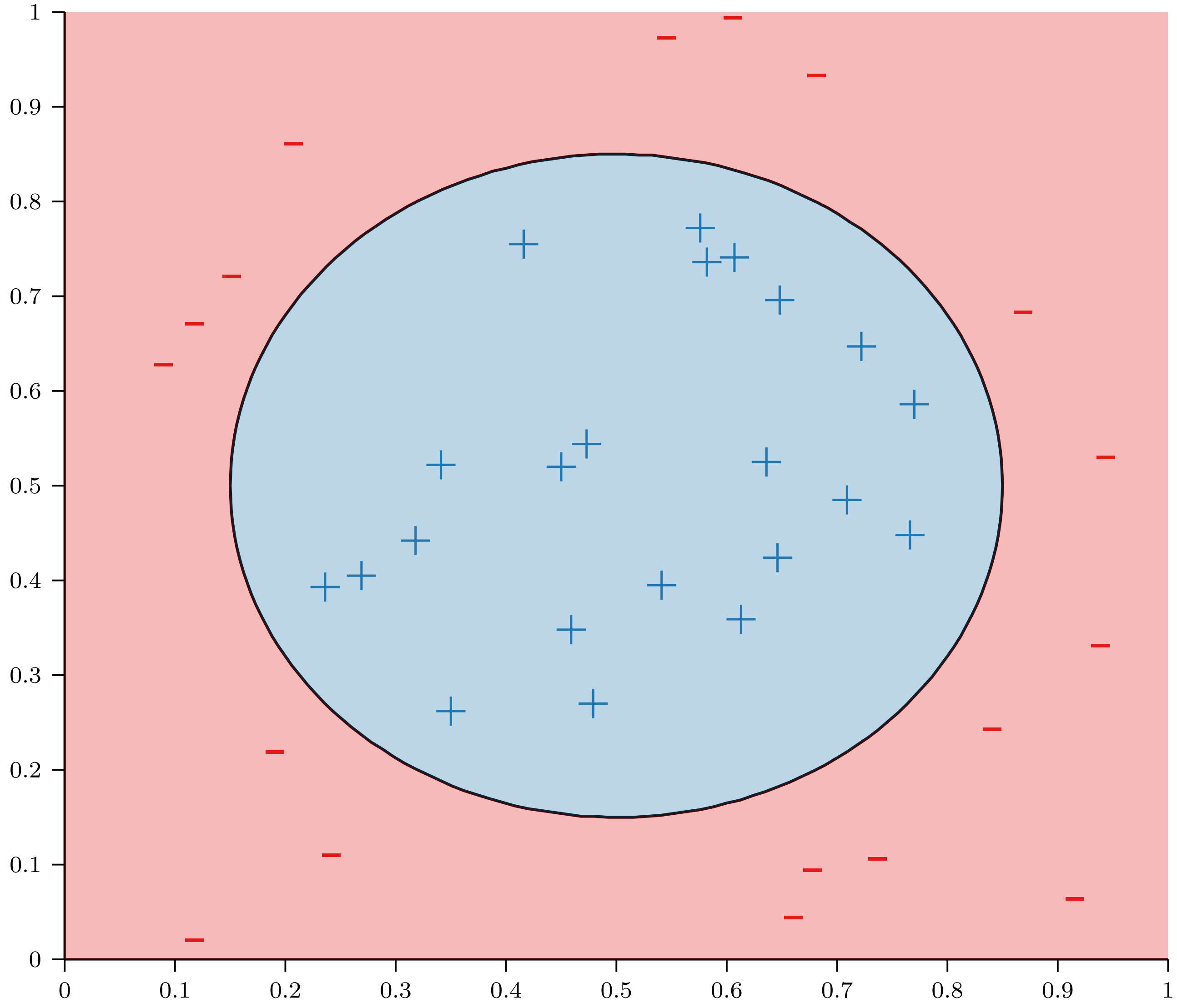
Linear Models?



Linear Models?



Nonlinear Models

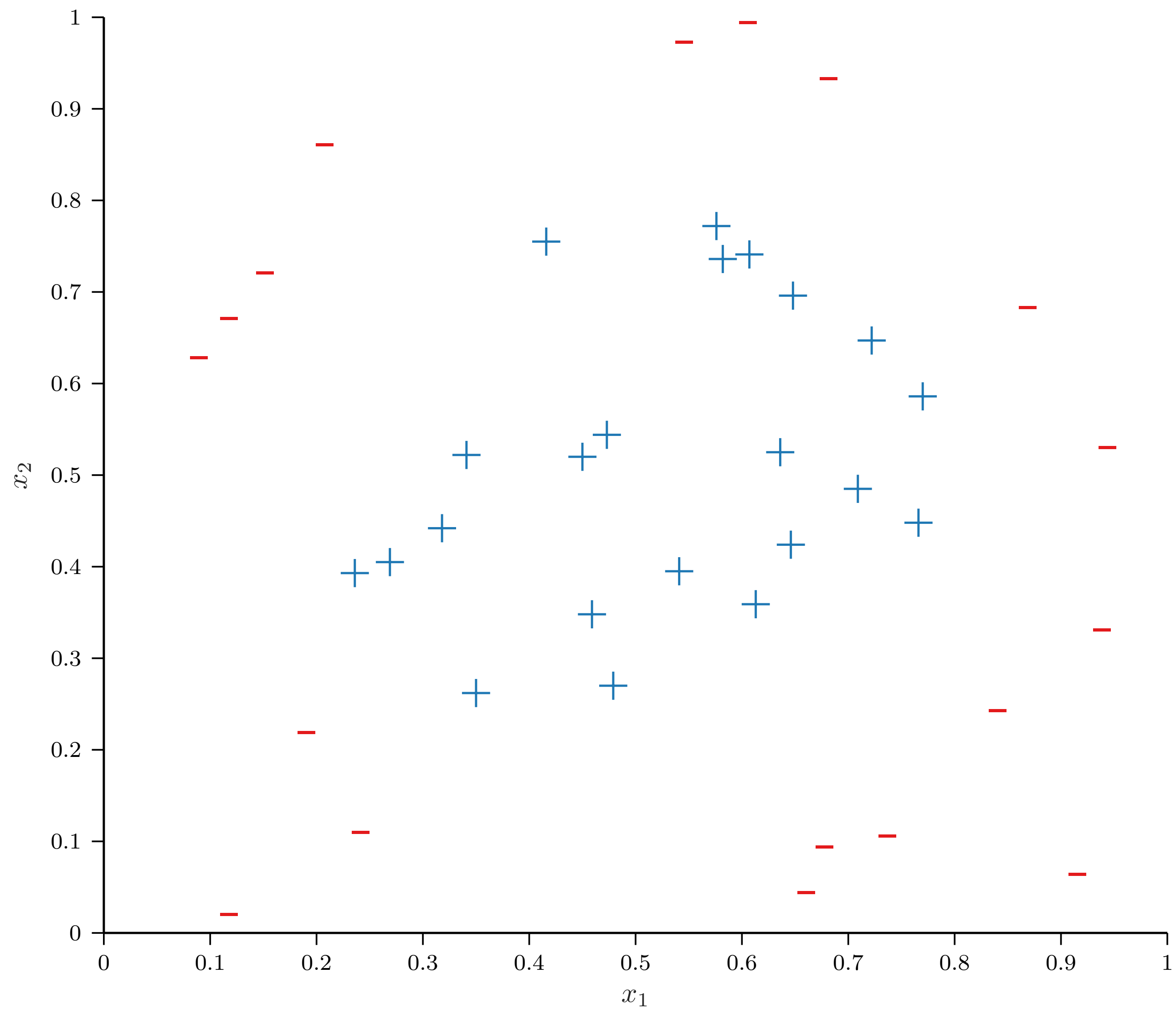


Feature Transforms

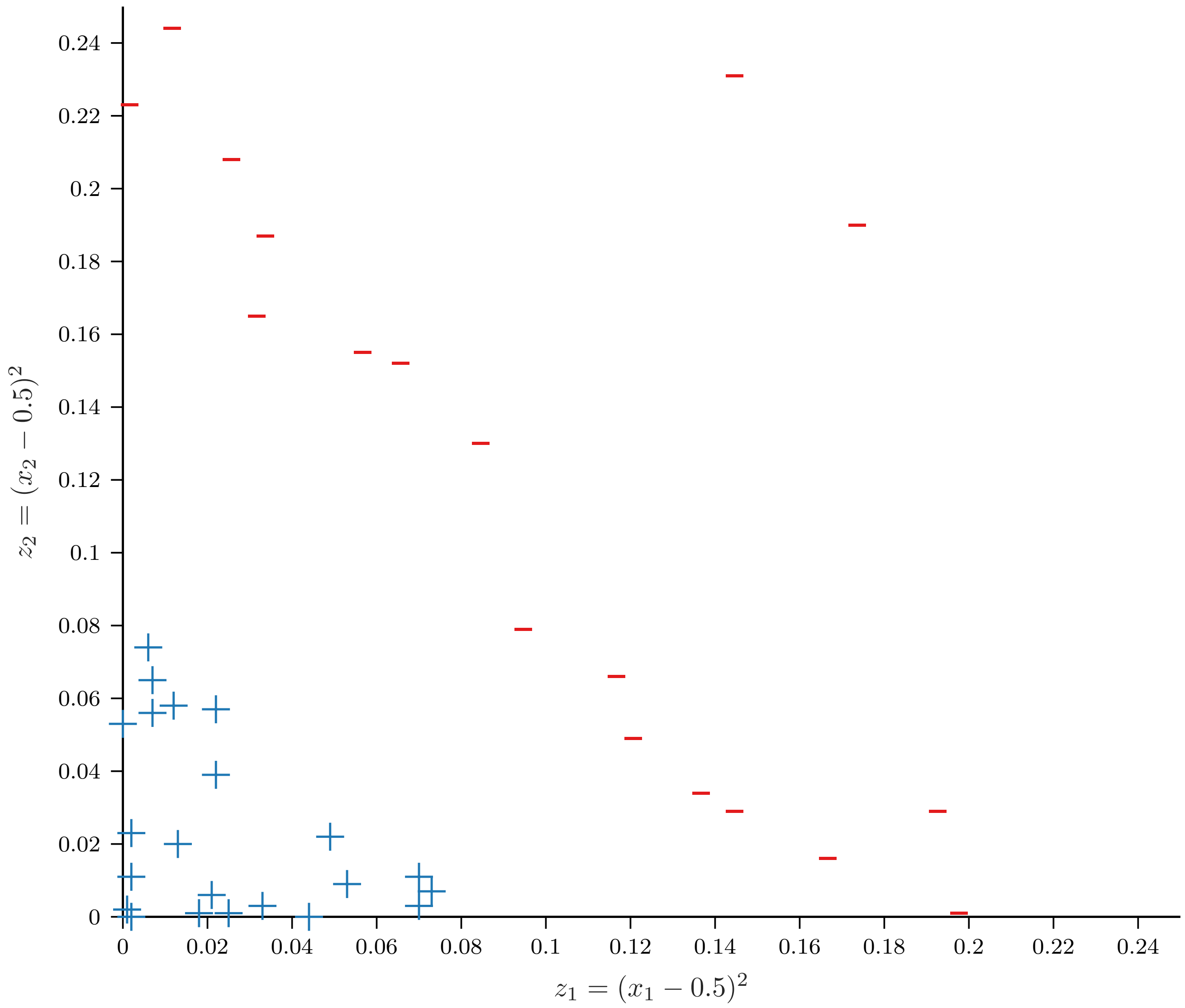
- Given D-dimensional inputs $\boldsymbol{x} = [x_1, \dots, x_D]$, first compute some transformation of our input, e.g.,

