10-301/601: Introduction to Machine Learning Lecture 3 – Decision Trees

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9/6/23

Front Matter

- Announcements:
  - HW1 released 9/1, due 9/6 (today!) at 11:59 PM
    - Reminder: we will grant (basically) any extension requests for this assignment!
  - HW2 released 9/6 (today!), due 9/15 at 11:59 PM
    - Unlike HW1, you will only have:
      - 1 submission for the written portion
      - 10 submissions of the programming portion to our autograder

Q & A:

How do these in-class polls work?

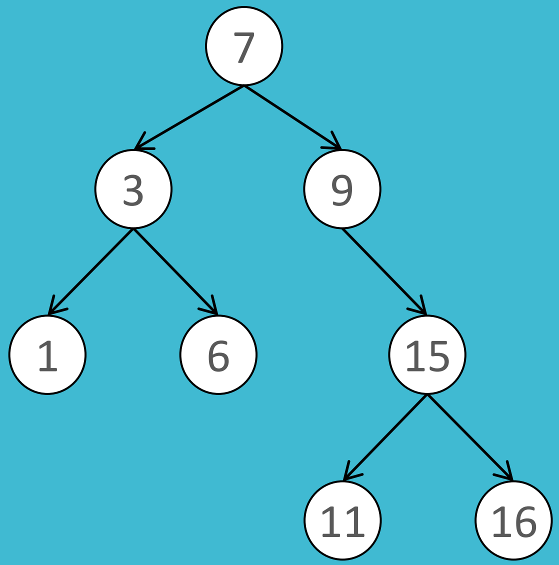
- Open the poll, either by clicking the [Poll] link on the schedule page of our course website or going to http://poll.mlcourse.org
- Sign into Google Forms using your **Andrew email**
- Answer all poll questions **during lecture for full credit** or **within 24 hours for half credit**
- Avoid the **toxic option** (will be clearly specified in lecture) which gives **negative poll points**
- You have 8 free “poll points” for the semester that will excuse you from all polls from a single lecture; you cannot use more than 3 poll points consecutively.

Poll Question 1:

Which of the following did you bring to class today? Select all that apply

A. A smartphone
B. A flip phone
C. A payphone
D. No phone

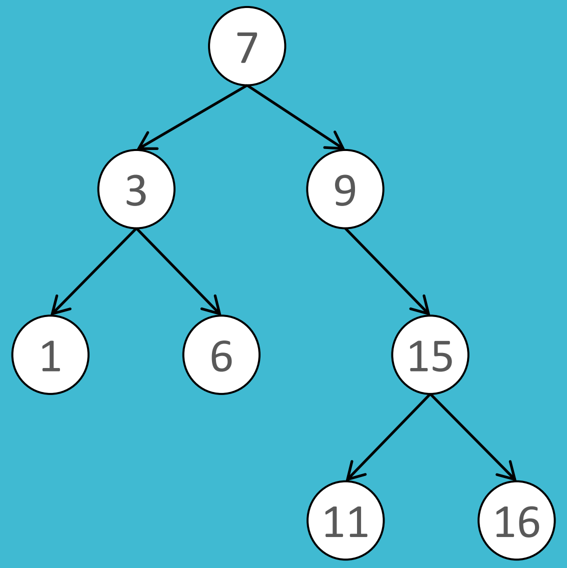
Background: Recursion



- A **binary search tree** (BST) consists of nodes, where each node:
  - has a value, v
  - up to 2 children, a left descendant and a right descendant
  - all its left descendants have values less than v and its right descendants have values greater than v
- We like BSTs because they permit search in O(log(n)) time, assuming n nodes in the tree

```
def contains_iterative(node, key):
    cur = node
    while true:
        if key < cur.value & cur.left != null:
            cur = cur.left
        else if cur.value < key & cur.right != null:
            cur = cur.right
        else:
            break
    return key == cur.value
```

Background: Recursion



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- We like BSTs because they permit search in O(log(n)) time, assuming n nodes in the tree

```
def contains_recursive(node, key):
    if key < node.value & node.left != null:
        return contains(node.left, key)
    else if node.value < key & node.right != null:
        return contains(node.right, key)
    else:
        return key == node.value
```

Recall: Decision Stumps

Algorithm 3: based on a single feature, x_d, predict the most common label in the training dataset among all data points that have the same value for x_d

	y	x_1	x_2	x_3	x_4
predictions	**allergic?**	**hives?**	**sneezing?**	**red eye?**	**has cat?**
−	−	Y	N	N	N
+	−	N	Y	N	N
+	+	Y	Y	N	N
−	−	Y	N	Y	Y
+	+	N	Y	Y	N

Nonzero training error, but perhaps still better than the memorizer

Example decision stump:

$$h(\mathbf{x}) = \begin{cases} + & \text{if sneezing} = Y \\ - & \text{otherwise} \end{cases}$$

But why would we only use just one feature?

