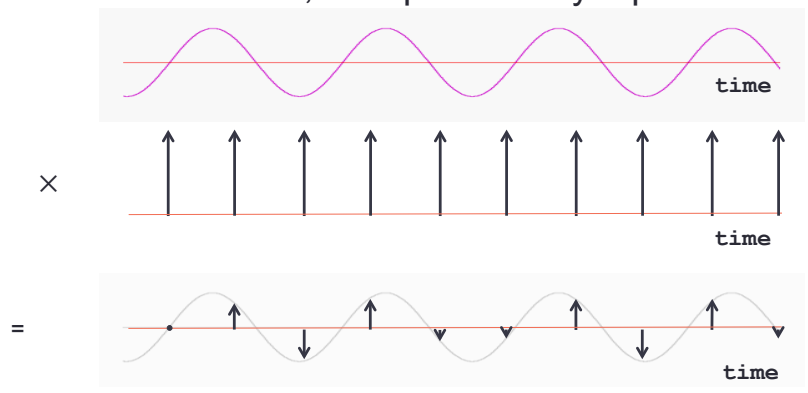
PERFECT SAMPLING

From continuous signals to discrete samples and back again

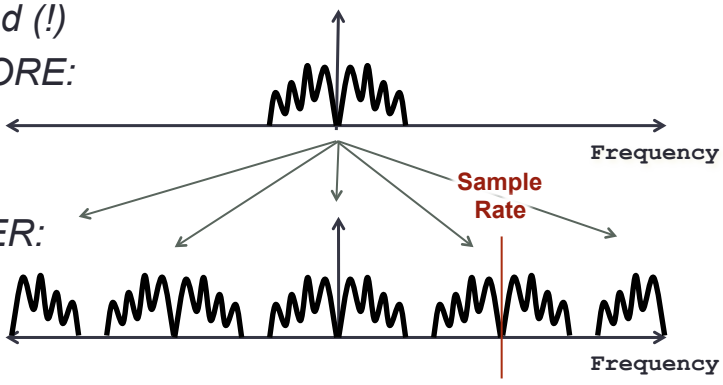
Sampling – Time Domain

- What happens when you sample a signal?
- In time domain, multiplication by a pulse train:



Sampling – Frequency Domain

- What happens when you sample a signal?
- In frequency domain, the spectrum is *copied and shifted (!)*
- *BEFORE:*

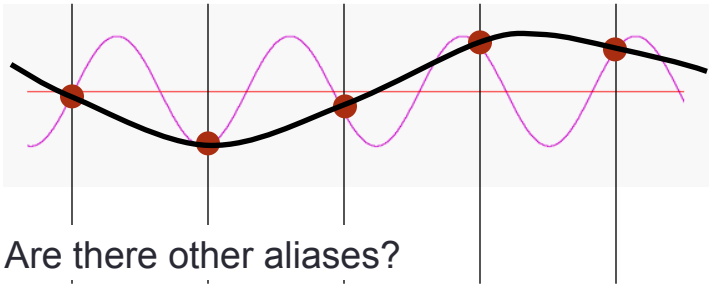


- *AFTER:*

An Aside

- Why copied and shifted?
- We're glossing over some details …
- Multiplication in the time domain is equivalent to convolution in the frequency domain.
- The transform of a pulse train is a pulse train(!)
- Convolution with a pulse train copies and shifts the spectrum.
- See text for more detail.
- Take linear systems for derivation and proof.

Aliasing: Time Domain View

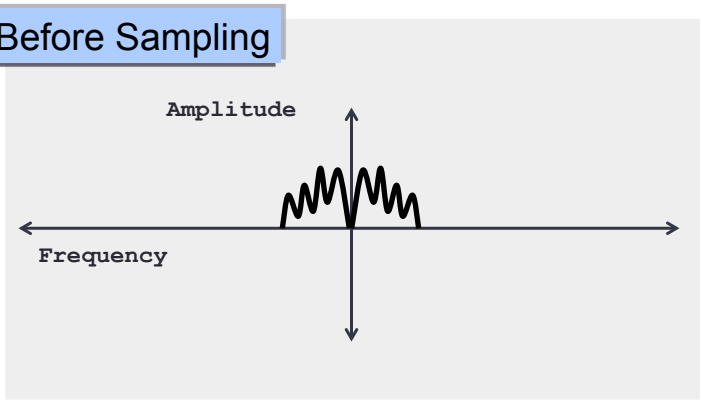


Are there other aliases?