$$\phi([x_1, x_2]) = [z_1 = (x_1 - 0.5)^2, z_2 = (x_2 - 0.5)^2]$$

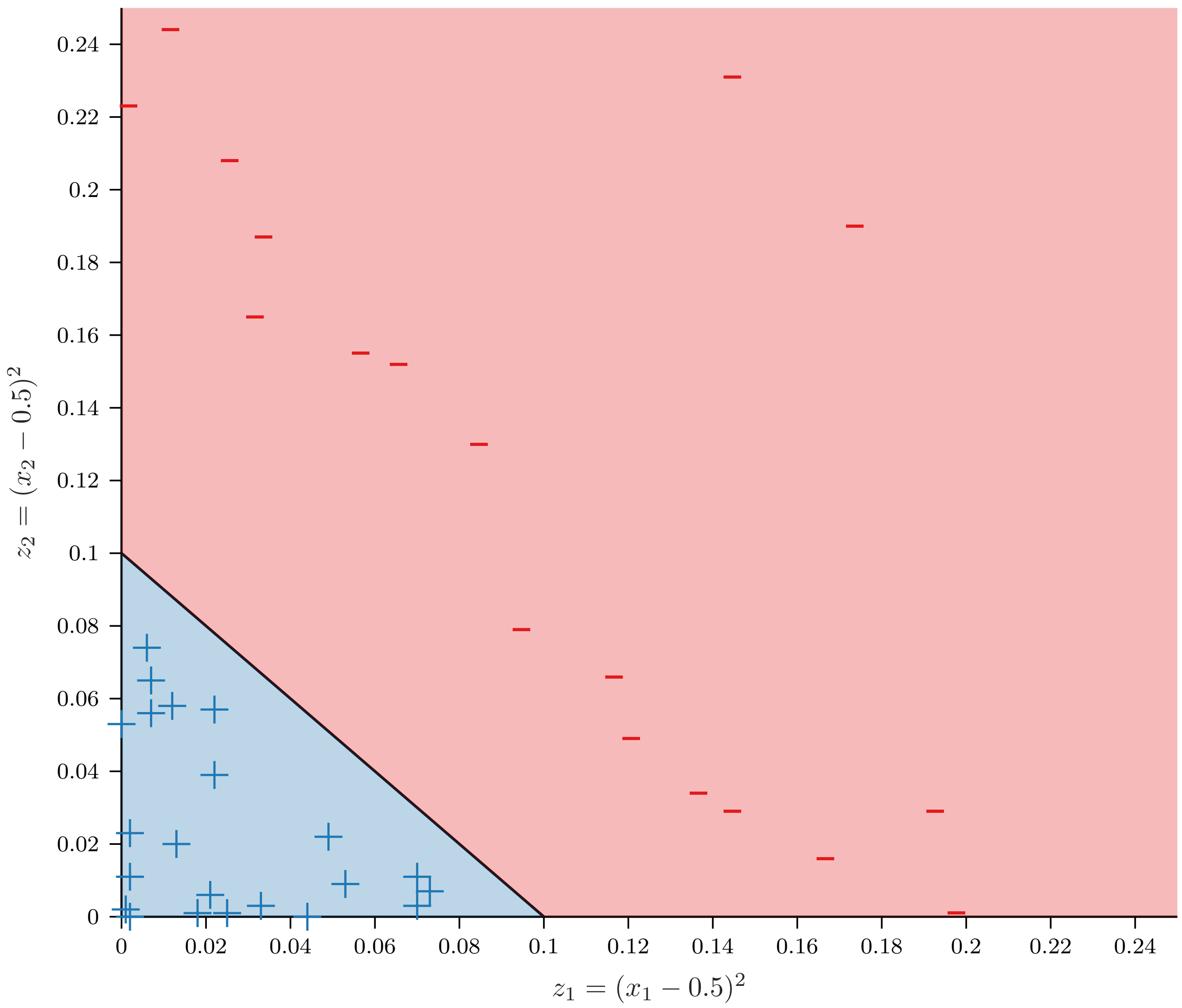
Nonlinear Models



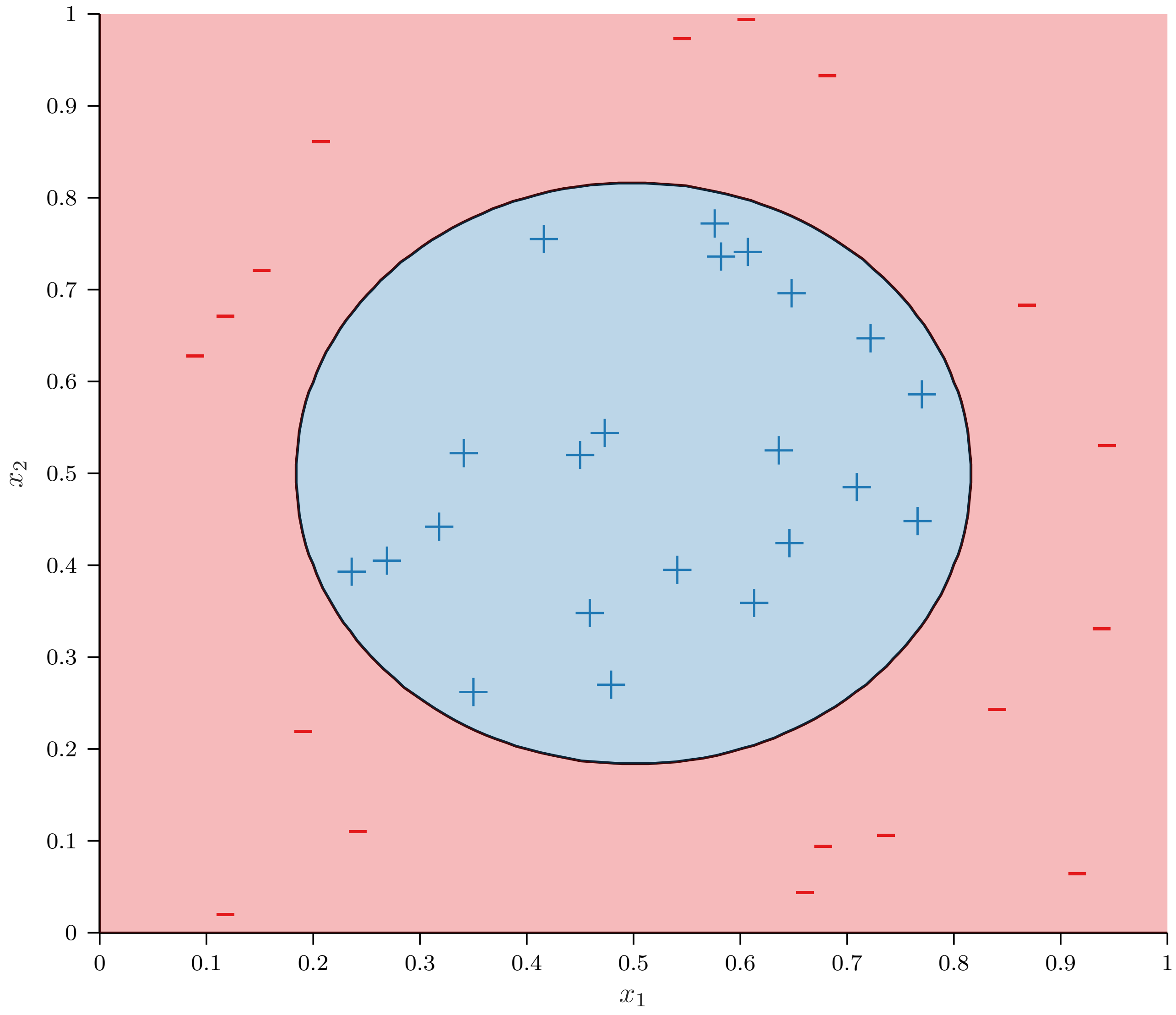
Nonlinear Models



Nonlinear Models



Nonlinear Models



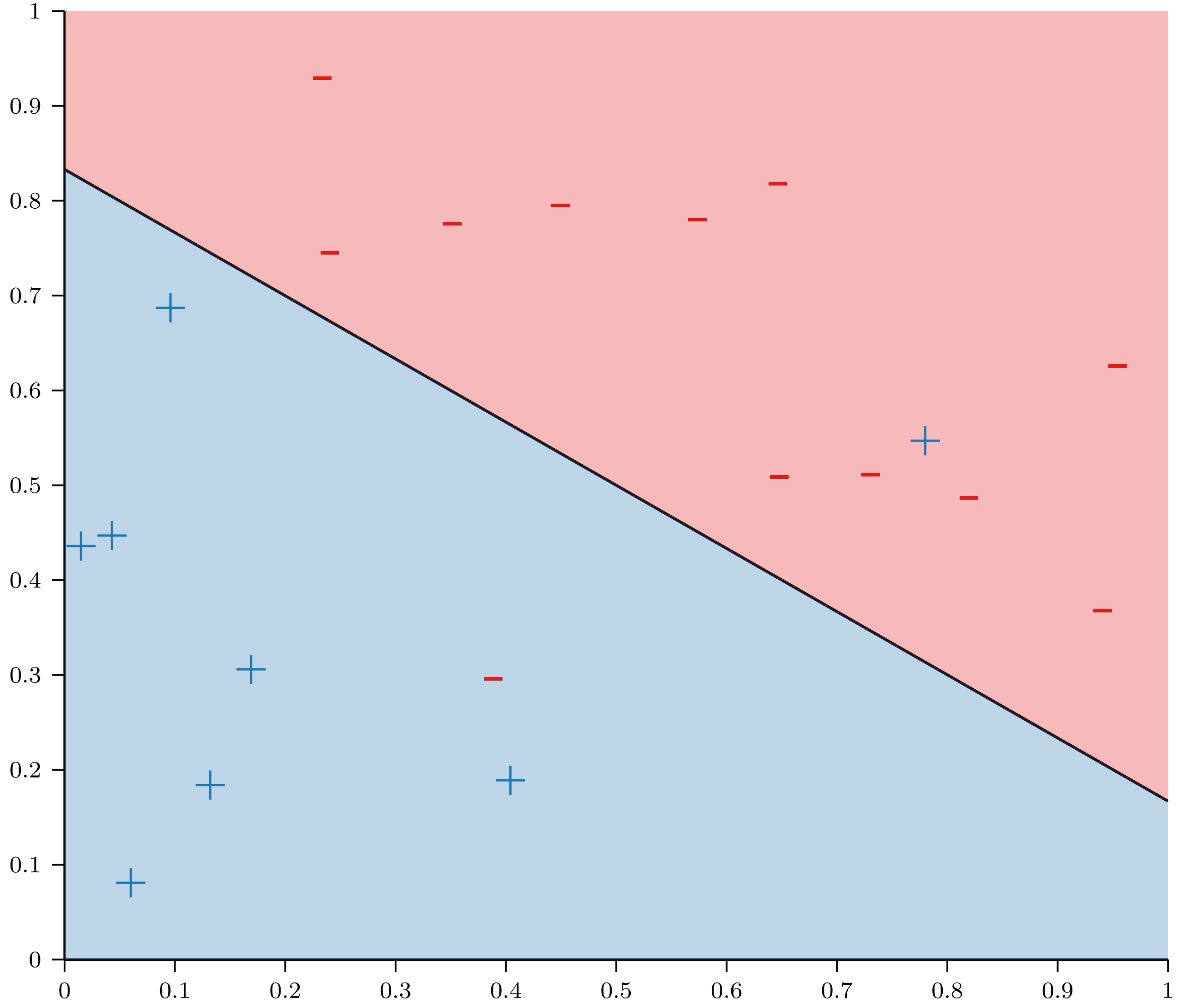
General Q^{th}-order Transforms

input size

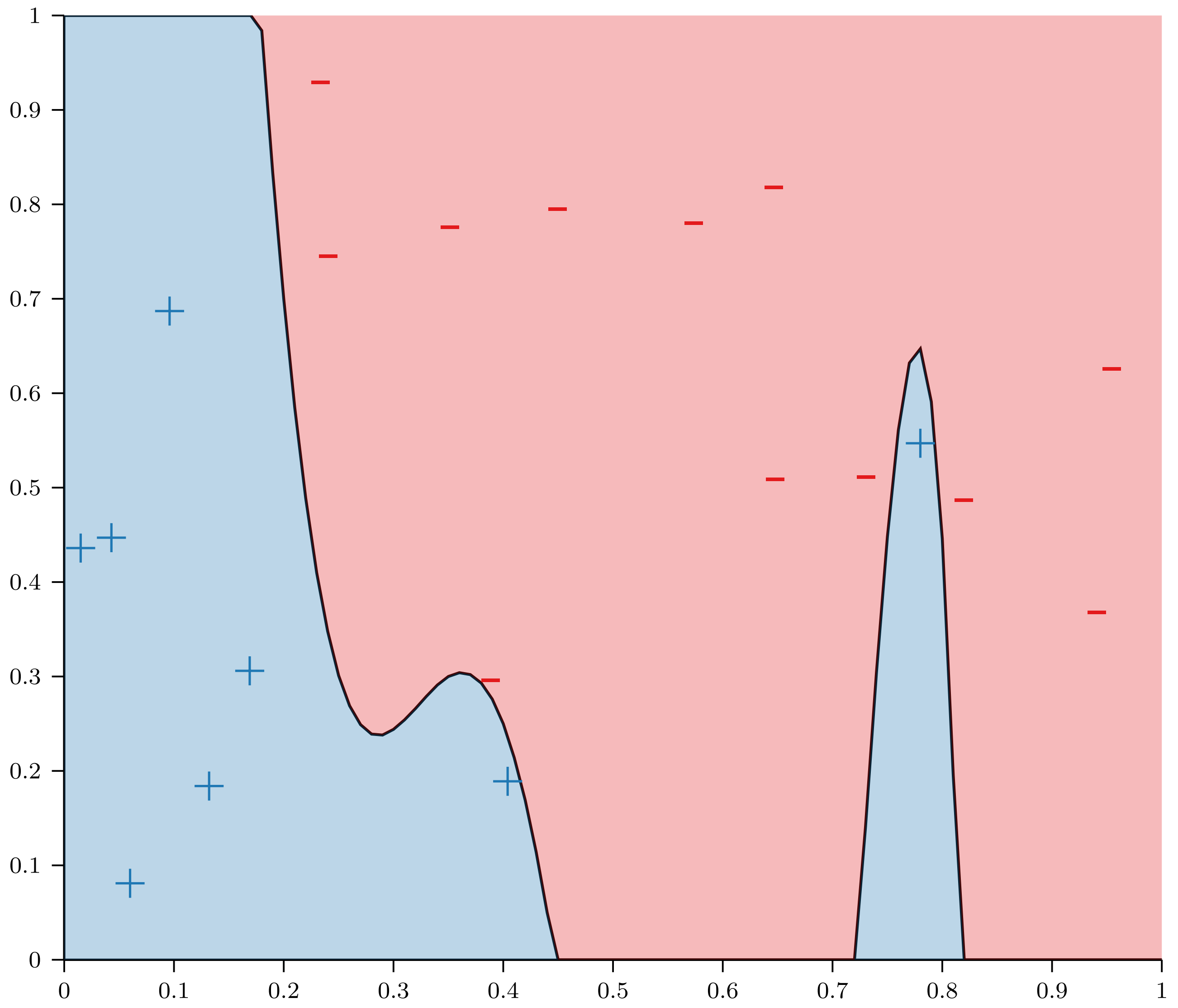
polynomial order

- $\phi_{2,2}([x_1, x_2]) = [x_1, x_2, x_1^2, x_1 x_2, x_2^2]$
- $\phi_{2,3}([x_1, x_2]) = [x_1, x_2, x_1^2, x_1 x_2, x_2^2, x_1^3, x_1^2 x_2, x_1 x_2^2, x_2^3]$
- $\phi_{2,4}([x_1, x_2]) = [x_1, x_2, x_1^2, x_1 x_2, x_2^2, x_1^3, x_1^2 x_2, x_1 x_2^2, x_2^3, x_1^4, x_1^3 x_2, x_1^2 x_2^2, x_1 x_2^3, x_2^4]$
- $\phi_{2,Q}$ maps a 2-dimensional input to a $\frac{Q(Q+3)}{2}$-dimensional output
- Scales even worse for higher-dimensional inputs…

Linear Models



Nonlinear Models?



Feature Transforms: Tradeoffs

