Logic and Mechanized Reasoning
Conflict-Driven Clause-Learning Solving

Marijn J.H. Heule

Carnegie Mellon University

First Midterm Exam

Monday February 19 at 12:30pm in NSH 1305 and GHC 4301

Material covered in the exam:

- All lectures up to (and including) February 7
- All homework through Assignment 4
- Textbook chapters 1-7, excluding Sections 6.3, 6.5, 7.4

Practice exam and solutions on course website

No new homework assigned this week

Extra office hours:

- Josh: Saturday from 5-6pm
- Tika: Sunday from 2-3pm
- Alex: Sunday from 6-7pm
- Joseph: Monday from 10:30-11:30am

The Satisfiability (SAT) problem

$$
\begin{array}{l}
(p_5 \lor p_8 \lor \neg p_2) \land (p_2 \lor \neg p_1 \lor \neg p_3) \land (\neg p_8 \lor \neg p_3 \lor \neg p_7) \land \\
(\neg p_5 \lor p_3 \lor p_8) \land (\neg p_6 \lor \neg p_1 \lor \neg p_5) \land (p_8 \lor \neg p_9 \lor p_3) \land \\
(p_2 \lor p_1 \lor p_3) \land (\neg p_1 \lor p_8 \lor p_4) \land (\neg p_9 \lor \neg p_6 \lor p_8) \land \\
(p_8 \lor p_3 \lor \neg p_9) \land (p_9 \lor \neg p_3 \lor p_8) \land (p_6 \lor \neg p_9 \lor p_5) \land \\
(p_2 \lor \neg p_3 \lor \neg p_8) \land (p_8 \lor \neg p_6 \lor \neg p_3) \land (p_8 \lor \neg p_3 \lor \neg p_1) \land \\
(\neg p_8 \lor p_6 \lor \neg p_2) \land (p_7 \lor p_9 \lor \neg p_2) \land (p_8 \lor \neg p_9 \lor p_2) \land \\
(\neg p_1 \lor \neg p_9 \lor p_4) \land (p_8 \lor p_1 \lor \neg p_2) \land (p_3 \lor \neg p_4 \lor \neg p_6) \land \\
(\neg p_1 \lor \neg p_7 \lor p_5) \land (\neg p_7 \lor p_1 \lor p_6) \land (\neg p_5 \lor p_4 \lor \neg p_6) \land \\
(\neg p_4 \lor p_9 \lor \neg p_8) \land (p_2 \lor p_9 \lor p_1) \land (p_5 \lor \neg p_7 \lor p_1) \land \\
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(p_5 \lor p_9 \lor p_3) \land (\neg p_5 \lor \neg p_7 \lor p_9) \land (p_2 \lor \neg p_8 \lor p_1) \land \\
(\neg p_7 \lor p_1 \lor p_5) \land (p_1 \lor p_4 \lor p_3) \land (p_1 \lor \neg p_9 \lor \neg p_4) \land \\
(p_3 \lor p_5 \lor p_6) \land (\neg p_6 \lor p_3 \lor \neg p_9) \land (\neg p_7 \lor p_5 \lor p_9) \land \\
(p_7 \lor \neg p_5 \lor \neg p_2) \land (p_4 \lor p_7 \lor p_3) \land (p_4 \lor \neg p_9 \lor \neg p_7) \land \\
(p_5 \lor \neg p_1 \lor p_7) \land (p_5 \lor \neg p_1 \lor p_7) \land (p_6 \lor p_7 \lor \neg p_3) \land \\
(\neg p_8 \lor \neg p_6 \lor \neg p_7) \land (p_6 \lor p_2 \lor p_3) \land (\neg p_8 \lor p_2 \lor p_5)
\end{array}
$$

Does there exist an assignment satisfying all clauses?

Search for a satisfying assignment (or proof none exists)

$$
\begin{array}{l}
(p_5 \vee p_8 \vee \neg p_2) \wedge (p_2 \vee \neg p_1 \vee \neg p_3) \wedge (\neg p_8 \vee \neg p_3 \vee \neg p_7) \wedge \\
(\neg p_5 \vee p_3 \vee p_8) \wedge (\neg p_6 \vee \neg p_1 \vee \neg p_5) \wedge (p_8 \vee \neg p_9 \vee p_3) \wedge \\
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(p_2 \vee \neg p_3 \vee \neg p_8) \wedge (p_8 \vee \neg p_6 \vee \neg p_3) \wedge (p_8 \vee \neg p_3 \vee \neg p_1) \wedge \\
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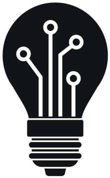
$$

SAT Solver Paradigms Overview



DPLL: Aims at finding a small search-tree by selecting effective splitting variables (e.g. via looking ahead).