Algorithm 3: based on a single feature, x_d, predict the most common label in the training dataset among all data points that have the same value for x_d

	y	x_1	x_2	x_3	x_4
predictions	**allergic?**	**hives?**	**sneezing?**	**red eye?**	**has cat?**
−	−	Y	N	N	N
+	−	N	Y	N	N
+	+	Y	Y	N	N
−	−	Y	N	Y	Y
+	+	N	Y	Y	N

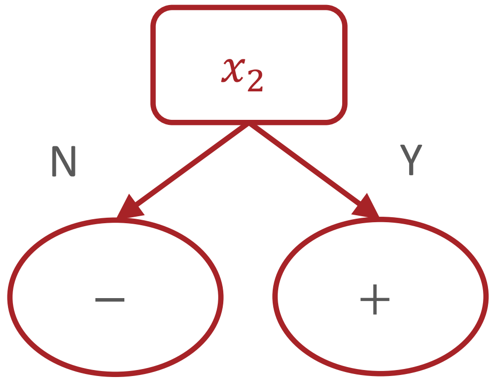
Nonzero training error, but perhaps still better than the memorizer

Example decision stump:

$$h(\mathbf{x}) = \begin{cases} + & \text{if sneezing = Y} \\ - & \text{otherwise} \end{cases}$$