	Low-Dimensional Input Space	High-Dimensional Input Space
Training Error	High	Low
Generalization	Good	Bad

Feature Transforms: Experiment

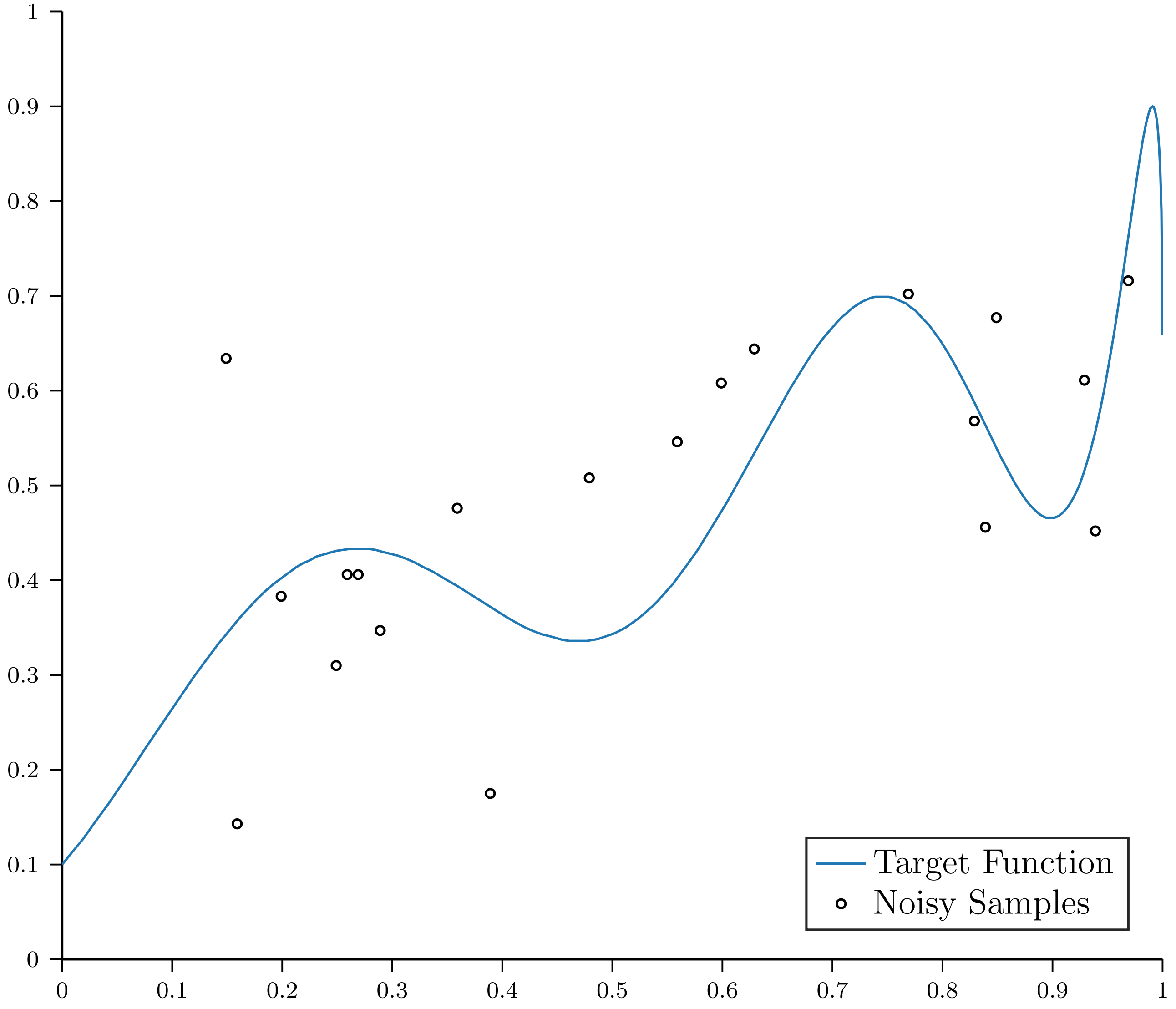
- $x \in \mathbb{R}$, $y \in \mathbb{R}$ and $N = 20$
- Targets are generated by a 10^{th}-order polynomial in x with additive Gaussian noise:

$$y = \sum_{d=0}^{10} a_d x^d + \epsilon \text{ where } \epsilon \sim N(0, \sigma^2)$$

- $\mathcal{H}_2 = 2^{\text{nd}}$-order polynomials
  - $\phi_{1,2}(x) = [x, x^2]$
- $\mathcal{H}_{10} = 10^{\text{th}}$-order polynomials
  - $\phi_{1,10}(x) = [x, x^2, x^3, x^4, x^5, x^6, x^7, x^8, x^9, x^{10}]$

Noisy Targets

- 10-dimensional target function with additive Gaussian noise
- $\mathcal{H}_2 = 2^{\text{nd}}$-order polynomial
- $\mathcal{H}_{10} = 10^{\text{th}}$-order polynomial



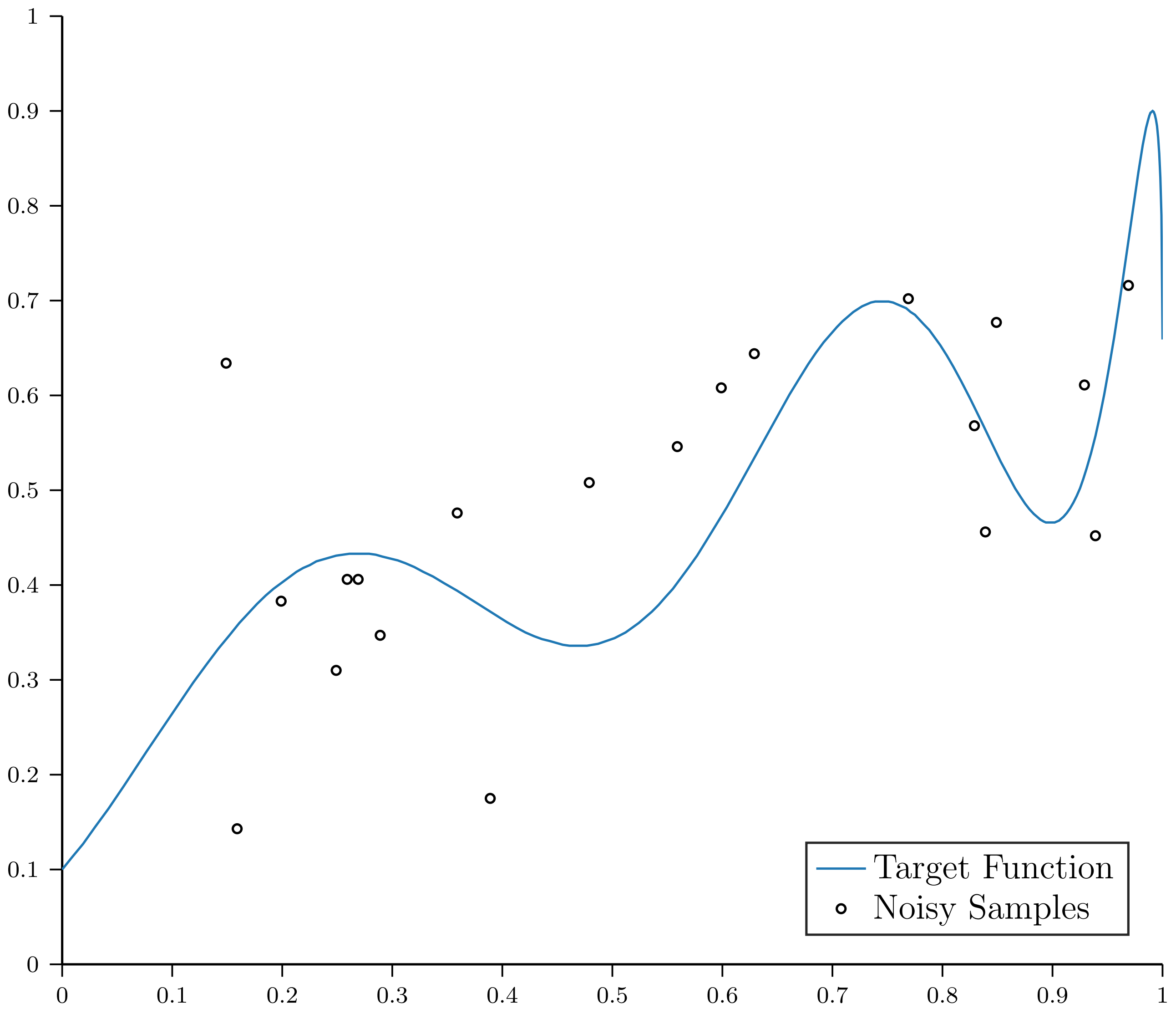
Poll Question 1

Which model do you think will have a lower true error?