Strength: Effective on small, hard formulas.

Weakness: Expensive.

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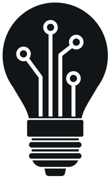
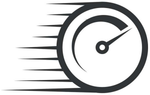


Local search: Given a full assignment for a formula Γ, flip the truth values of variables until satisfying Γ.

Strength: Can quickly find solutions for hard formulas.

Weakness: Cannot prove unsatisfiability.



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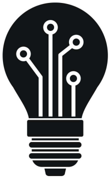

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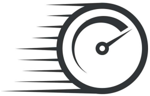
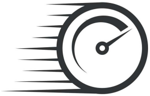


Conflict-driven clause learning (CDCL): Makes fast decisions and converts conflicts into learned clauses.

Strength: Effective on large, "easy" formulas.

Weakness: Hard to parallelize.



Conflict-driven Clause Learning Highlights

- Most successful architecture

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- Superior on industrial benchmarks

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  - Addition conflict clauses
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Conflict-driven Clause Learning Highlights

- Most successful architecture
- Superior on industrial benchmarks
- Brute-force?
  - Addition conflict clauses
  - Fast unit propagation
- Complete local search (for a refutation)?
- State-of-the-art (sequential) CDCL solvers: CaDiCaL, Glucose, CryptoMiniSAT

Clause Learning

Data-structures

Heuristics

Proofs of Unsatisfiability

Clause Learning

Data-structures

Heuristics

Proofs of Unsatisfiability

Conflict-driven SAT solvers: Search and Analysis

$(p_1 \vee p_4) \wedge$
$(p_3 \vee \neg p_4 \vee p_5) \wedge$
$(\neg p_2 \vee \neg p_3 \vee \neg p_4) \wedge$
Γ_{extra}

0

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$$
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(p_1 \lor p_4) \land \\
(p_3 \lor \neg p_4 \lor p_5) \land \\
(\neg p_2 \lor \neg p_3 \lor \neg p_4) \land \\
\Gamma_{\mathrm{extra}}
\end{array}
$$

$$
\begin{tikzcd}
0 \arrow[d, "p_5 = \perp"'] \\
1
\end{tikzcd}
$$

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$(p_3 \vee \neg p_4 \vee p_5) \wedge$
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Γ_{extra}

$$\begin{tikzcd} \textcircled{0} \arrow[d, "p_5 = \bot"'] \\ \textcircled{1} \arrow[d, "p_2 = \top"'] \\ \textcircled{2} \end{tikzcd}$$

$(p_1 \lor p_4) \land$
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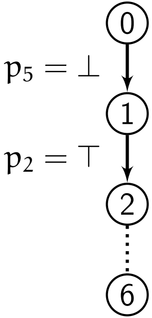

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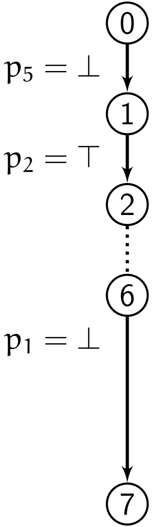

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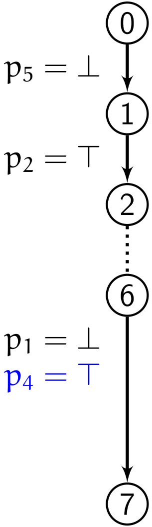

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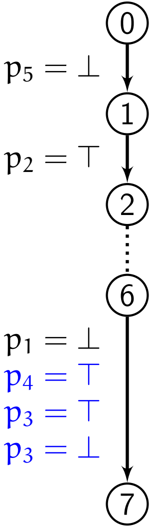

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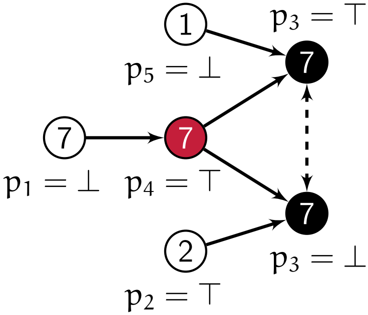

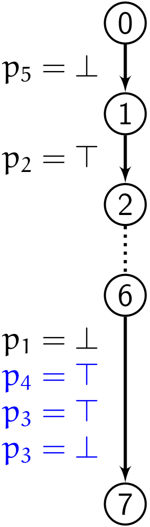

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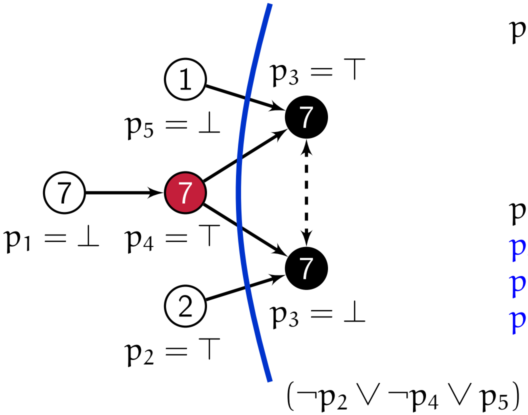