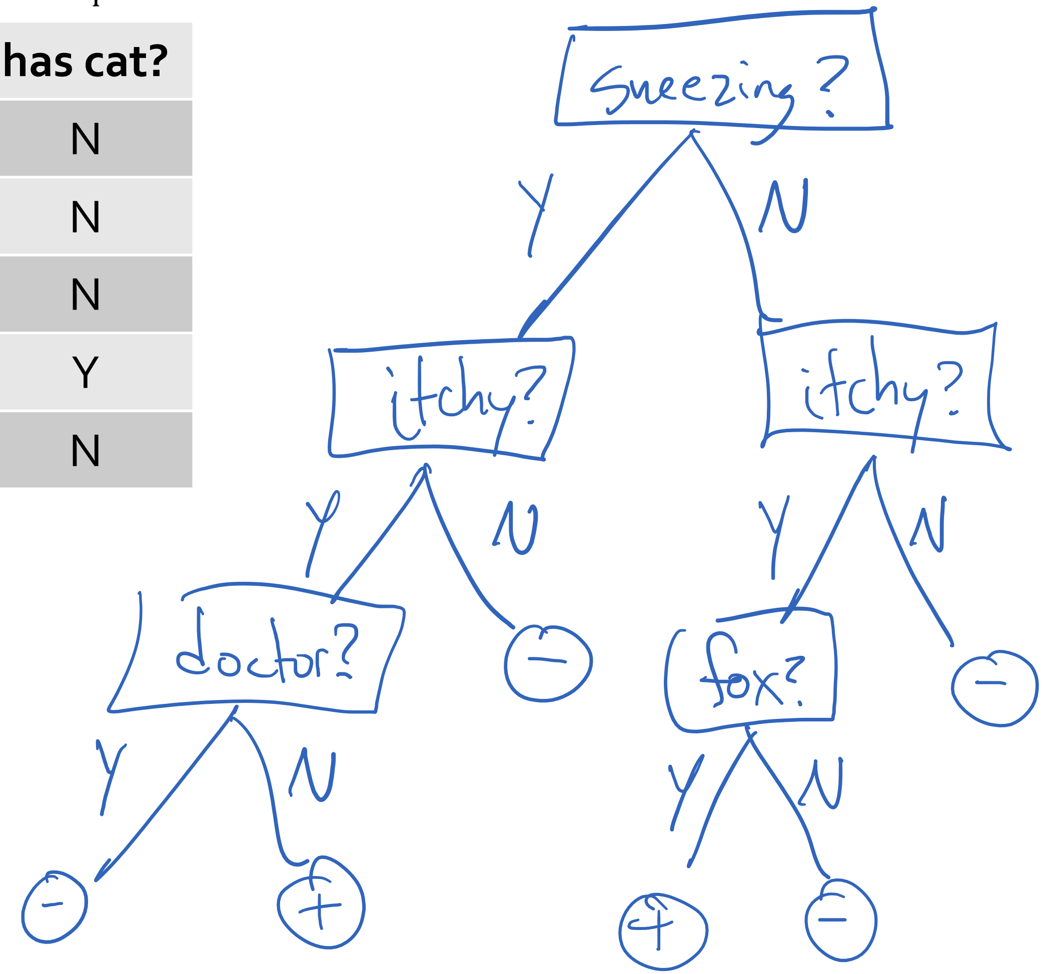
From Decision Stump …

y	x_1	x_2	x_3	x_4
allergic?	hives?	sneezing?	red eye?	has cat?
−	Y	N	N	N
−	N	Y	N	N
+	Y	Y	N	N
−	Y	N	Y	Y
+	N	Y	Y	N



From Decision Stump to Decision Tree

y allergic?	x_1 hives?	x_2 sneezing?	x_3 red eye?	x_4 has cat?
−	Y	N	N	N
−	N	Y	N	N
+	Y	Y	N	N
−	Y	N	Y	Y
+	N	Y	Y	N



Decision Tree: Pseudocode

def $h(\boldsymbol{x}')$:

Decision Tree: Example

Learned from medical records of 1000 women

Negative examples are C-sections

```
[833+,167-] .83+ .17-
Fetal_Presentation = 1: [822+,116-] .88+ .12-
| Previous_Csection = 0: [767+,81-] .90+ .10-
| | Primiparous = 0: [399+,13-] .97+ .03-
| | Primiparous = 1: [368+,68-] .84+ .16-
| | | Fetal_Distress = 0: [334+,47-] .88+ .12-
| | | Fetal_Distress = 1: [34+,21-] .62+ .38-
| Previous_Csection = 1: [55+,35-] .61+ .39-
Fetal_Presentation = 2: [3+,29-] .11+ .89-
Fetal_Presentation = 3: [8+,22-] .27+ .73-
```


Figure courtesy of Tom Mitchell

Decision Tree Questions

1. How can we pick which feature to split on?

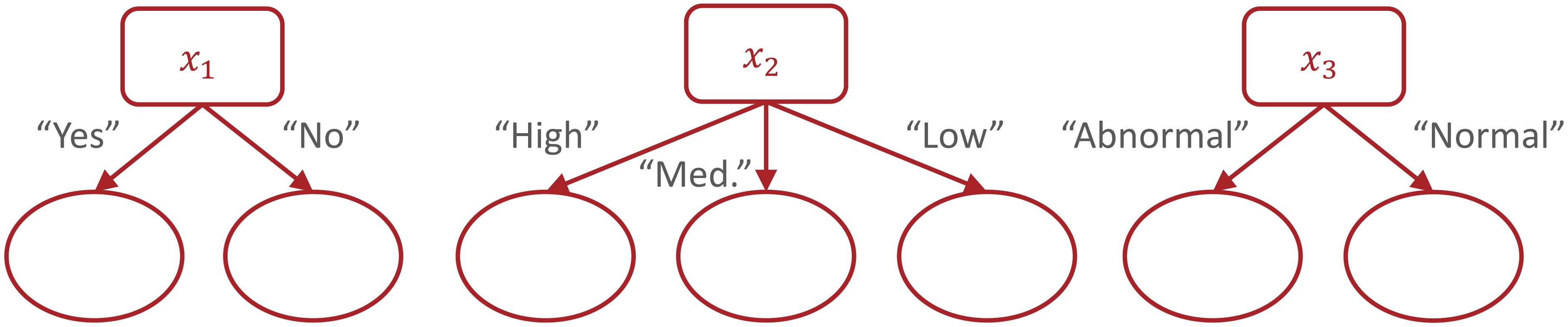
2. How do we pick the order of the splits?

