THE FREQUENCY DOMAIN

An alternative to waveforms (the time domain)

The Frequency Domain

- Examples of Simple Spectra
- Fourier Transform vs Short-Term Fourier Transform
- DFT – Discrete Fourier Transform
- FFT – Fast Fourier Transform
- Windowing

Formal Definition

$$R(\omega) = \int_{-\infty}^{\infty} f(t)\cos\omega t \, dt$$

$$X(\omega) = -\int_{-\infty}^{\infty} f(t)\sin\omega t \, dt$$

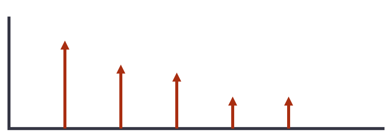
$$F(\omega) = \int_{-\infty}^{\infty} f(t)e^{-j\omega t} \, dt$$

Simple Spectra Examples

- Sinusoid

- Noise

- Tone with harmonics



More Examples

- Narrow Band Noise

- Impulse

Negative Frequencies

- Recall that FT is defined for negative as well as positive frequencies. What does this mean?
- $cos(\omega t) = cos(-\omega t)$, $sin(\omega t) = -sin(-\omega t)$
- For FT of *real* signals,
  - Imaginary part has odd symmetry: $X(\omega) = -X(-\omega)$
  - The real part has even symmetry: $R(\omega) = R(-\omega)$
- Therefore, the negative frequencies contain redundant information. That's why we've mostly ignored them.

Fourier Transform vs Short-Term Transform

- In practice, we can't do an infinite integral, so do a finite integral: the short term FT (STFT)

$$F(\omega) = \int_a^b f(t)e^{-j\omega t}\,dt$$

- In general, the interesting properties of true FT hold for STFT, but with annoying artifacts

Discrete Fourier Transform

- Since we work with samples rather than continuous data,
- We need a discrete version of FT: DFT
- DFT is essentially just like FT, except band limited and computable
- I'm glossing over many derivations, proofs, and details here.

Fast Fourier Transform

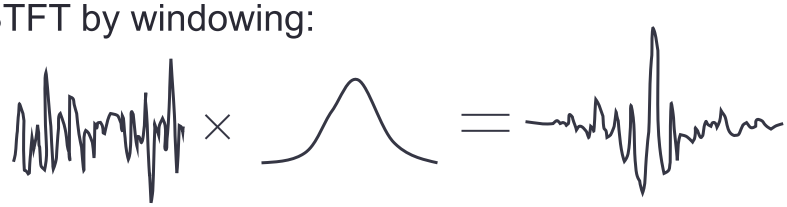
- Replacing integral with a sum, you would think computing $R(\omega)$ would be an $O(n^2)$ problem

$$F_k = \sum_{n=a}^{b} f_n e^{-j2\pi kn/N}$$

- Interestingly, there is an O(n log n) algorithm, the Fast Fourier Transform, or FFT

Windowing

- Typically, you can reduce the artifacts of the STFT by windowing:



- Different windows optimize different criteria: Hamming, Hanning, Blackman, etc.

More Examples Using Audacity