Logic and Mechanized Reasoning
Conflict-Driven Clause-Learning Solving

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The Satisfiability (SAT) problem

$$
\begin{gathered}
(p_5 \vee p_8 \vee \neg p_2) \wedge (p_2 \vee \neg p_1 \vee \neg p_3) \wedge (\neg p_8 \vee \neg p_3 \vee \neg p_7) \wedge \\
(\neg p_5 \vee p_3 \vee p_8) \wedge (\neg p_6 \vee \neg p_1 \vee \neg p_5) \wedge (p_8 \vee \neg p_9 \vee p_3) \wedge \\
(p_2 \vee p_1 \vee p_3) \wedge (\neg p_1 \vee p_8 \vee p_4) \wedge (\neg p_9 \vee \neg p_6 \vee p_8) \wedge \\
(p_8 \vee p_3 \vee \neg p_9) \wedge (p_9 \vee \neg p_3 \vee p_8) \wedge (p_6 \vee \neg p_9 \vee p_5) \wedge \\
(p_2 \vee \neg p_3 \vee \neg p_8) \wedge (p_8 \vee \neg p_6 \vee \neg p_3) \wedge (p_8 \vee \neg p_3 \vee \neg p_1) \wedge \\
(\neg p_8 \vee p_6 \vee \neg p_2) \wedge (p_7 \vee p_9 \vee \neg p_2) \wedge (p_8 \vee \neg p_9 \vee p_2) \wedge \\
(\neg p_1 \vee \neg p_9 \vee p_4) \wedge (p_8 \vee p_1 \vee \neg p_2) \wedge (p_3 \vee \neg p_4 \vee \neg p_6) \wedge \\
(\neg p_1 \vee \neg p_7 \vee p_5) \wedge (\neg p_7 \vee p_1 \vee p_6) \wedge (\neg p_5 \vee p_4 \vee \neg p_6) \wedge \\
(\neg p_4 \vee p_9 \vee \neg p_8) \wedge (p_2 \vee p_9 \vee p_1) \wedge (p_5 \vee \neg p_7 \vee p_1) \wedge \\
(\neg p_7 \vee \neg p_9 \vee \neg p_6) \wedge (p_2 \vee p_5 \vee p_4) \wedge (p_8 \vee \neg p_4 \vee p_5) \wedge \\
(p_5 \vee p_9 \vee p_3) \wedge (\neg p_5 \vee \neg p_7 \vee p_9) \wedge (p_2 \vee \neg p_8 \vee p_1) \wedge \\
(\neg p_7 \vee p_1 \vee p_5) \wedge (p_1 \vee p_4 \vee p_3) \wedge (p_1 \vee \neg p_9 \vee \neg p_4) \wedge \\
(p_3 \vee p_5 \vee p_6) \wedge (\neg p_6 \vee p_3 \vee \neg p_9) \wedge (\neg p_7 \vee p_5 \vee p_9) \wedge \\
(p_7 \vee \neg p_5 \vee \neg p_2) \wedge (p_4 \vee p_7 \vee p_3) \wedge (p_4 \vee \neg p_9 \vee \neg p_7) \wedge \\
(p_5 \vee \neg p_1 \vee p_7) \wedge (p_5 \vee \neg p_1 \vee p_7) \wedge (p_6 \vee p_7 \vee \neg p_3) \wedge \\
(\neg p_8 \vee \neg p_6 \vee \neg p_7) \wedge (p_6 \vee p_2 \vee p_3) \wedge (\neg p_8 \vee p_2 \vee p_5)
\end{gathered}
$$

Does there exist an assignment satisfying all clauses?

Search for a satisfying assignment (or proof none exists)

$$
\begin{gathered}
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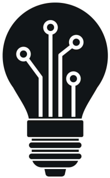
\end{gathered}
$$

SAT Solver Paradigms Overview



DPLL: Aims at finding a small search-tree by selecting effective splitting variables (e.g. via looking ahead).

Strength: Effective on small, hard formulas.

Weakness: Expensive.

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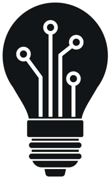
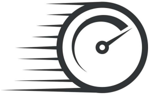


Local search: Given a full assignment for a formula Γ, flip the truth values of variables until satisfying Γ.

Strength: Can quickly find solutions for hard formulas.