At 16kHz SR,
Sine tones at:
1000 Hz
3010 Hz
5020 Hz
7030 Hz
9040 Hz
11060 Hz
13070 Hz
15080 Hz

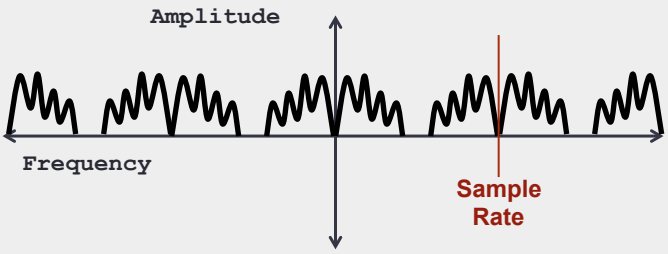
Aliasing: Frequency Domain View

Before Sampling



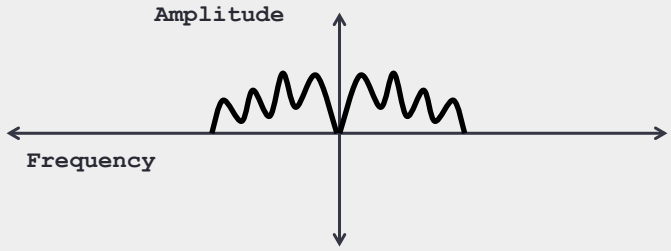
Frequency Domain View (2)

After Sampling

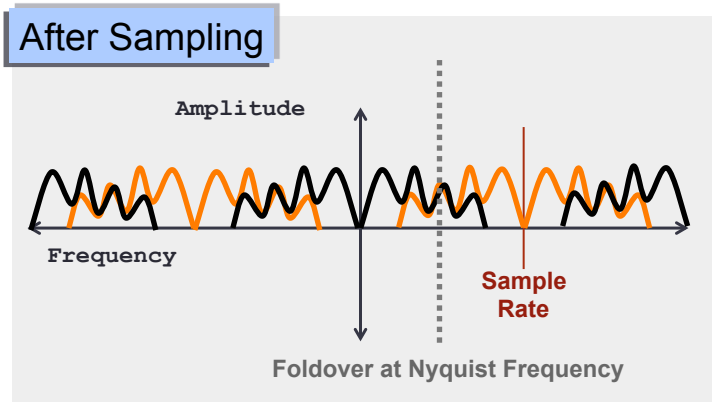


A Signal With Higher Frequency Components

Before Sampling



A Signal With Higher Frequency Components



Bandwidth

What sample rate should we use?
Why does it matter?

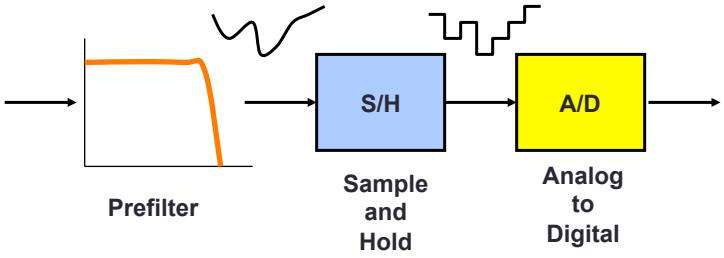
Bandwidth

The frequency range (bandwidth) is determined by the sample rate!

Sampling Without Aliasing

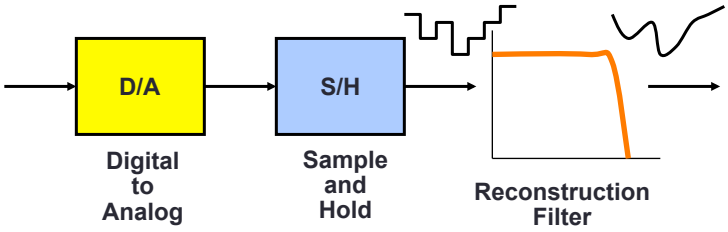
How do we **convert** analog to/from digital?