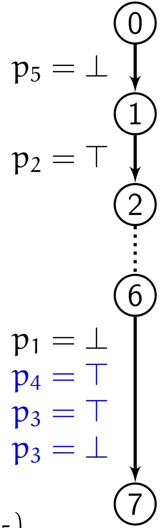

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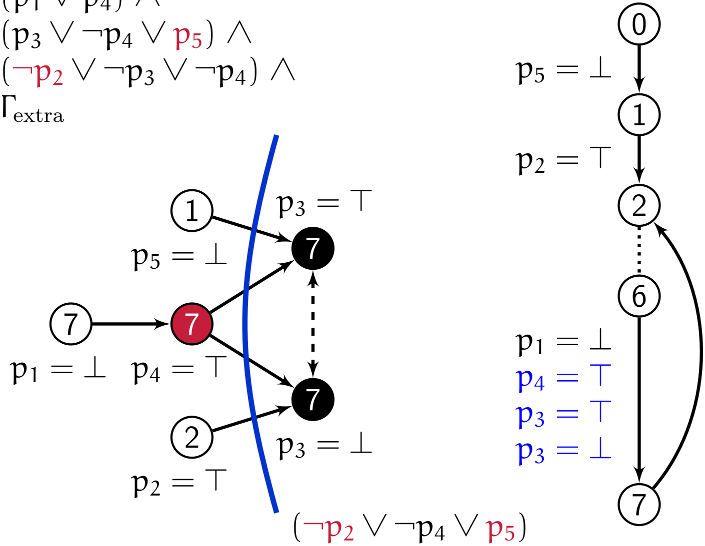

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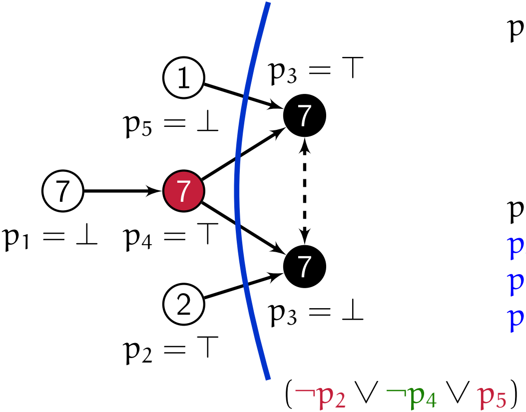

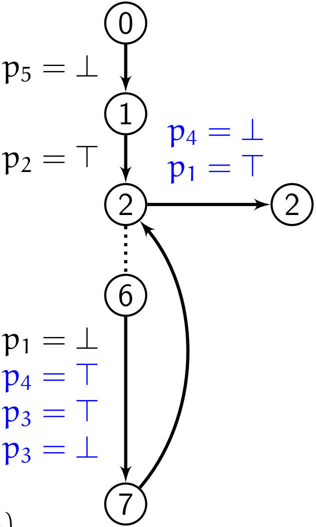

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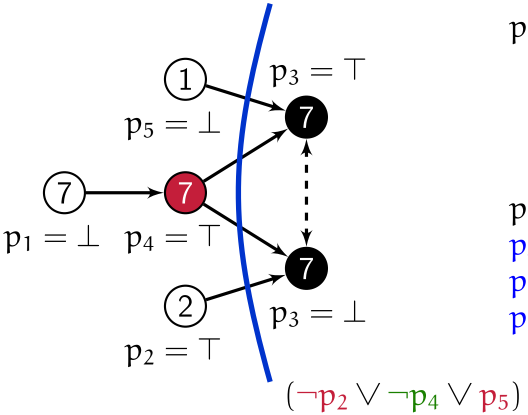

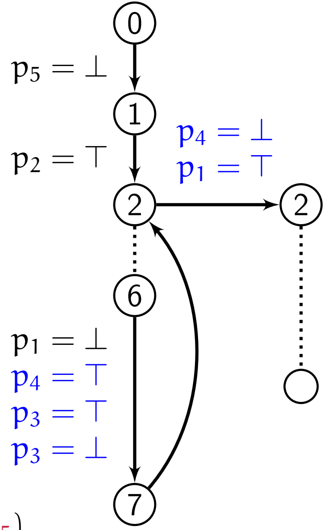

Reverse Unit Propagation

Let Γ be a formula. A clause C is implied by Γ via unit propagation (UP) if UP on $\Gamma \wedge \neg C$ results in a conflict.