Weakness: Cannot prove unsatisfiability.



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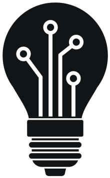

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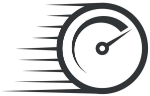
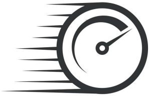


Conflict-driven clause learning (CDCL): Makes fast decisions and converts conflicts into learned clauses.

Strength: Effective on large, "easy" formulas.

Weakness: Hard to parallelize.



Conflict-driven Clause Learning Highlights

- Most successful architecture

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- Superior on industrial benchmarks

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Conflict-driven Clause Learning Highlights

- Most successful architecture
- Superior on industrial benchmarks
- Brute-force?
  - Addition conflict clauses
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- Complete local search (for a refutation)?
- State-of-the-art (sequential) CDCL solvers: CaDiCaL, Glucose, CryptoMiniSAT

Clause Learning

Data-structures

Heuristics

Proofs of Unsatisfiability

Clause Learning

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Proofs of Unsatisfiability

Conflict-driven SAT solvers: Search and Analysis

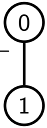
$$
\begin{aligned}
&(p_1 \vee p_4) \wedge \\
&(p_3 \vee \neg p_4 \vee \neg p_5) \wedge \\
&(\neg p_3 \vee \neg p_2 \vee \neg p_4) \wedge \\
&\Gamma_{\text{extra}}
\end{aligned}
$$

0

Conflict-driven SAT solvers: Search and Analysis

$(p_1 \lor p_4) \land$
$(p_3 \lor \neg p_4 \lor \neg p_5) \land$
$(\neg p_3 \lor \neg p_2 \lor \neg p_4) \land$
Γ_{extra}

$p_5 = \top$



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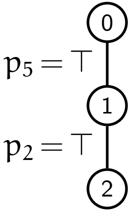

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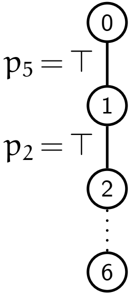

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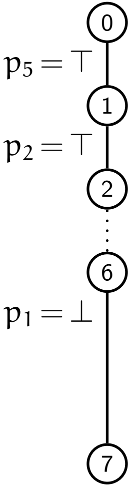

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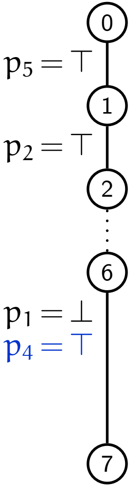

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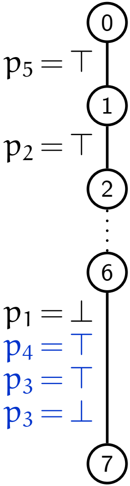

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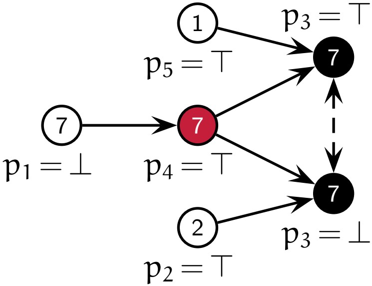

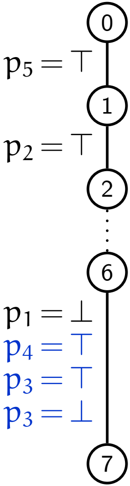

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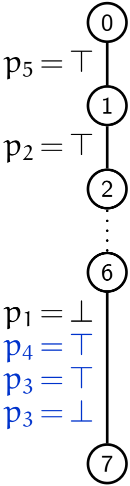

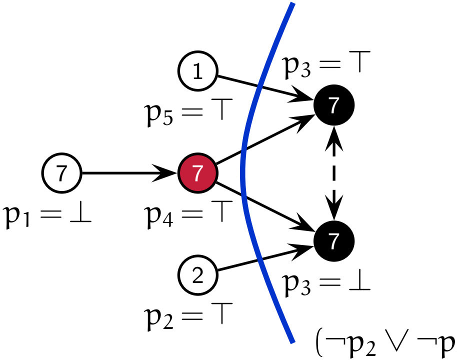

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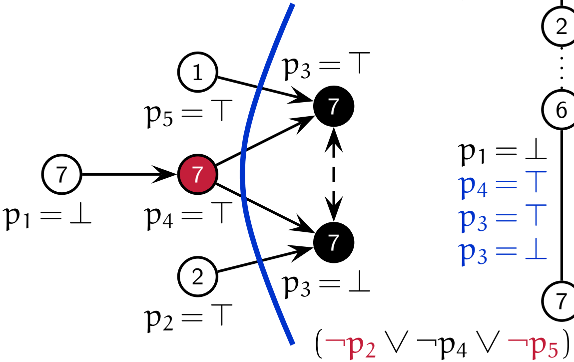