Splitting Criterion

- A **splitting criterion** is a function that measures how good or useful splitting on a particular feature is *for a specified dataset*
- Idea: when deciding which feature to split on, use the one that optimizes the splitting criterion

Training Error Rate as a Splitting Criterion

x_1 Family History	x_2 Resting Blood Pressure	x_3 Cholesterol	y Heart Disease?
Yes	Low	Normal	No
No	Medium	Normal	No
No	Low	Abnormal	Yes
Yes	Medium	Normal	Yes
Yes	High	Abnormal	Yes



Training error rate:

Training error rate:

Training error rate:

Poll Question 2:

Which feature would you split on using training error rate as the splitting criterion?

x_1	x_2	y
1	0	0
1	0	0
1	0	1
1	0	1
1	1	1
1	1	1
1	1	1
1	1	1

A. x_1

B. x_2

C. Either x_1 or x_2

D. Neither x_1 nor x_2

Splitting Criterion

- A **splitting criterion** is a function that measures how good or useful splitting on a particular feature is *for a specified dataset*
- Idea: when deciding which feature to split on, use the one that optimizes the splitting criterion
- Potential splitting criteria:
  - Training error rate (minimize)
  - Gini impurity (minimize) → CART algorithm
  - Mutual information (maximize) → ID3 algorithm

Entropy

- The **entropy** of a *random variable* describes the uncertainty of its outcome: the higher the entropy, the less certain we are about what the outcome will be.

$$H(X) = -\sum_{v \in V(X)} P(X = v) \log_2\big(P(X = v)\big)$$

where X is a (discrete) random variable

$V(X)$ is the set of possible values X can take on

Entropy

- The **entropy** of a *set* describes how uniform or pure it is: the higher the entropy, the more impure or "mixed-up" the set is

$$H(S) = -\sum_{v \in V(S)} \frac{|S_v|}{|S|} \log_2\left(\frac{|S_v|}{|S|}\right)$$

where S is a collection of values,

$V(S)$ is the set of unique values in S

S_v is the collection of elements in S with value v

- If all the elements in S are the same, then

Entropy

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$$H(S) = -\sum_{v \in V(S)} \frac{|S_v|}{|S|} \log_2\left(\frac{|S_v|}{|S|}\right)$$

where S is a collection of values,

$V(S)$ is the set of unique values in S

S_v is the collection of elements in S with value v

- If S is split fifty-fifty between two values, then

Mutual Information

- The **mutual information** between *two random variables* describes how much clarity knowing the value of one random variables provides about the other

$$I(Y;X) = H(Y) - H(Y|X)$$

$$= H(Y) - \sum_{v \in V(X)} P(X = v)H(Y|X = v)$$

where X and Y are random variables

$V(X)$ is the set of possible values X can take on

$H(Y|X = v)$ is the conditional entropy of Y given $X = v$

Mutual Information

- The **mutual information** between *a feature and the label* describes how much clarity knowing the feature provides about the label

$$I(y; x_d) = H(y) - H(y|x_d)$$

$$= H(y) - \sum_{v \in V(x_d)} f_v \left(H\left(Y_{x_d=v}\right)\right)$$

where x_d is a feature and y is the set of all labels

$V(x_d)$ is the set of possible values x_d can take on

f_v is the fraction of data points where $x_d = v$

$Y_{x_d=v}$ is the set of all labels where $x_d = v$

Mutual Information: Example

x_d	y
1	1
1	1
0	0
0	0

Mutual Information: Example

x_d	y
1	1
0	1
1	0
0	0

Poll Question 3:

Which feature would you split on using mutual information as the splitting criterion?

x_1	x_2	y
1	0	0
1	0	0
1	0	1
1	0	1
1	1	1
1	1	1
1	1	1
1	1	1

A. x_1

B. x_2

C. Either x_1 or x_2

D. Neither x_1 nor x_2

Decision Tree Questions

1. How can we pick which feature to split on?

2. How do we pick the order of the splits?

Decision Tree: Pseudocode

```
def train(𝒟):


def tree_recurse(𝒟′):
    q = new node()
    base case – if (SOME CONDITION):
    recursion – else:







    return q
```

Decision Tree: Pseudocode

```
def train(𝒟):
    store root = tree_recurse(𝒟)
def tree_recurse(𝒟′):
    q = new node()
    base case – if




    recursion – else:
    return q
```