Example

$$\Gamma = (p_1 \vee p_4) \wedge (p_3 \vee \neg p_4 \vee p_5) \wedge (\neg p_2 \vee \neg p_3 \vee \neg p_4) \wedge \ldots$$

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clause
units $\neg p_1 \wedge p_2 \wedge \neg p_5$

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clause	$(p_1 \vee p_4)$
units $\neg p_1 \wedge p_2 \wedge \neg p_5$	p_4

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clause		$(p_1 \lor p_4)$	$(p_3 \lor \neg p_4 \lor p_5)$	$(\neg p_2 \lor \neg p_3 \lor \neg p_4)$
units	$\neg p_1 \land p_2 \land \neg p_5$	p_4	p_3	$\perp$

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clause		$(p_1 \lor p_4)$	$(p_3 \lor \neg p_4 \lor p_5)$	$(\neg p_2 \lor \neg p_3 \lor \neg p_4)$
units	$\neg p_1 \land p_2 \land \neg p_5$	p_4	p_3	$\bot$

$$\dfrac{\dfrac{(\neg p_2 \lor \neg p_3 \lor \neg p_4) \qquad (p_3 \lor \neg p_4 \lor p_5)}{(\neg p_2 \lor \neg p_4 \lor p_5)} \qquad (p_1 \lor p_4)}{(p_1 \lor \neg p_2 \lor p_5)}$$

CDCL Overview

CDCL in a nutshell:

1. Main loop combines efficient problem simplification with cheap, but effective decision heuristics; ($> 90\%$ of time)
2. Reasoning kicks in if the current state is conflicting;
3. The current state is analyzed and turned into a constraint;
4. The constraint is added to the problem, the heuristics are updated, and the algorithm (partially) restarts.

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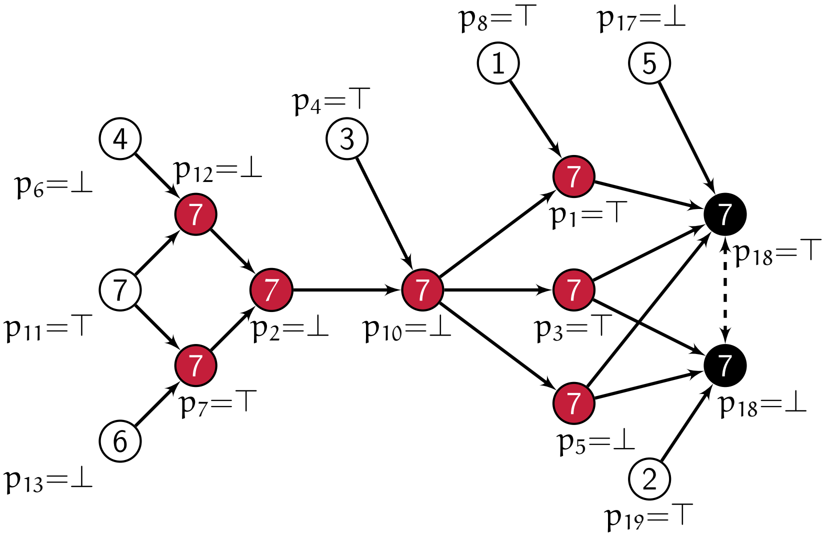
However, it has three weaknesses:

- CDCL is notoriously hard to parallelize;
- the representation impacts CDCL performance; and
- CDCL has exponential runtime on some "simple" problems.

Conflict-driven Clause Learning: Pseudo-code

while TRUE **do**
    l_{decision} := Decide ()
    If no l_{decision} **then return** `satisfiable`
    τ := Simplify $(\tau \cup (l_{\text{decision}} = \top), \Gamma)$
    while $[\![\Gamma]\!]_\tau$ contains $C_{\text{falsified}}$ **do**
        C_{conflict} := Analyze $(C_{\text{falsified}}, \tau)$
        If $C_{\text{conflict}} = \bot$ **then return** `unsatisfiable`
        $\Gamma := \Gamma \cup \{C_{\text{conflict}}\}$
        τ := BackTrack $(\tau, C_{\text{conflict}})$
        τ := Simplify (τ, Γ)
    end while
end while

Learning conflict clauses [Marques-Silva,Sakallah'96]