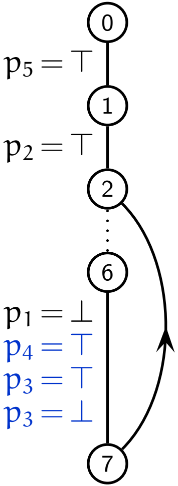

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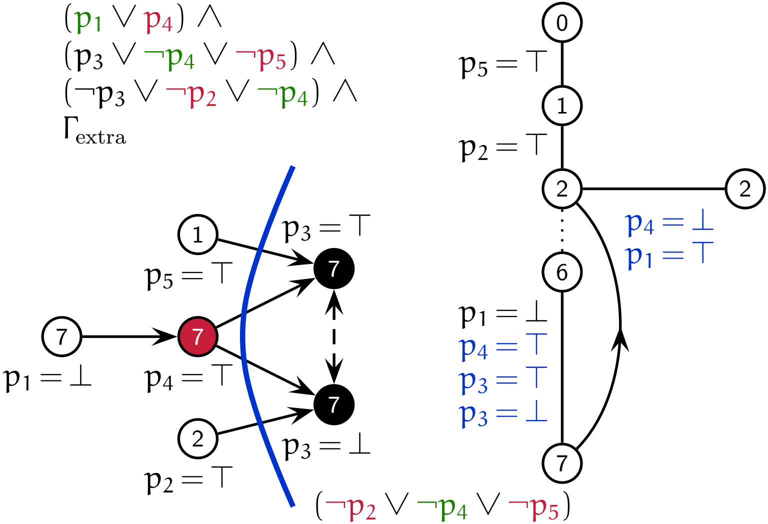

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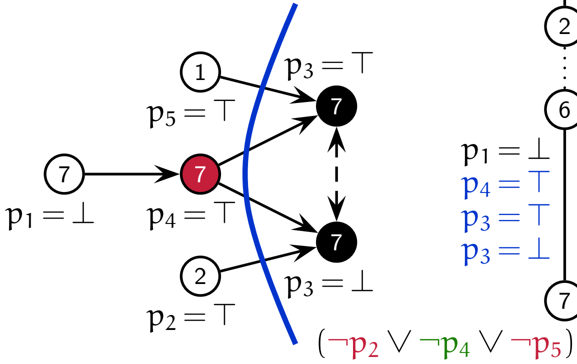

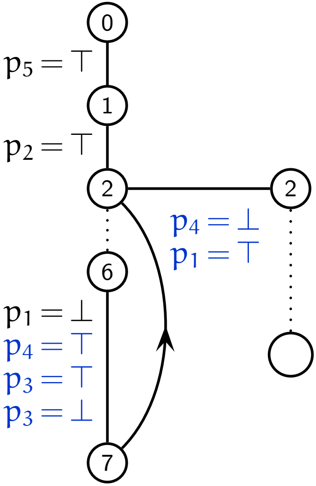

Implication graph [Marques-SilvaSakallah '96]

CDCL in a nutshell:

1. Main loop combines efficient problem simplification with cheap, but effective decision heuristics; ($> 90\%$ of time)
2. Reasoning kicks in if the current state is conflicting;
3. The current state is analyzed and turned into a constraint;
4. The constraint is added to the problem, the heuristics are updated, and the algorithm (partially) restarts.

