Assignment 3

due Wednesday, September 14, 2022

Remember that homework is due at midnight on the due date.

Problem 1 (3 points)

Use structural induction to prove that for every ℓ we have

$$\text{length}(\text{reverse}(\ell)) = \text{length}(\ell).$$

You can use either the definition of reverse or reverse′ from the textbook.

Problem 2 (3 points)

In class we defined size(t) and depth(t) for any (extended) binary tree t. Using those definitions, prove by structural recursion that for any tree t, we have $\text{size}(t) \le 2^{\text{depth}(t)} - 1$.

Problem 3 (4 points)

Recall the MU puzzle by Douglas Hofstadter from the textbook and the lecture. It concerns strings consisting of the letters M, I, and U. Starting with the string MI, we are allowed to apply any of the following rules:

1. Replace xI by xIU, that is, add a U to the end of any string that ends with I.
2. Replace Mx by Mxx, that is, double the string after the initial M.
3. Replace xIIIy by xUy, that is, replace any three consecutive Is with a U.
4. Replace xUUy by xy, that is, delete any consecutive pair of Us.

Prove that you can produce any string MI...I, that is, M followed by only Is, if the number of Is is not $0 \pmod 3$ when starting with string MI.

Problem 4 (3 points)

Using operations on List, write a Lean function that for every n returns the list of all the divisors of n that are less than n.

Problem 5 (4 points)

A natural number n is *perfect* if it is equal to the sum of the divisors less than n. Write a Lean function (with return type Bool) that determines whether a number n is perfect. Use it to find all the perfect numbers less than 1,000.

Problem 6 (3 points)

Define a recursive function sublists(ℓ) in Lean that, for every list ℓ, returns a list of all the sublists of ℓ. For example, given the list $[1, 2, 3]$, it should compute the list

$$[[], [1], [2], [3], [1, 2], [1, 3], [2, 3], [1, 2, 3]].$$

The elements need not be listed in that same order.