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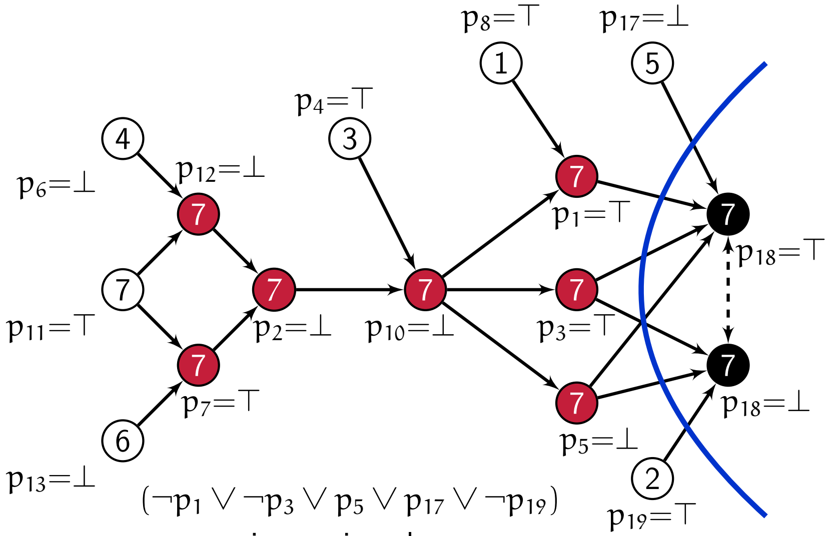

$(\neg p_1 \vee \neg p_3 \vee p_5 \vee p_{17} \vee \neg p_{19})$

tri-asserting clause

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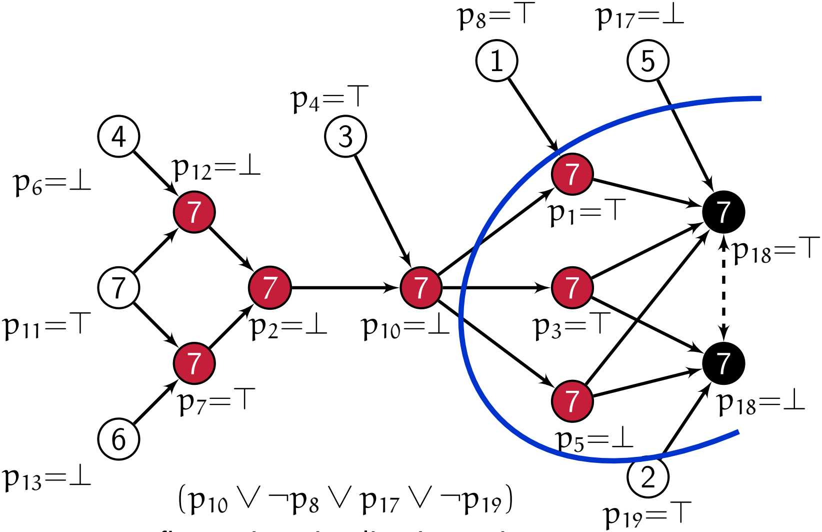

$(p_{10} \vee \neg p_8 \vee p_{17} \vee \neg p_{19})$

first unique implication point

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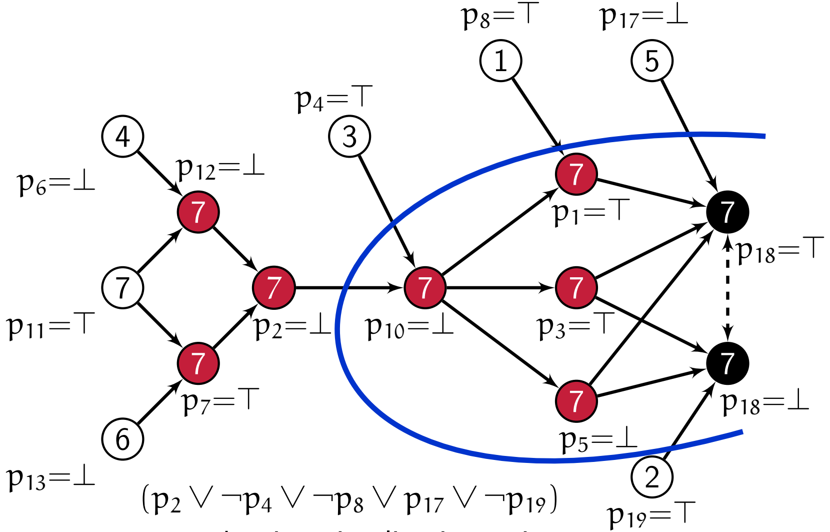
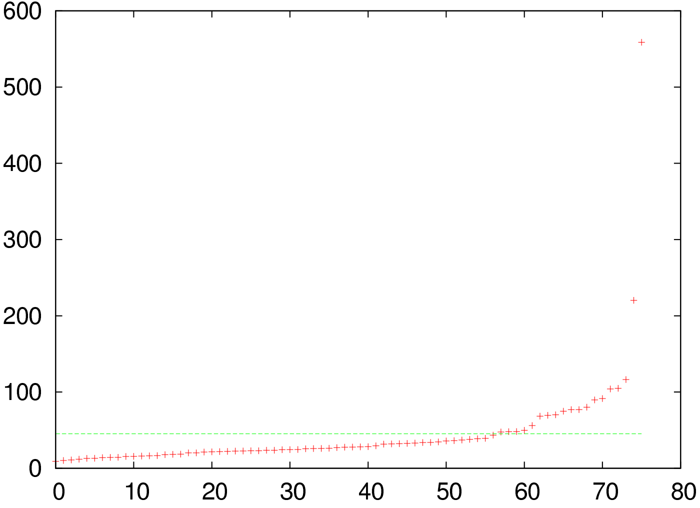