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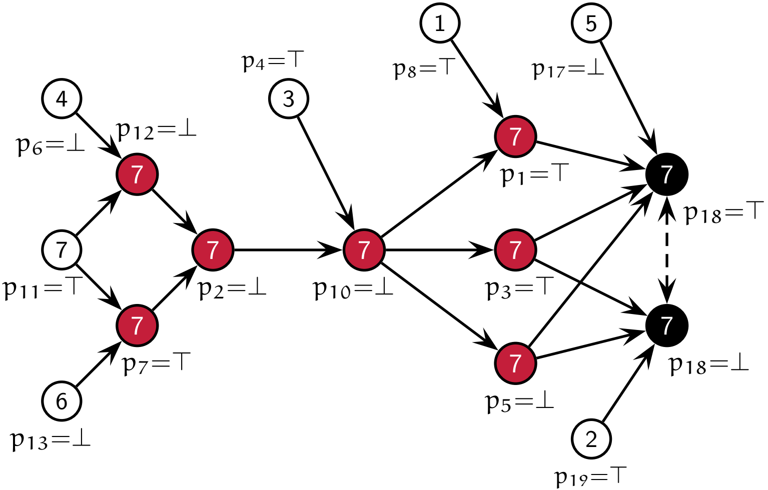
However, it has three weaknesses:

- CDCL is notoriously hard to parallelize;
- the representation impacts CDCL performance; and
- CDCL has exponential runtime on some "simple" problems.

Conflict-driven Clause Learning: Pseudo-code

while TRUE **do**
    $l_{\text{decision}} :=$ Decide ()
    If no l_{decision} **then return** `satisfiable`
    $\tau :=$ Simplify $(\tau \cup (l_{\text{decision}} = \top), \Gamma)$
    while $[\![\Gamma]\!]_\tau$ contains $C_{\text{falsified}}$ **do**
        $C_{\text{conflict}} :=$ Analyze $(C_{\text{falsified}}, \tau)$
        If $C_{\text{conflict}} = \bot$ **then return** `unsatisfiable`
        $\Gamma := \Gamma \cup \{C_{\text{conflict}}\}$
        $\tau :=$ BackTrack $(\tau, C_{\text{conflict}})$
        $\tau :=$ Simplify (τ, Γ)
    end while
end while

Learning conflict clauses [Marques-SilvaSakallah'96]



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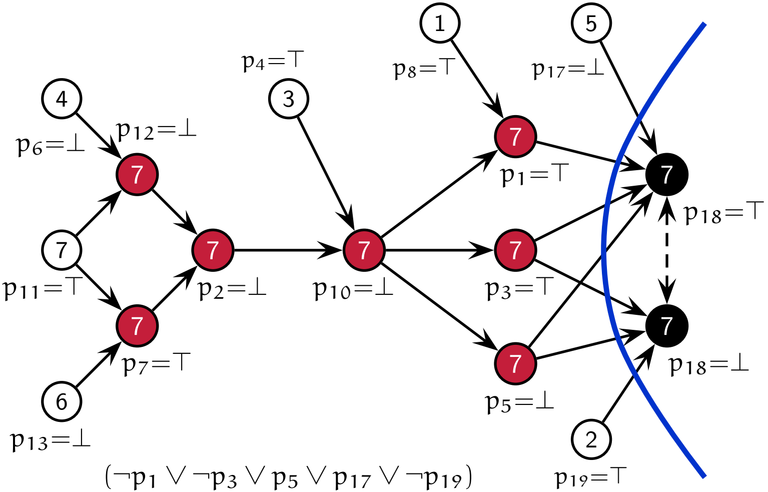

$(\neg p_1 \vee \neg p_3 \vee p_5 \vee p_{17} \vee \neg p_{19})$

tri-asserting clause

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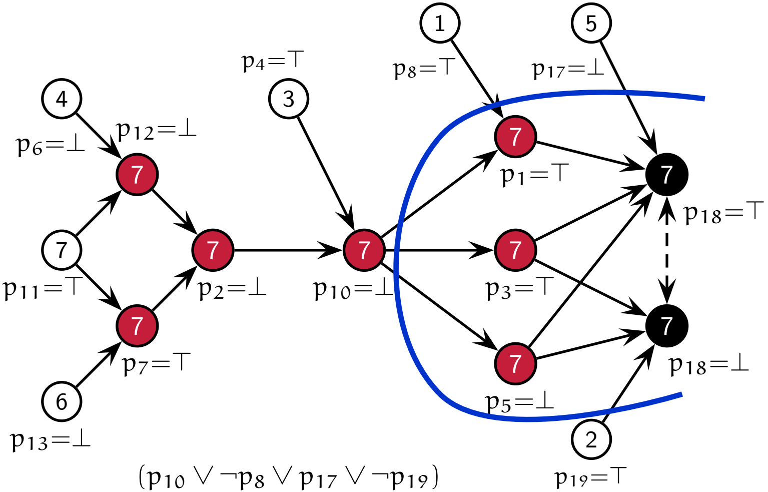