A. $\mathcal{H}_2$

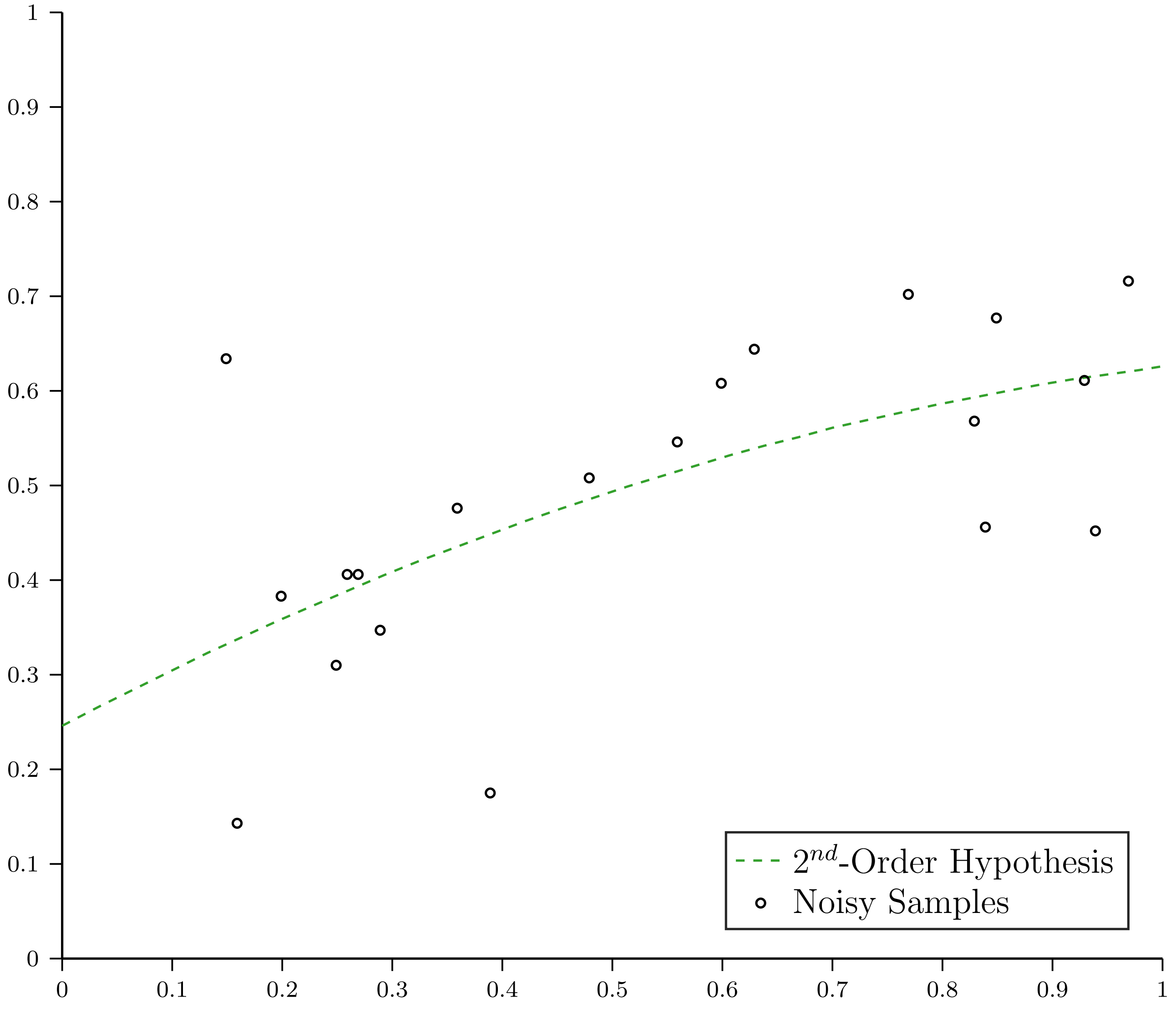
B. TOXIC

C. $\mathcal{H}_{10}$



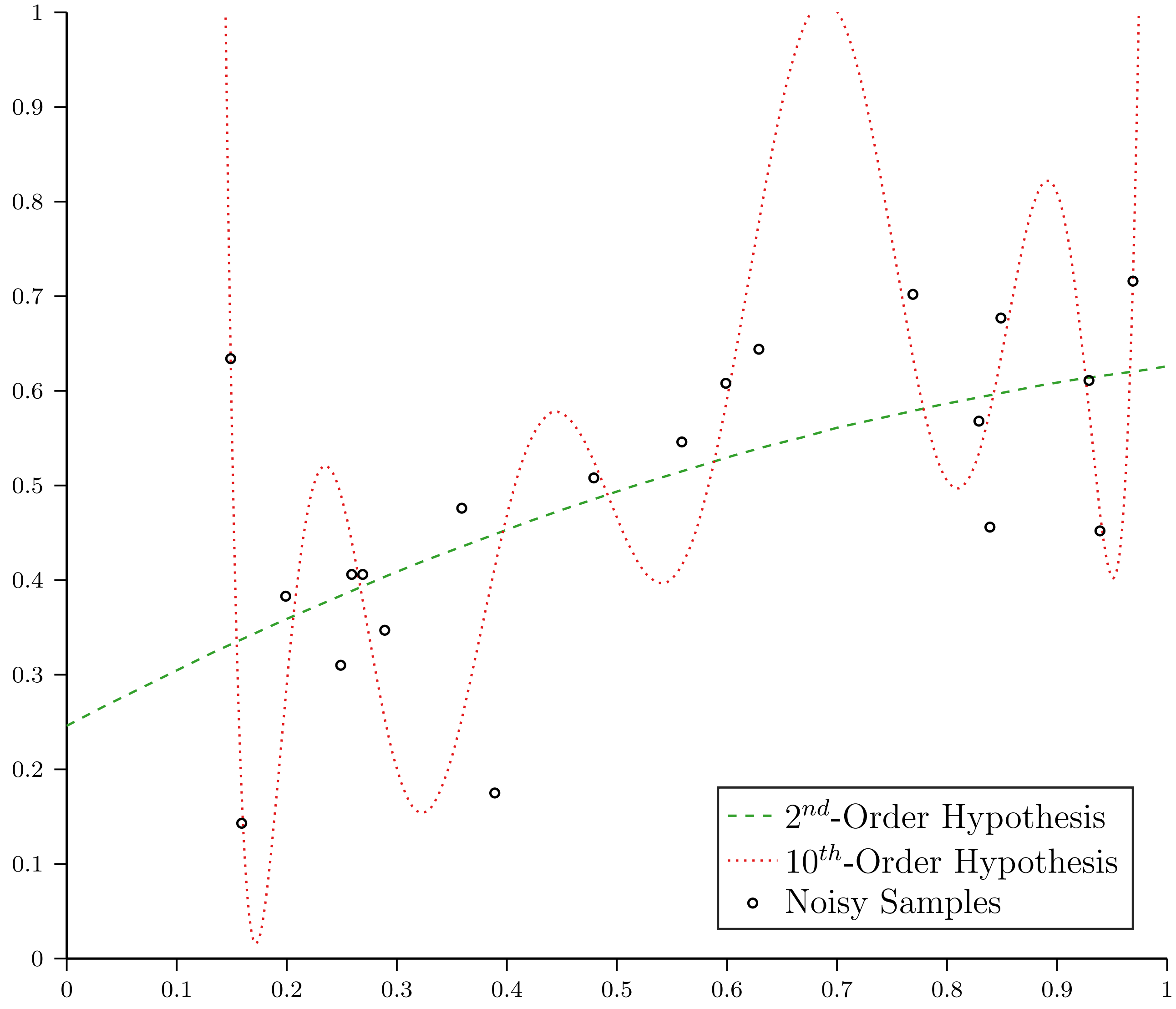
Noisy Targets

- 10-dimensional target function with additive Gaussian noise
- $\mathcal{H}_2 = 2^{\text{nd}}$-order polynomial
- $\mathcal{H}_{10} = 10^{\text{th}}$-order polynomial



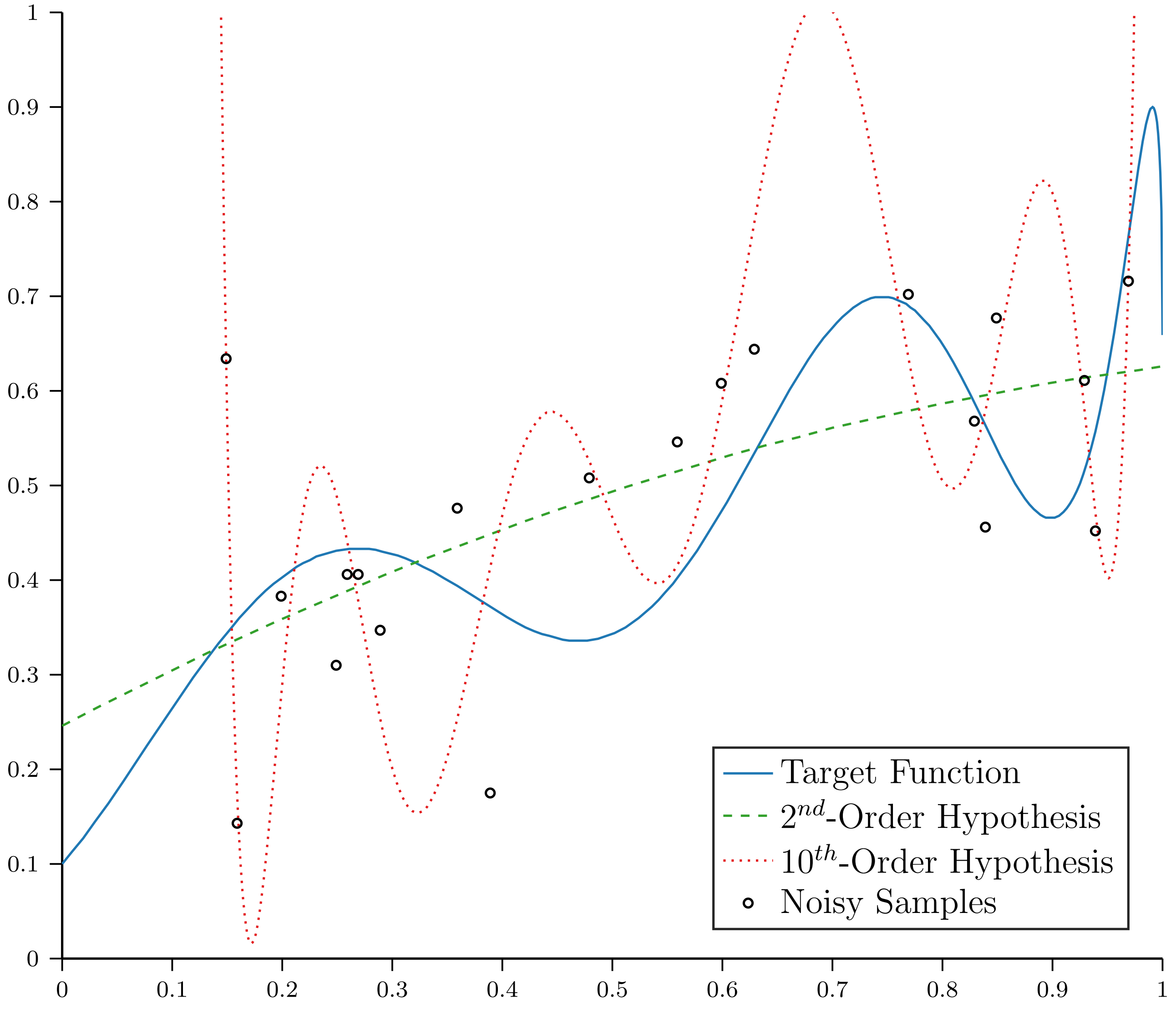
Noisy Targets

- 10-dimensional target function with additive Gaussian noise
- $\mathcal{H}_2 = 2^{\text{nd}}$-order polynomial
- $\mathcal{H}_{10} = 10^{\text{th}}$-order polynomial