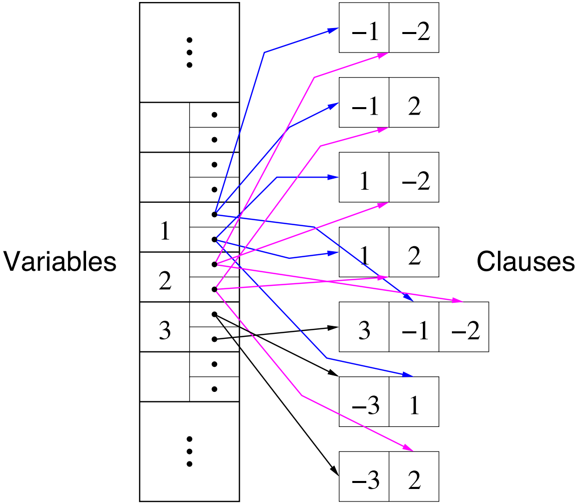
$(p_2 \vee \neg p_4 \vee \neg p_8 \vee p_{17} \vee \neg p_{19})$

second unique implication point

Average Learned Clause Length



Simple data structure for unit propagation



Conflict-driven: Watch pointers (1) [MoskewiczMZZM'01]

$$\tau = \{p_1 = *, p_2 = *, p_3 = *, p_4 = *, p_5 = *, p_6 = *\}$$

$\neg p_1$	p_2	$\neg p_3$	$\neg p_5$	p_6

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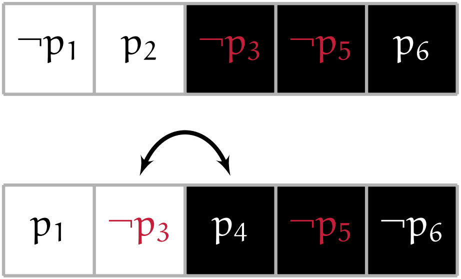

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Conflict-driven: Watch pointers (2) [MoskewiczMZZM'01]

Only examine (get in the cache) a clause when both

- a watch pointer gets falsified
- the other one is not satisfied

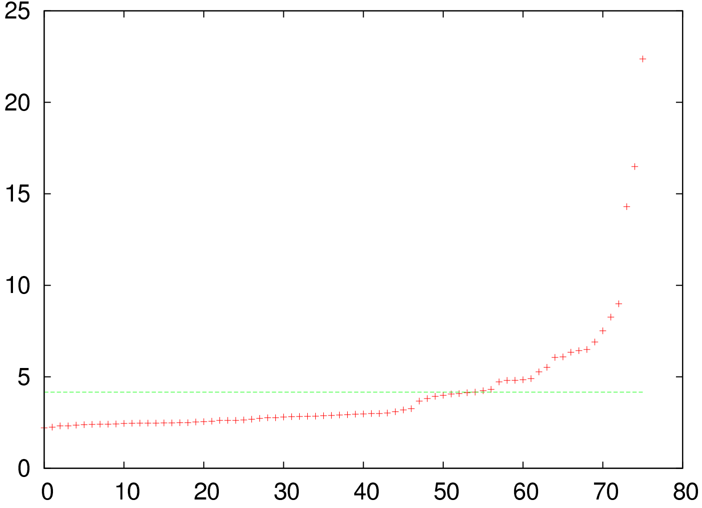
While backjumping, just unassign variables