$(p_{10} \vee \neg p_8 \vee p_{17} \vee \neg p_{19})$

first unique implication point

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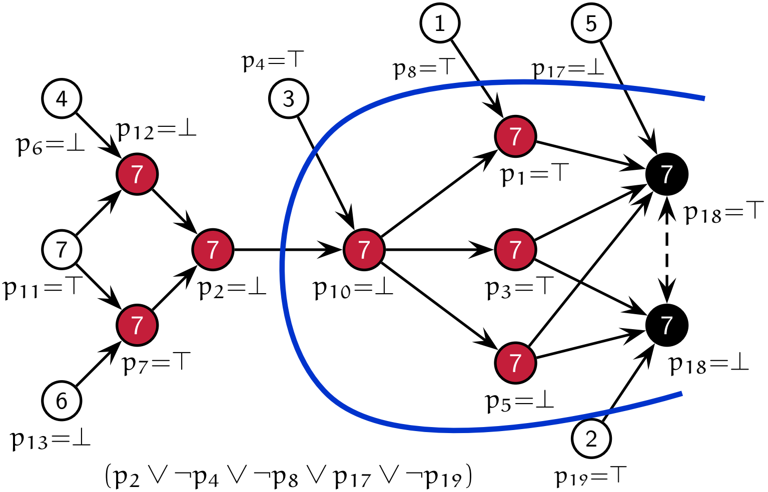
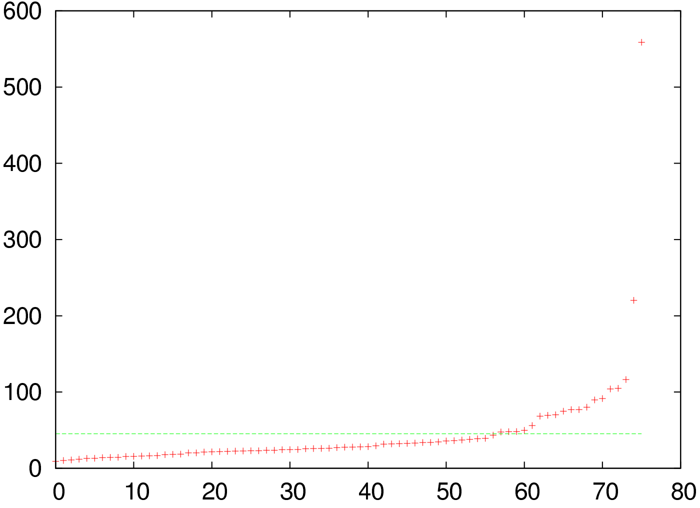


$(p_2 \vee \neg p_4 \vee \neg p_8 \vee p_{17} \vee \neg p_{19})$

second unique implication point

Average Learned Clause Length



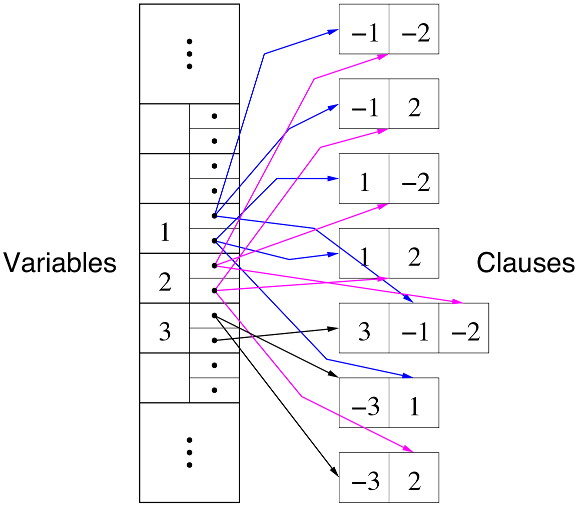
Clause Learning

Data-structures

Heuristics

Proofs of Unsatisfiability

Simple data structure for unit propagation



Conflict-driven: Watch pointers (1) [MoskewiczMZZM'01]

$$\tau = \{p_1 = *, p_2 = *, p_3 = *, p_4 = *, p_5 = *, p_6 = *\}$$

$\neg p_1$	p_2	$\neg p_3$	$\neg p_5$	p_6

p_1	$\neg p_3$	p_4	$\neg p_5$	$\neg p_6$

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$$\tau = \{p_1 = *, p_2 = *, p_3 = *, p_4 = *, p_5 = \top, p_6 = *\}$$