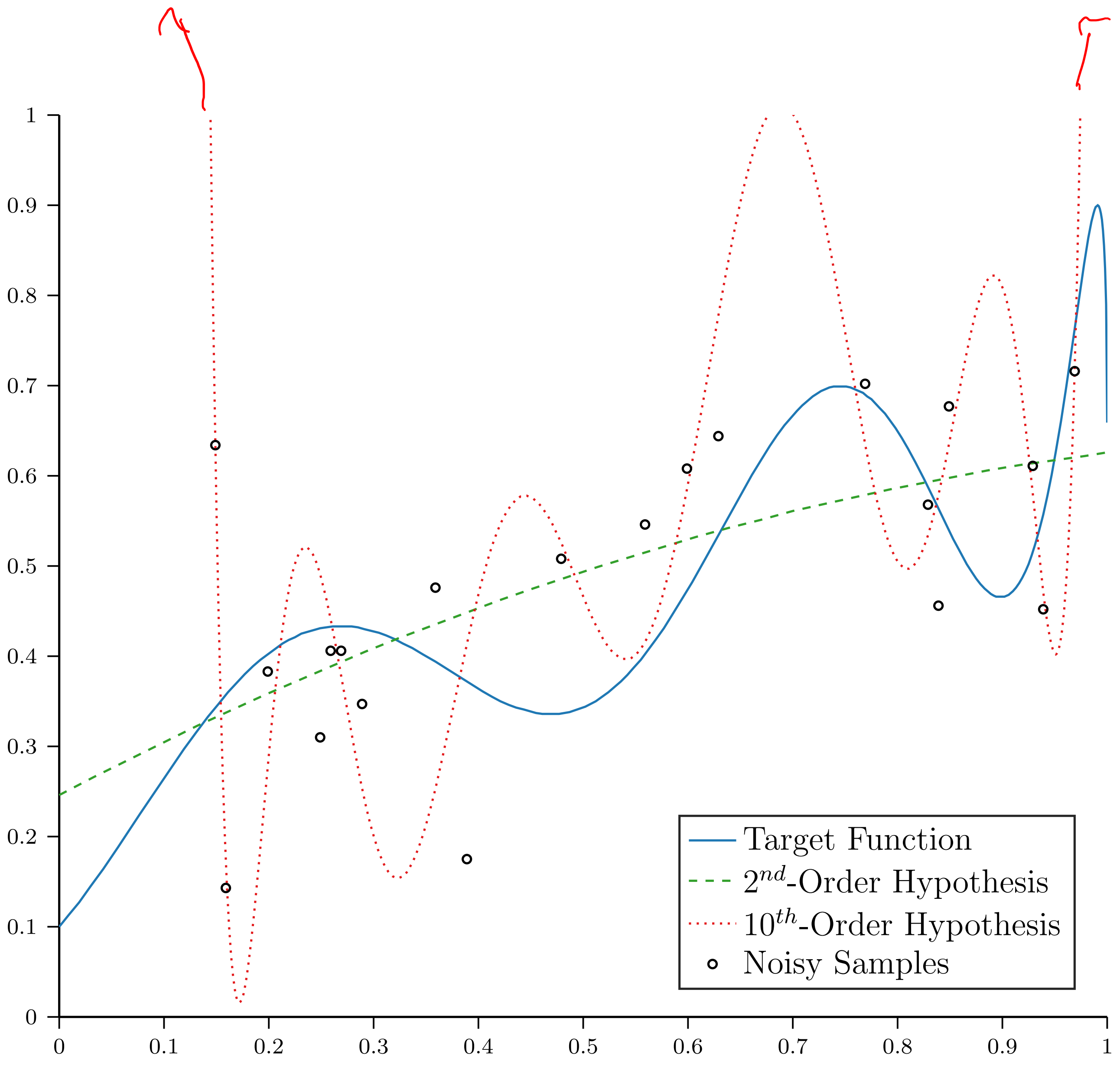
Noisy Targets

- 10-dimensional target function with additive Gaussian noise
- $\mathcal{H}_2 = 2^{\text{nd}}$-order polynomial
- $\mathcal{H}_{10} = 10^{\text{th}}$-order polynomial



Noisy Targets

	$\mathcal{H}_2$	$\mathcal{H}_{10}$
Training Error	0.016	0.011
True Error	0.009	3797



Feature Transforms: Experiment

- $x \in \mathbb{R}$, $y \in \mathbb{R}$ and $N = 100$
- Targets are generated by a 10^{th}-order polynomial in x with additive Gaussian noise:

$$y = \sum_{d=0}^{10} a_d x^d + \epsilon \text{ where } \epsilon \sim N(0, \sigma^2)$$

- $\mathcal{H}_2 = 2^{\text{nd}}$-order polynomials
  - $\phi_{1,2}(x) = [x, x^2]$
- $\mathcal{H}_{10} = 10^{\text{th}}$-order polynomials
  - $\phi_{1,10}(x) = [x, x^2, x^3, x^4, x^5, x^6, x^7, x^8, x^9, x^{10}]$

Poll Question 2

Now wich model do you think will have a lower true error?

A. TOXIC

B. $\mathcal{H}_2$

C. $\mathcal{H}_{10}$

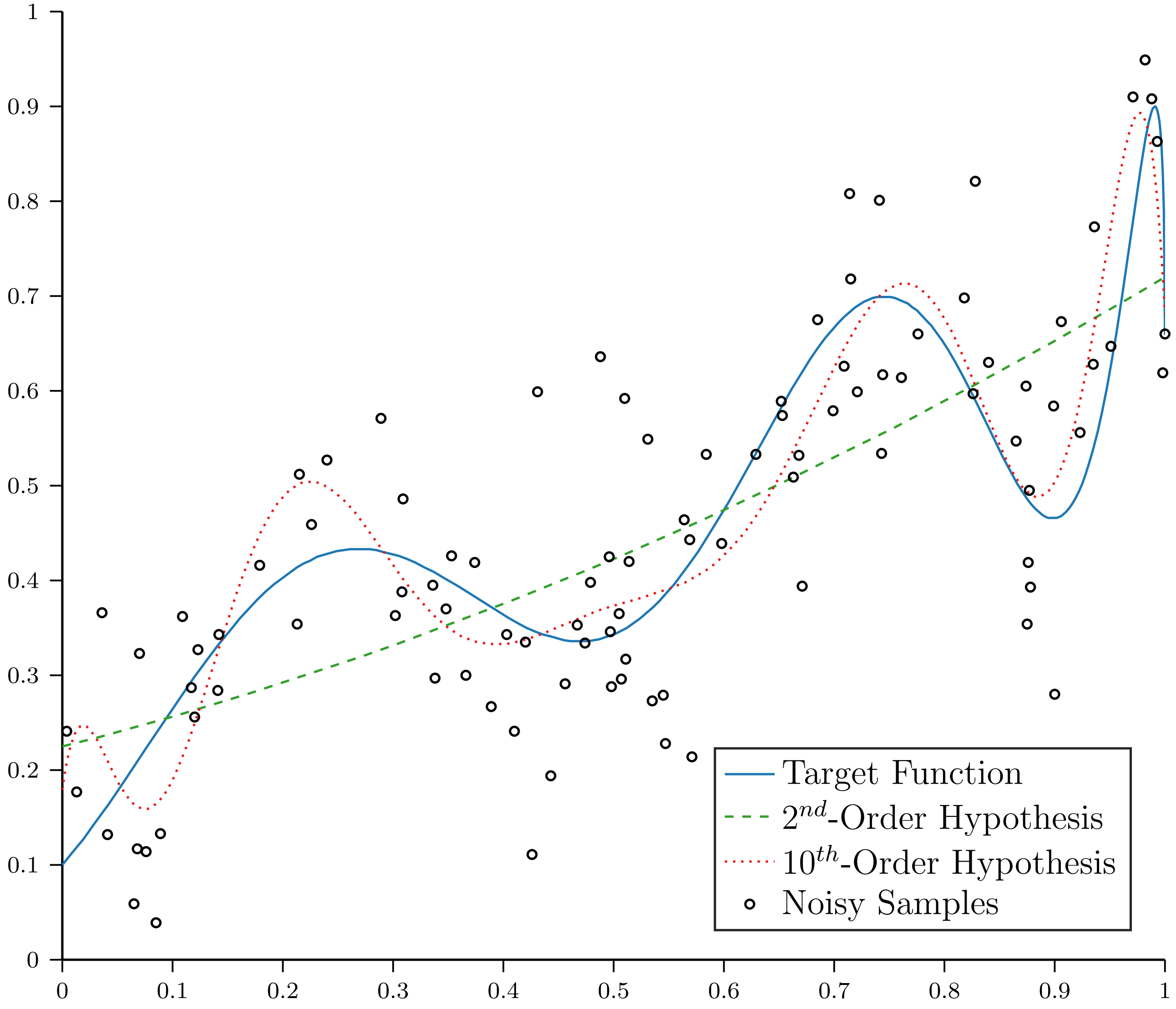
- $x \in \mathbb{R}$, $y \in \mathbb{R}$ and $N = 100$
- Targets are generated by a 10^{th}-order polynomial in x with additive Gaussian noise:

$$y = \sum_{d=0}^{10} a_d x^d + \epsilon \text{ where } \epsilon \sim N(0, \sigma^2)$$

- $\mathcal{H}_2 = 2^{\text{nd}}$-order polynomials
  - $\phi_{1,2}(x) = [x, x^2]$
- $\mathcal{H}_{10} = 10^{\text{th}}$-order polynomials
  - $\phi_{1,10}(x) = [x, x^2, x^3, x^4, x^5, x^6, x^7, x^8, x^9, x^{10}]$

Noisy Targets

	$\mathcal{H}_2$	$\mathcal{H}_{10}$
Training Error	0.018	0.010
True Error	0.009	0.003



Regularization

- Constrain models to prevent them from overfitting
- Learning algorithms are optimization problems and regularization imposes constraints on the optimization

Hard Constraints

- $\mathcal{H}_{10}$ = 10^{th}-order polynomials
  - $\phi_{1,10}(x) = [x, x^2, x^3, x^4, x^5, x^6, x^7, x^8, x^9, x^{10}]$
- Given $\mathrm{X} = \begin{bmatrix} 1 & \phi_{1,10}\left(x^{(1)}\right) \\ 1 & \phi_{1,10}\left(x^{(2)}\right) \\ \vdots & \vdots \\ 1 & \phi_{1,10}\left(x^{(N)}\right) \end{bmatrix}$ and $\boldsymbol{y} = \begin{bmatrix} y^{(1)} \\ y^{(2)} \\ \vdots \\ y^{(N)} \end{bmatrix}$ find

  $\boldsymbol{\theta} = [\theta_0, \theta_1, \theta_2, \theta_3, \theta_4, \theta_5, \theta_6, \theta_7, \theta_8, \theta_9, \theta_{10}]$
  that minimizes

$$(X\boldsymbol{\theta} - \boldsymbol{y})^T (X\boldsymbol{\theta} - \boldsymbol{y})$$

- Subject to

$$\theta_3 = \theta_4 = \theta_5 = \theta_6 = \theta_7 = \theta_8 = \theta_9 = \theta_{10} = 0$$

Hard Constraints

- $\mathcal{H}_{10}$ = 10^{th}-order polynomials
  - $\phi_{1,10}(x) = [x, x^2, x^3, x^4, x^5, x^6, x^7, x^8, x^9, x^{10}]$
- Given $\mathrm{X} = \begin{bmatrix} 1 & \phi_{1,10}\left(x^{(1)}\right) \\ 1 & \phi_{1,10}\left(x^{(2)}\right) \\ \vdots & \vdots \\ 1 & \phi_{1,10}\left(x^{(N)}\right) \end{bmatrix}$ and $\boldsymbol{y} = \begin{bmatrix} y^{(1)} \\ y^{(2)} \\ \vdots \\ y^{(N)} \end{bmatrix}$ find

  $\boldsymbol{\theta} = [\theta_0, \theta_1, \theta_2, \theta_3, \theta_4, \theta_5, \theta_6, \theta_7, \theta_8, \theta_9, \theta_{10}]$
  that minimizes

$$\sum_{n=1}^{N} \left(\left(\sum_{d=0}^{10} x_d^{(n)} \theta_d \right) - y^{(n)} \right)^2$$

- Subject to

$$\theta_3 = \theta_4 = \theta_5 = \theta_6 = \theta_7 = \theta_8 = \theta_9 = \theta_{10} = 0$$

Hard Constraints

- $\mathcal{H}_{10} = 10^{\text{th}}$-order polynomials
  - $\phi_{1,10}(x) = [x, x^2, x^3, x^4, x^5, x^6, x^7, x^8, x^9, x^{10}]$
- Given $\mathrm{X} = \begin{bmatrix} 1 & \phi_{1,10}\left(x^{(1)}\right) \\ 1 & \phi_{1,10}\left(x^{(2)}\right) \\ \vdots & \vdots \\ 1 & \phi_{1,10}\left(x^{(N)}\right) \end{bmatrix}$ and $\boldsymbol{y} = \begin{bmatrix} y^{(1)} \\ y^{(2)} \\ \vdots \\ y^{(N)} \end{bmatrix}$ find

  $\boldsymbol{\theta} = [\theta_0, \theta_1, \theta_2, \theta_3, \theta_4, \theta_5, \theta_6, \theta_7, \theta_8, \theta_9, \theta_{10}]$ that minimizes

$$\sum_{n=1}^{N} \left(\left(\sum_{d=0}^{2} x_d^{(n)} \theta_d \right) - y^{(n)} \right)^2$$

- Subject to nothing!