Conflict clauses $\longrightarrow$ watch pointers

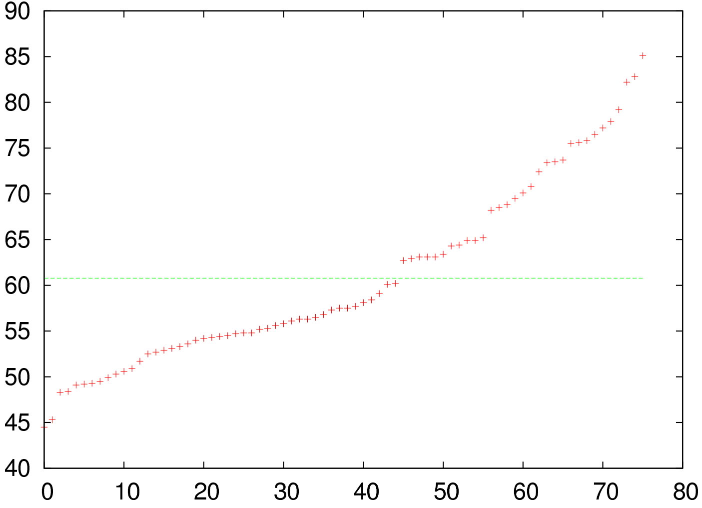
No detailed information available

Not used for binary clauses

Average Number Clauses Visited Per Propagation



Percentage visited clauses with other watched literal true



Clause Learning

Data-structures

Heuristics

Proofs of Unsatisfiability

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Variable selection heuristics

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Restart strategies

- aim: avoid heavy-tail behavior [GomesSelmanCrato'97]
- plus: focus search on recent conflicts when combined with dynamic heuristics

Variable selection heuristics

Based on the occurrences in the (reduced) formula

- examples: Jeroslow-Wang, Maximal Occurrence in clauses of Minimal Size (MOMS), look-aheads
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Variable State Independent Decaying Sum (VSIDS)

- original idea (zChaff): for each conflict, increase the score of involved variables by 1, half all scores each 256 conflicts [MoskewiczMZZM'01]
- improvement (MiniSAT): for each conflict, increase the score of involved variables by δ and increase $\delta := 1.05\delta$ [EenSörensson'03]

Visualization of VSIDS in PicoSAT

http://www.youtube.com/watch?v=MOjhFywLre8

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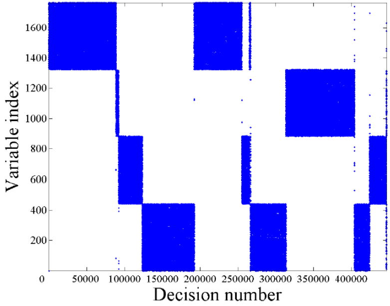
- negative branching (early MiniSAT) [EenSörensson'03]

Based on the last implied value (phase-saving)

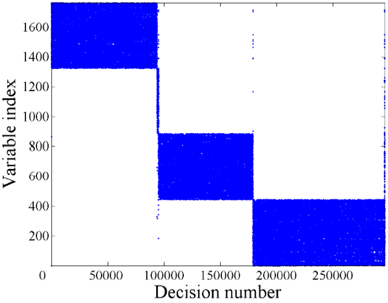
- introduced to CDCL [PipatsrisawatDarwiche'07]
- already used in local search [HirschKojevnikov'01]

Heuristics: Phase-saving [PipatsrisawatDarwiche'07]

Selecting the last implied value remembers solved components



negative branching



phase-saving

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Restarts in CDCL solvers:

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Restart strategies: [Walsh'99, LubySinclairZuckerman'93]

- Geometrical restart: e.g. $100, 150, 225, 333, 500, 750, \ldots$
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Rapid restarts by reusing trail: [vanderTakHeuleRamos'11]

- Partial restart same effect as full restart
- Optimal strategy Luby-1: $1, 1, 2, 1, 1, 2, 4, \ldots$

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Proofs of Unsatisfiability

Motivation for Proofs of Unsatisfiability

SAT solvers may have errors and only return yes/no.

- Documented bugs in SAT, SMT, and QSAT solvers;
  [Brummayer and Biere, 2009; Brummayer et al., 2010]
- Competition winners have contradictory results
  (HWMCC winners from 2011 and 2012)
- Implementation errors often imply conceptual errors;
- Proofs now mandatory for the annual SAT Competitions;
- Mathematical results require a stronger justification than a simple yes/no by a solver. UNSAT must be verifiable.

Clausal Proofs of Unsatisfiability

Reduce the size of the proof by only storing added clauses

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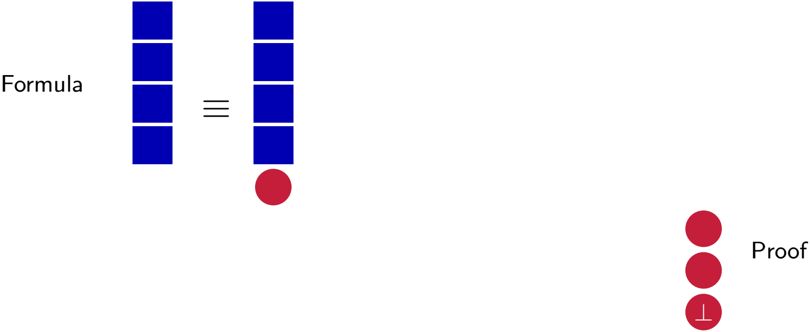