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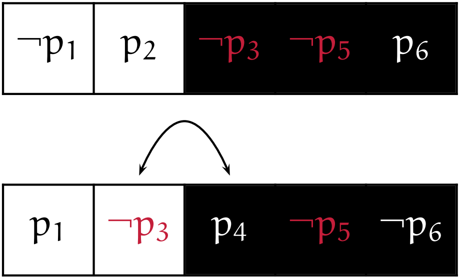

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p_6	p_2	$\neg p_3$	$\neg p_5$	$\neg p_1$

p_1	p_4	$\neg p_3$	$\neg p_5$	$\neg p_6$

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p_6	p_2	$\neg p_3$	$\neg p_5$	$\neg p_1$

p_1	p_4	$\neg p_3$	$\neg p_5$	$\neg p_6$

Conflict-driven: Watch pointers (1) [MoskewiczMZZM'01]

$$\tau = \{p_1 = \top, p_2 = \bot, p_3 = \top, p_4 = \bot, p_5 = \top, p_6 = *\}$$

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Conflict-driven: Watch pointers (2) [MoskewiczMZZM'01]

Only examine (get in the cache) a clause when both

- a watch pointer gets falsified
- the other one is not satisfied

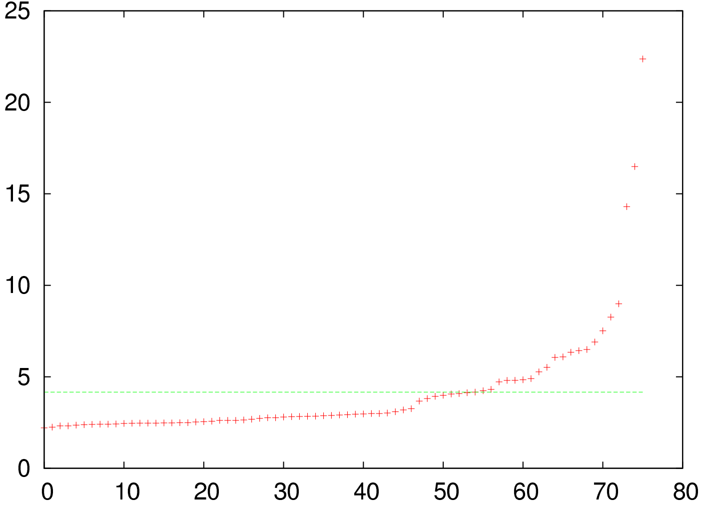
While backjumping, just unassign variables

Conflict clauses $\rightarrow$ watch pointers

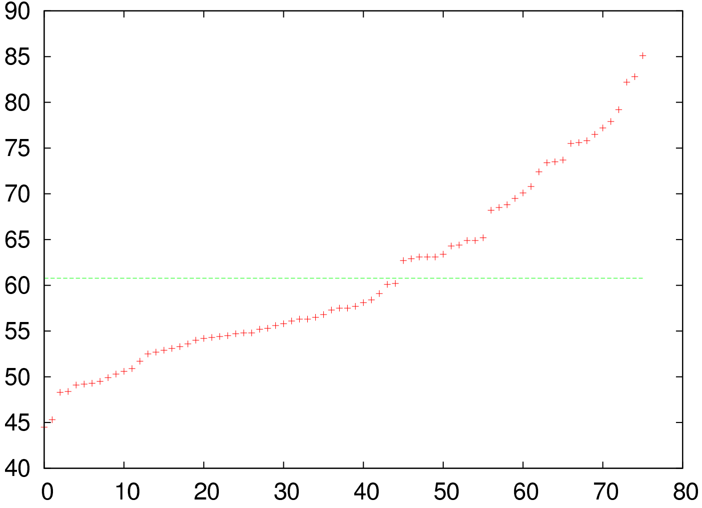
No detailed information available

Not used for binary clauses

Average Number Clauses Visited Per Propagation



Percentage visited clauses with other watched literal true



Clause Learning

Data-structures

Heuristics

Proofs of Unsatisfiability

Most important CDCL heuristics

Variable selection heuristics

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Restart strategies

- aim: avoid heavy-tail behavior [GomesSelmanCrato'97]
- plus: focus search on recent conflicts when combined with dynamic heuristics