Hard Constraints

- $\mathcal{H}_2 = 2^{\mathrm{nd}}$-order polynomials
  - $\phi_{1,2}(x) = [x, x^2]$
- Given $\mathrm{X} = \begin{bmatrix} 1 & \phi_{1,2}(x^{(1)}) \\ 1 & \phi_{1,2}(x^{(2)}) \\ \vdots & \vdots \\ 1 & \phi_{1,2}(x^{(N)}) \end{bmatrix}$ and $\boldsymbol{y} = \begin{bmatrix} y^{(1)} \\ y^{(2)} \\ \vdots \\ y^{(N)} \end{bmatrix}$ find

  $\boldsymbol{\theta} = [\theta_0, \theta_1, \theta_2]$
  that minimizes

$$(X\boldsymbol{\theta} - \boldsymbol{y})^T (X\boldsymbol{\theta} - \boldsymbol{y})$$

- Subject to nothing!

Soft Constraints

- More generally, ϕ can be any nonlinear transformation, e.g., exp, log, sin, sqrt, etc...
- Given $X = \begin{bmatrix} 1 & \phi_1(\boldsymbol{x}^{(1)}) & \cdots & \phi_m(\boldsymbol{x}^{(1)}) \\ \vdots & \vdots & \ddots & \vdots \\ 1 & \phi_1(\boldsymbol{x}^{(N)}) & \cdots & \phi_m(\boldsymbol{x}^{(N)}) \end{bmatrix}$ and $\boldsymbol{y} = \begin{bmatrix} y^{(1)} \\ y^{(2)} \\ \vdots \\ y^{(N)} \end{bmatrix}$,

  find $\boldsymbol{\omega}$ that minimizes

$$(X\boldsymbol{\theta} - \boldsymbol{y})^T(X\boldsymbol{\theta} - \boldsymbol{y})$$

- Subject to:

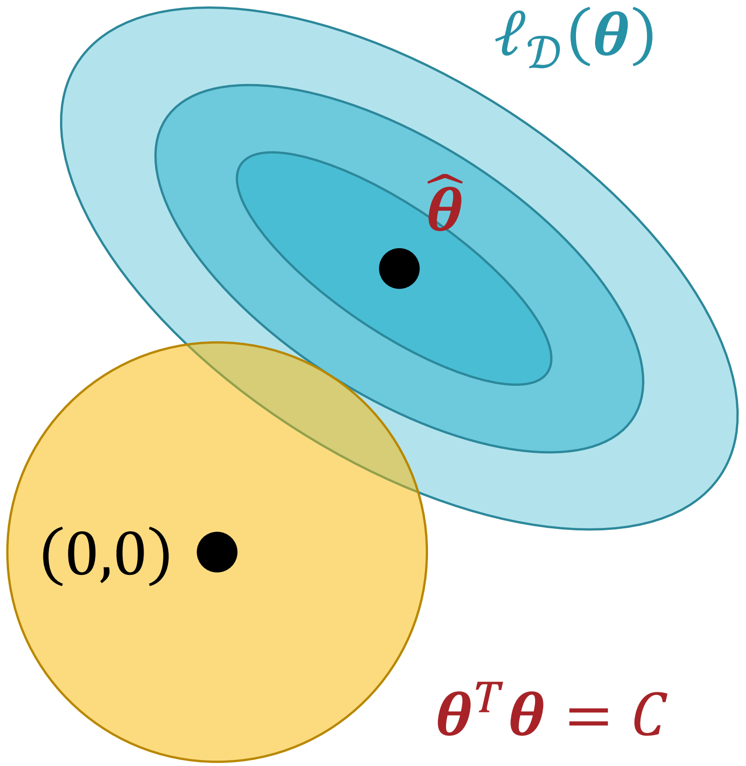
$$\|\boldsymbol{\theta}\|_2^2 = \boldsymbol{\theta}^T\boldsymbol{\theta} = \sum_{d=0}^{D} \theta_d^2 \leq C$$

Soft Constraints

minimize $\ell_{\mathcal{D}}(\boldsymbol{\theta}) = (X\boldsymbol{\theta} - \boldsymbol{y})^T (X\boldsymbol{\theta} - \boldsymbol{y})$

$(\theta_0^2 + \theta_1^2) \leq C$

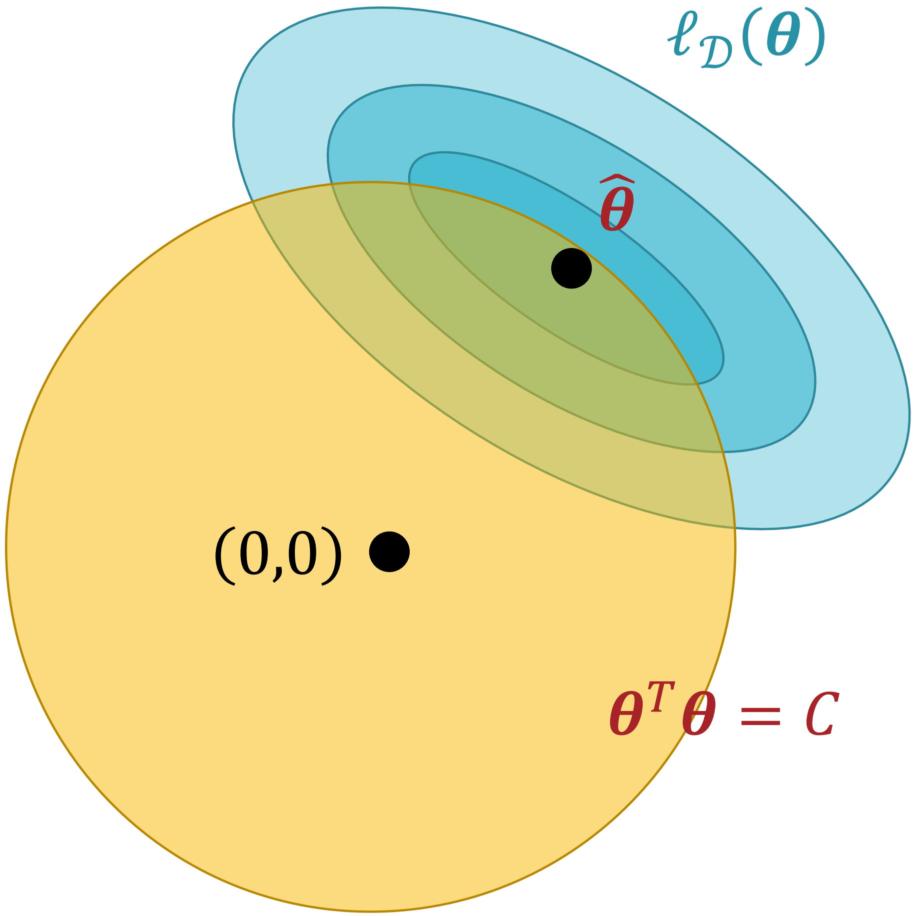
subject to $\boldsymbol{\theta}^T \boldsymbol{\theta} \leq C$



Soft Constraints

minimize $\ell_{\mathcal{D}}(\boldsymbol{\theta}) = (X\boldsymbol{\theta} - \boldsymbol{y})^T(X\boldsymbol{\theta} - \boldsymbol{y})$

subject to $\boldsymbol{\theta}^T\boldsymbol{\theta} \leq C$



Soft Constraints

minimize $\ell_{\mathcal{D}}(\boldsymbol{\theta}) = (X\boldsymbol{\theta} - \boldsymbol{y})^T(X\boldsymbol{\theta} - \boldsymbol{y})$

subject to $\boldsymbol{\theta}^T\boldsymbol{\theta} \le C$

proportional to

$\nabla_\theta \ell_D(\hat{\theta}_{REG}) \propto -\hat{\theta}_{REG}$

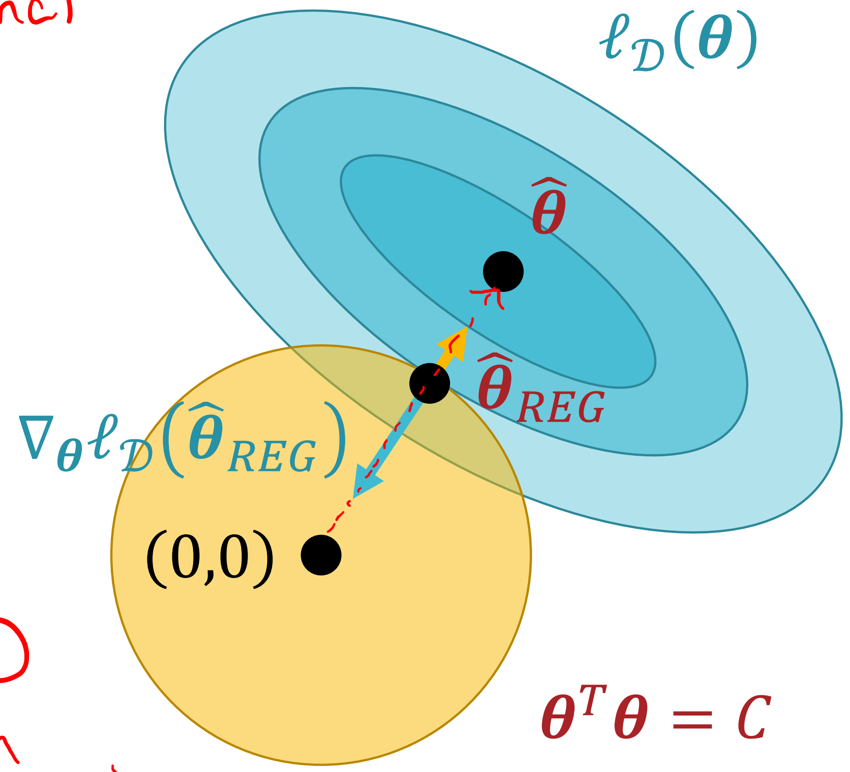
$\rightarrow \nabla_\theta \ell_D(\hat{\theta}_{REG}) = -2\lambda_c \hat{\theta}_{REG}$

$\lambda_c \ge 0$

$\Rightarrow \nabla_\theta \ell_D(\hat{\theta}_{REG}) + 2\lambda_c \hat{\theta}_{REG} = 0$

$\Rightarrow \nabla_\theta \left(\ell_D(\hat{\theta}_{REG}) + \lambda_c \hat{\theta}_{REG}^T \hat{\theta}_{REG}\right) = 0$

min. $\ell_D(\theta) + \lambda_c \theta^T\theta$



Soft Constraints: Solving for $\widehat{\boldsymbol{\theta}}_{REG}$

minimize $\ell_{\mathcal{D}}(\boldsymbol{\theta}) = (X\boldsymbol{\theta} - \boldsymbol{y})^T(X\boldsymbol{\theta} - \boldsymbol{y})$

subject to $\boldsymbol{\theta}^T\boldsymbol{\theta} \leq C$

$$\Updownarrow$$

minimize $\ell_{\mathcal{D}}^{AUG}(\boldsymbol{\theta}) = \ell_{\mathcal{D}}(\boldsymbol{\theta}) + \lambda_C \boldsymbol{\theta}^T\boldsymbol{\theta}$

s.t. $\lambda_c \geq 0$

Ridge Regression

$$\text{minimize } \ell_{\mathcal{D}}^{AUG}(\boldsymbol{\theta}) = \ell_{\mathcal{D}}(\boldsymbol{\theta}) + \lambda_C \boldsymbol{\theta}^T \boldsymbol{\theta}$$

$\rightarrow (X^T\theta - y)^T (X^T\theta - y)$

$$\nabla_\theta \ell_D^{AUG}(\theta) = 2 X^T X \theta - 2X^T y + 2\lambda_c \theta$$

$$\Rightarrow 2X^TX\hat{\theta}_{REG} - 2X^Ty + 2\lambda_c\hat{\theta}_{REG} = 0$$

$$\Rightarrow X^TX\hat{\theta}_{REG} + \lambda_c\hat{\theta}_{REG} = X^Ty$$

$$\Rightarrow (X^TX + \lambda_c I_{D+1})\hat{\theta}_{REG} = X^Ty$$

$$\Rightarrow \hat{\theta}_{REG} = (X^TX + \lambda_c I_{D+1})^{-1} X^T y$$

$(\lambda_c \geq 0)$

Poll Question 3

- Suppose we are minimizing $\ell_{\mathcal{D}}^{AUG}(\boldsymbol{\theta}) = \ell_{\mathcal{D}}(\boldsymbol{\theta}) + \lambda_C r(\boldsymbol{\theta})$.

