

Assignment 8

due Wednesday, November 2, 2022

Problem 1 (20 points)

The game Number Mind is a variant of the well known game Master Mind by Project Euler.¹ Instead of colored pegs, you have to guess a secret sequence of digits. After each guess you are only told in how many places you have guessed the correct digit. So, if the sequence was 1234 and you guessed 2036, you would be told that you have one correct digit; however, you would **not** be told that you also have another digit in the wrong place.

For instance, given the following guesses for a 5-digit secret sequence,

```
90342 ;2 correct
70794 ;0 correct
39458 ;2 correct
34109 ;1 correct
51545 ;2 correct
12531 ;1 correct
```

The correct sequence 39542 is unique.

Based on the following guesses,

```
5616185650518293 ;2 correct
3847439647293047 ;1 correct
5855462940810587 ;3 correct
9742855507068353 ;3 correct
4296849643607543 ;3 correct
3174248439465858 ;1 correct
4513559094146117 ;2 correct
7890971548908067 ;3 correct
8157356344118483 ;1 correct
2615250744386899 ;2 correct
8690095851526254 ;3 correct
6375711915077050 ;1 correct
6913859173121360 ;1 correct
6442889055042768 ;2 correct
2321386104303845 ;0 correct
2326509471271448 ;2 correct
5251583379644322 ;2 correct
1748270476758276 ;3 correct
4895722652190306 ;1 correct
3041631117224635 ;3 correct
1841236454324589 ;3 correct
2659862637316867 ;2 correct
```

the goal of the assignment is to find the unique 16-digit secret sequence.

¹<https://projecteuler.net/problem=185>

A) (6 points) Write in Lean a function that takes as input a `List Lit` and a natural number k and returns a propositional formula of Lean data type `CnfForm` that is satisfiable if and only if exactly k literals of list are satisfied. (Hint: The easiest way to encode that exactly k literals are true is by splitting it in at-least k are true and at-most k are true. No auxiliary propositional variables are required.)

B) (5 points) Write a Lean program that can solve Number Mind puzzles. Use propositional variables $p_{i,j}$ which are true if and only if at position i there is the digit j . No other variables are required.

The input is a list of exactly- k constraints with a constraint being a list of literals and a number k . The meaning of a constraint is that exactly k of the literals are true. The program should return a formula of Lean data type `CnfForm` that is satisfiable if and only if all constraints are satisfied. Note that Number Mind puzzles require i) constraints enforcing how many digits are correct **and** ii) constraints enforcing that for each position exactly 1 digit can be present in a solution.

The template file `assignment8.lean` provides encodings of the two games above.

C) (4 points) Run the program to solve the Number Mind puzzle shown above with 16 digits using the CaDiCaL interface. Transform the solution into a sequence of digits.

D) (5 points) The solution of the Number Mind puzzle shown above is claimed to be unique. Show that this claim is correct. Please explain your approach, implement it, and solve it. (Hint: showing uniqueness cannot be done using a single propositional formula. You will need to construct an additional, but closely related, formula of the one solved in C).