Variable selection heuristics

Based on the occurrences in the (reduced) formula

- examples: Jeroslow-Wang, Maximal Occurrence in clauses of Minimal Size (MOMS), look-aheads
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Variable State Independent Decaying Sum (VSIDS)

- original idea (zChaff): for each conflict, increase the score of involved variables by 1, half all scores each 256 conflicts [MoskewiczMZZM'01]
- improvement (MiniSAT): for each conflict, increase the score of involved variables by δ and increase $\delta := 1.05\delta$ [EenSörensson'03]

Visualization of VSIDS in PicoSAT

http://www.youtube.com/watch?v=MOjhFywLre8

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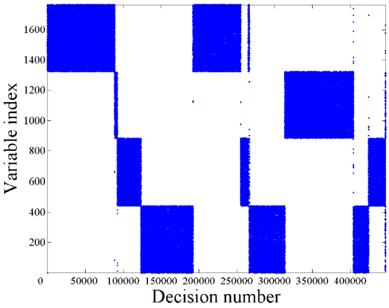
- negative branching (early MiniSAT) [EenSörensson'03]

Based on the last implied value (phase-saving)

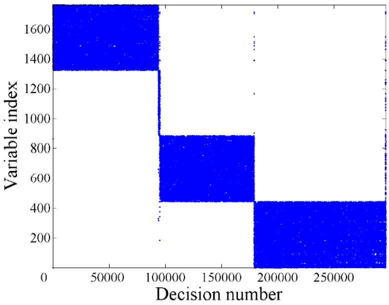
- introduced to CDCL [PipatsrisawatDarwiche'07]
- already used in local search [HirschKojevnikov'01]

Heuristics: Phase-saving [PipatsrisawatDarwiche'07]

Selecting the last implied value remembers solved components



negative branching



phase-saving

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Restarts in CDCL solvers:

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- Geometrical restart: e.g. $100, 150, 225, 333, 500, 750, \ldots$
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Rapid restarts by reusing trail: [vanderTakHeuleRamos'11]

- Partial restart same effect as full restart
- Optimal strategy Luby-1: $1, 1, 2, 1, 1, 2, 4, \ldots$

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Proofs of Unsatisfiability

Motivation for Proofs of Unsatisfiability

SAT solvers may have errors and only return yes/no.

- Documented bugs in SAT, SMT, and QSAT solvers;
  [Brummayer and Biere, 2009; Brummayer et al., 2010]
- Competition winners have contradictory results
  (HWMCC winners from 2011 and 2012)
- Implementation errors often imply conceptual errors;
- Proofs now mandatory for the annual SAT Competitions;
- Mathematical results require a stronger justification than a simple yes/no by a solver. UNSAT must be verifiable.

Clausal Proofs of Unsatisfiability

Reduce the size of the proof by only storing added clauses

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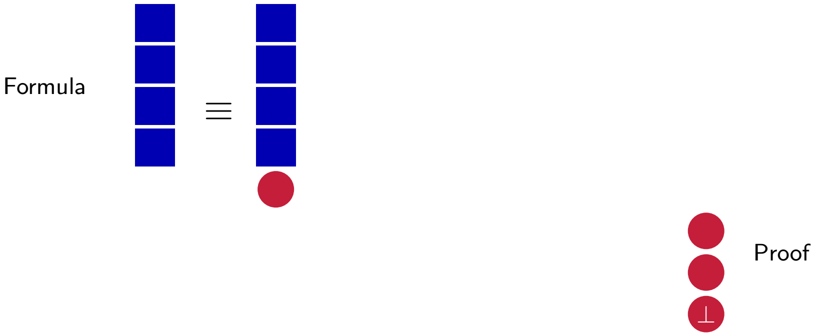