  As λ_C increases, the minimum of $\ell_{\mathcal{D}}^{AUG}$ …

A. … moves towards the midpoint of $\ell_{\mathcal{D}}$ and r

B. … moves towards the minimum of $\ell_{\mathcal{D}}$

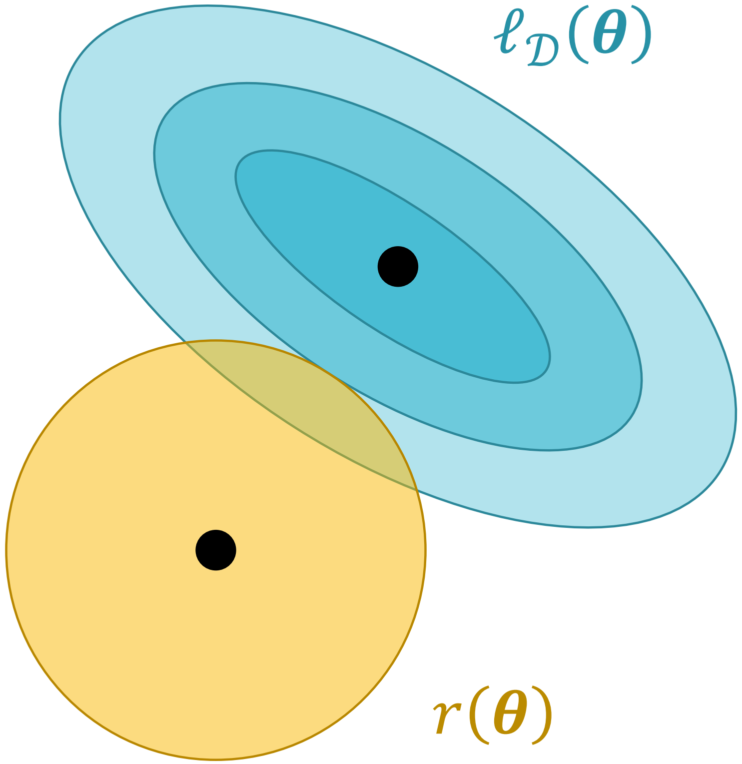
C. … moves towards the minimum of r

D. … moves towards the vector of all infinities

E. … moves towards the vector of all ones

F. … stays the same

TOXIC



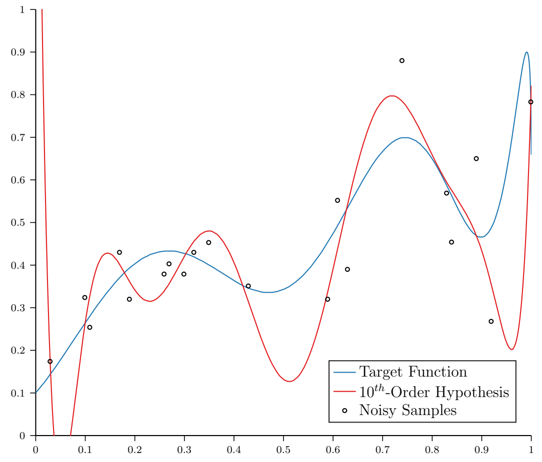
Regularization: Q & A

- Should we regularize the bias/intercept parameter, θ_0?

- Is feature scale a concern with regularization?

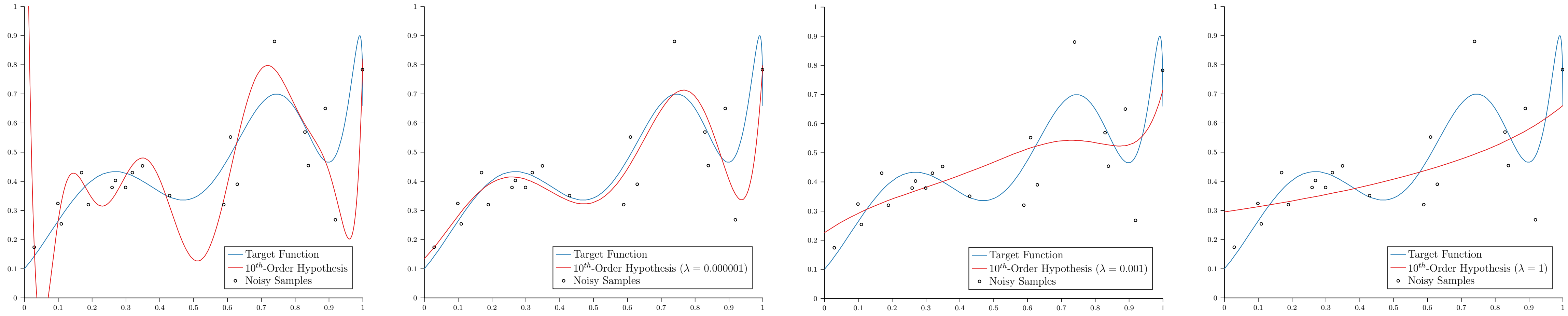
Regularization: Best Practices

- Should we regularize the bias/intercept parameter, θ_0?
  - No!
  - Regularizers typically avoid penalizing this term so that our classifiers can adapt to shifts in the y values
- Is feature scale a concern with regularization?
  - Yes!
  - Features at dramatically different scales might have vastly different coefficient values
  - When using regularization, it is common to *standardize* the features first by subtracting the mean and dividing by the standard deviation



Ridge Regression

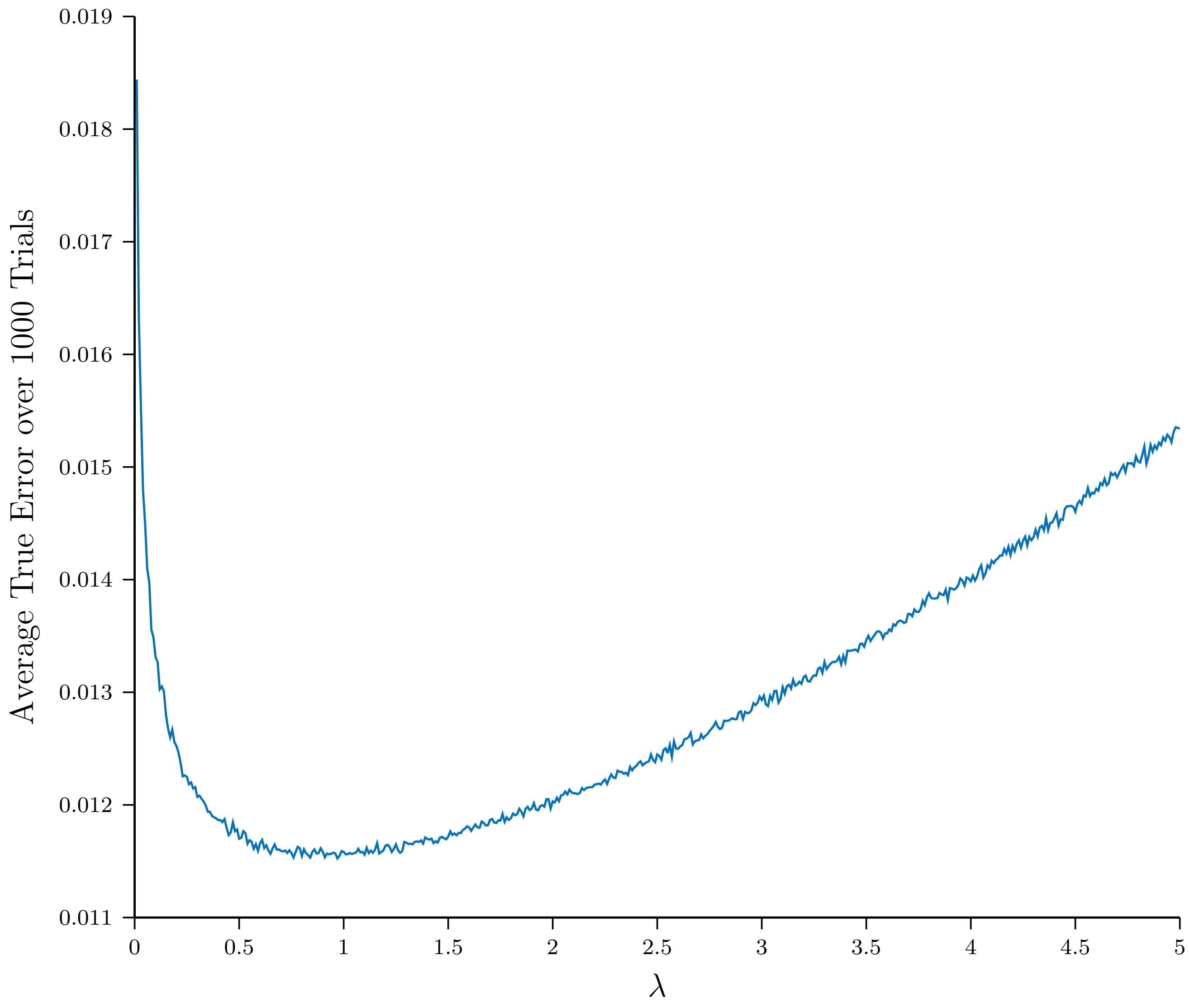
- 10-dimensional target function with additive Gaussian noise
- $\mathcal{H}_{10} = 10^{\text{th}}$-order polynomial



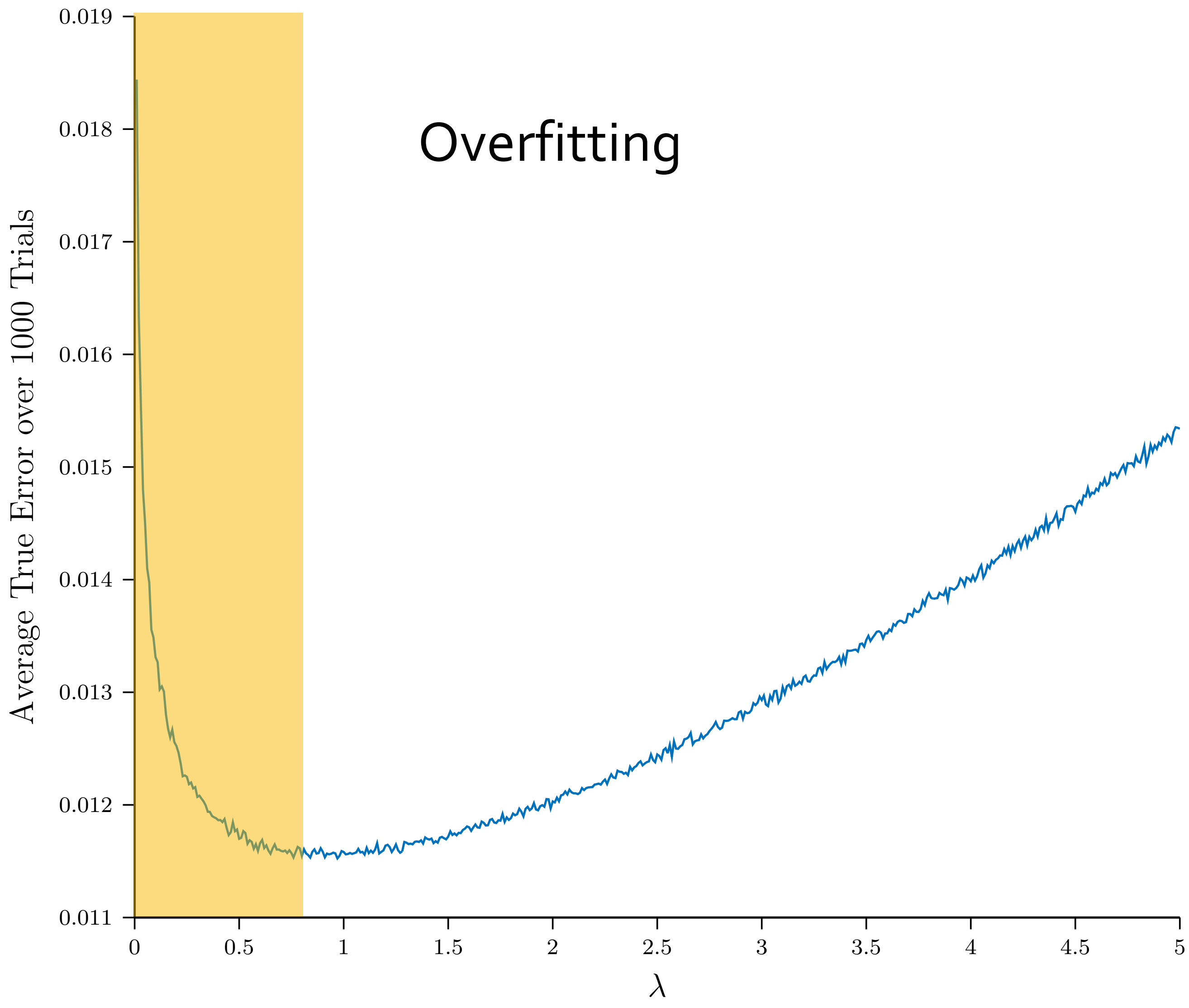
Ridge Regression

	$\lambda_C = 0$	$\lambda_C = 10^{-6}$	$\lambda_C = 10^{-3}$	$\lambda_C = 1$
True Error	0.059	0.006	0.008	0.011
	Overfit	Nice!	Wait...	Underfit