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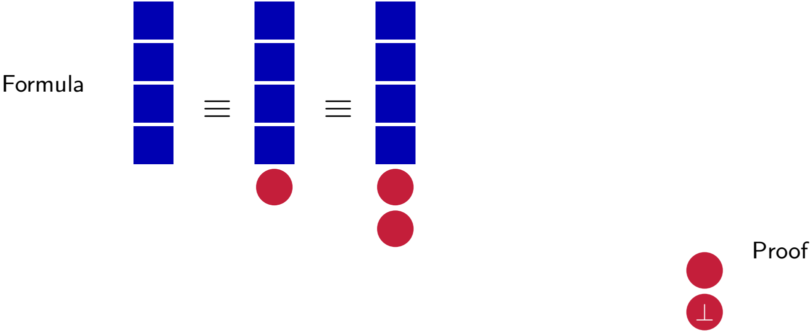

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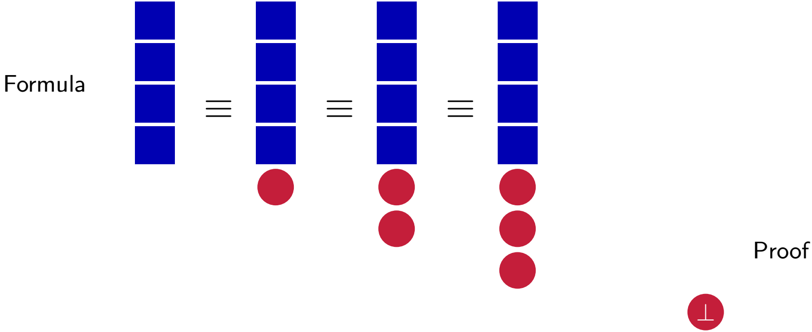

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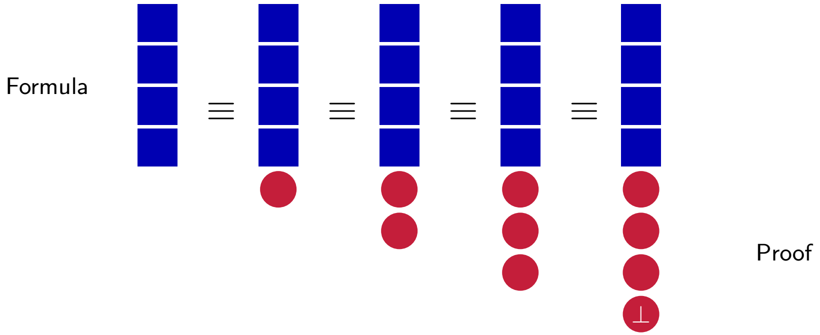

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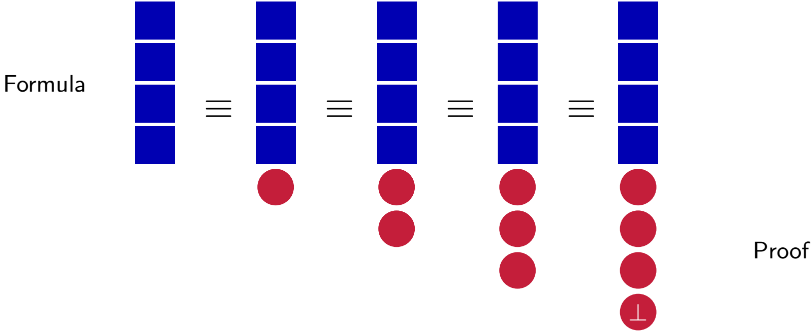

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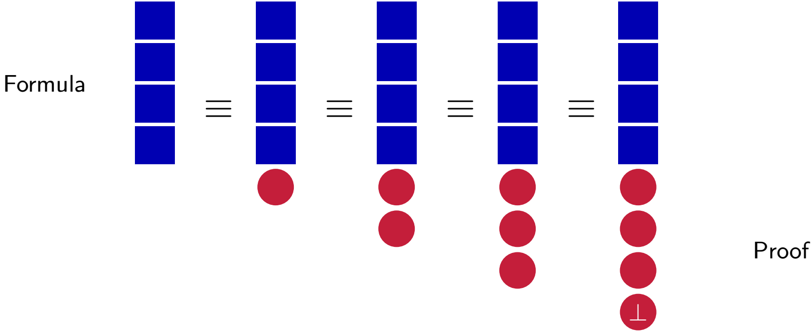

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- Checking redundancy should be efficient.
- Proof systems for this purpose in upcoming lectures.