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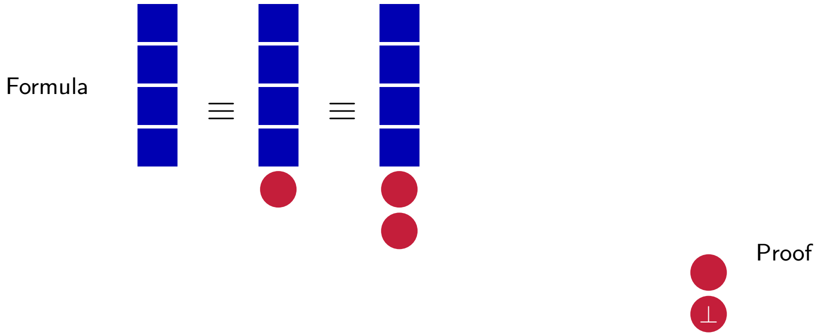

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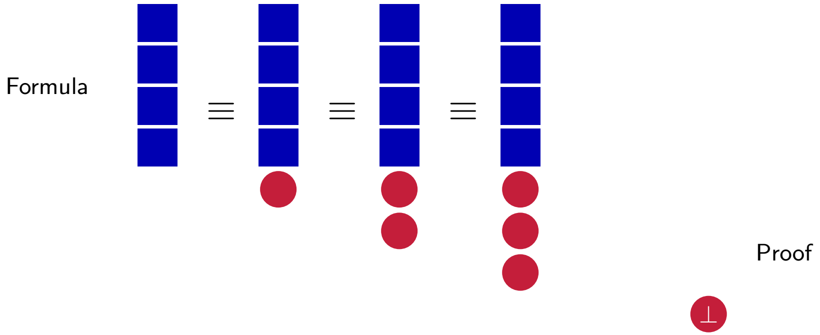

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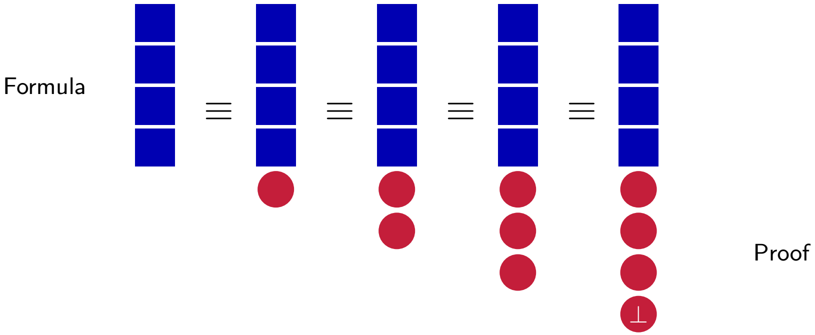

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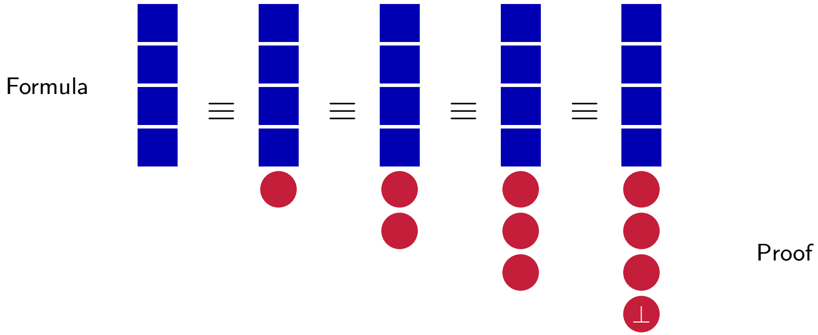

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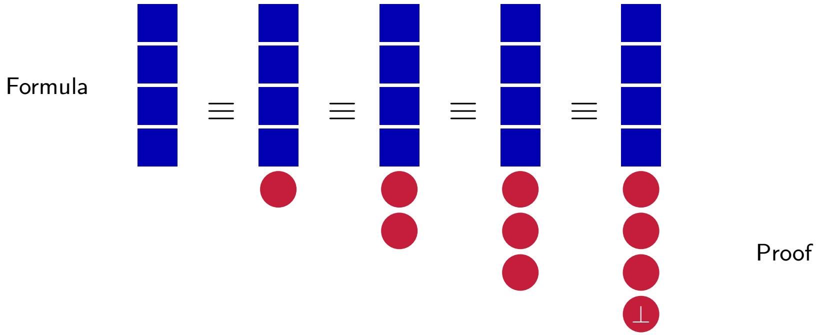

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- Checking redundancy should be efficient.
- Proof systems for this purpose in upcoming lectures.