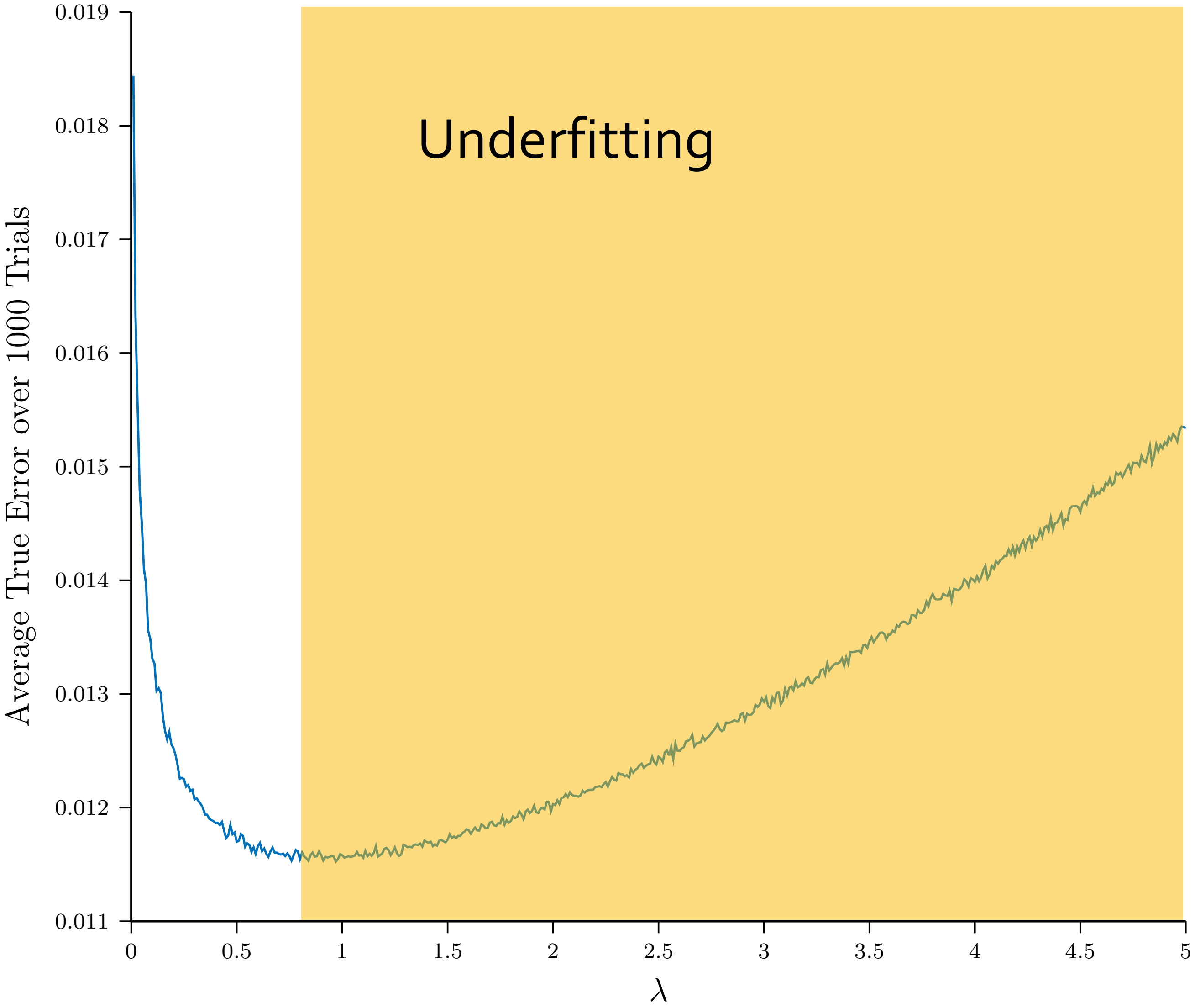
Setting λ



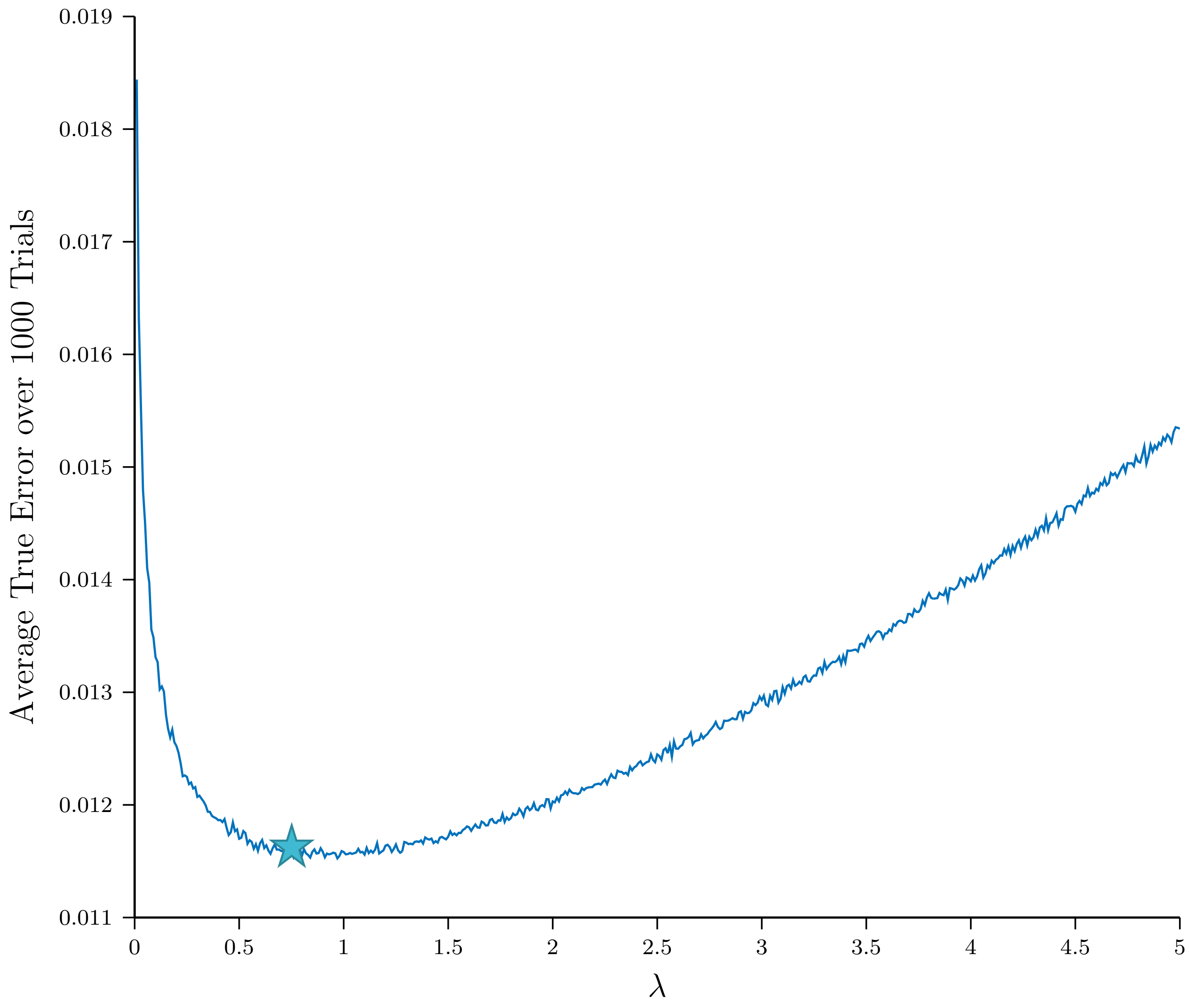
Setting λ



Setting λ



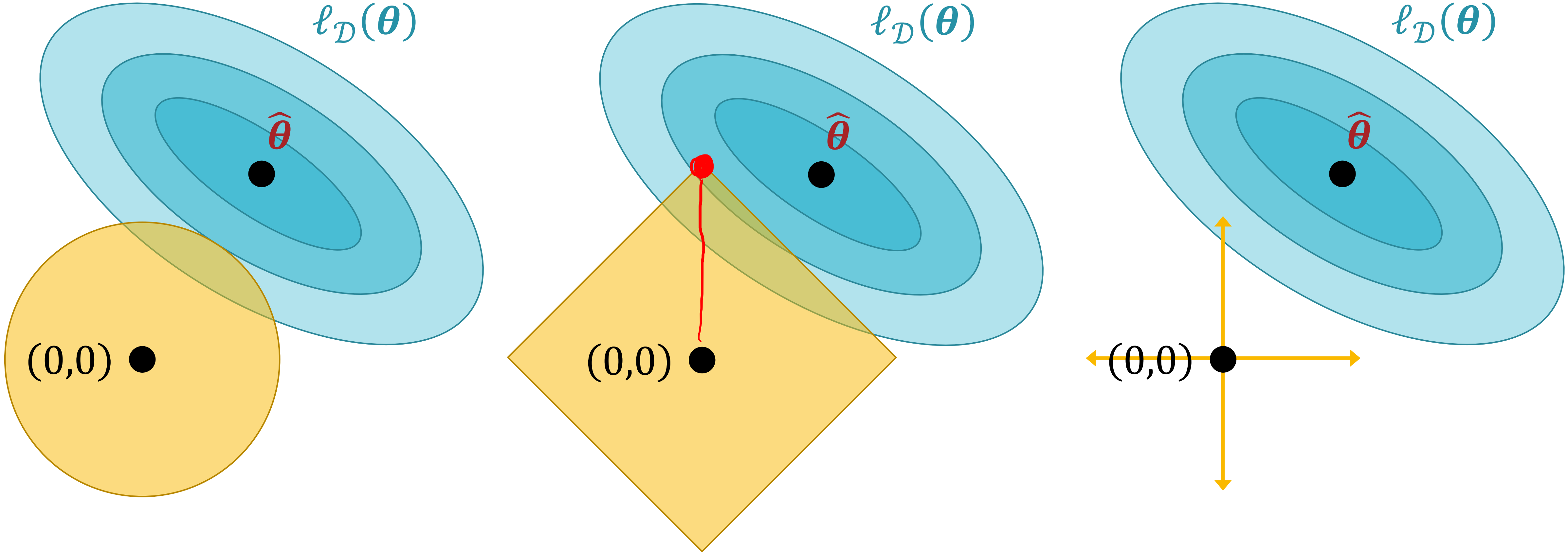
Setting λ



Other Regularizers

intractable to solve in closed-form

$\ell_{\mathcal{D}}(\boldsymbol{\theta}) + \lambda r(\theta)$		
Ridge or $L2$	$r(\boldsymbol{\theta}) = \|\boldsymbol{\theta}\|_2^2 = \sum_{d=0}^{D} \theta_d^2$	Encourages small weights
Lasso or $L1$	$r(\boldsymbol{\theta}) = \|\boldsymbol{\theta}\|_1 = \sum_{d=0}^{D} \lvert\theta_d\rvert$	Encourages sparsity
$L0$	$r(\boldsymbol{\theta}) = \|\boldsymbol{\theta}\|_0 = \sum_{d=0}^{D} \mathbb{1}(\theta_d \neq 0)$	Encourages sparsity (intractable)



Ridge or $L2$ | Lasso or $L1$ | $L0$

Other Regularizers

Regularization Learning Objectives

You should be able to…

- Identify when a model is overfitting
- Add a regularizer to an existing objective in order to combat overfitting
- Explain why we should not regularize the bias term
- Convert linearly inseparable dataset to a linearly separable dataset in higher dimensions