Sampling Without Aliasing



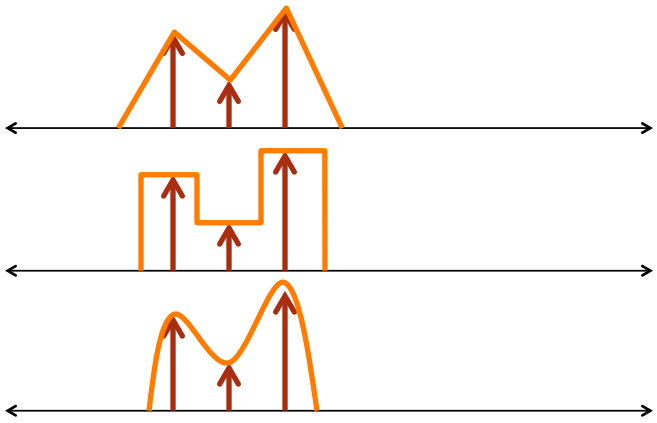
Prefilter removes all frequencies above 1/2 sampling rate (the Nyquist Frequency)

Conversion to Analog

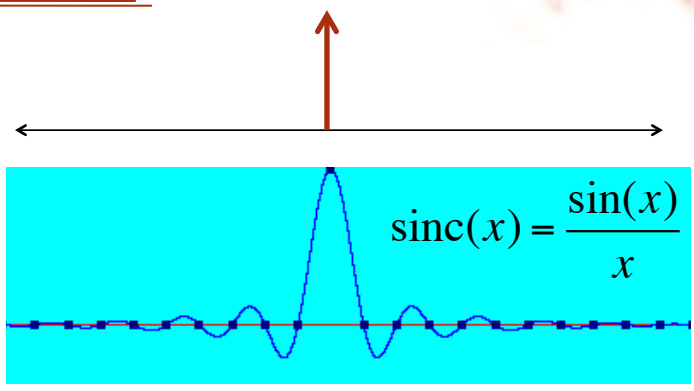


Reconstruction filter removes all frequencies above 1/2 sampling rate (the Nyquist Frequency)

What Does a Sample "Mean"?

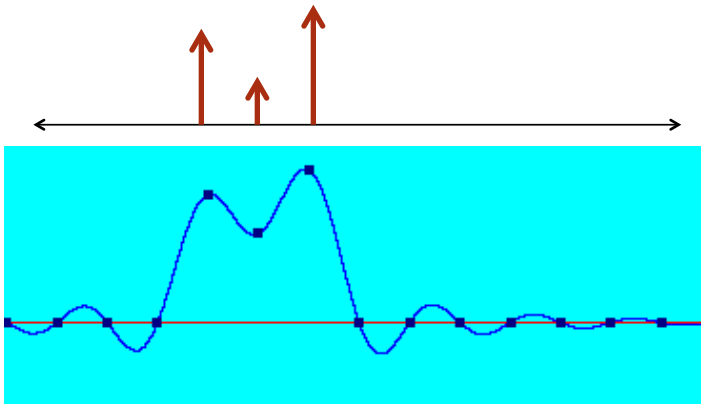


What Does a Sample "Mean"? (2)



Note: The time axis (x) is scaled so that the zeros of sinc(x) fall exactly on the times of other samples.

What Does a Sample "Mean"? (3)



Why sinc function?

- An impulse has infinite bandwidth.
- If you perfectly cut the bandwidth down to half the sample rate (the Nyquist frequency), you get a sinc function!
- When you reconstruct the signal, replacing impulses with sinc functions, you get the entire continuous band limited signal.
- Samples uniquely determined by signal, signal uniquely determined by samples.
- Bijective (for Klaus ☺)
- AMAZING.

Interpolation/Reconstruction

How can we interpolate samples to recover the sampled signal?

Interpolation/Reconstruction

- *Convolve* with a sinc function
- In other words, form the superposition of sinc functions shifted by the sample times and scaled by the sample values.
- Requires infinite lookahead and infinite computation!
- But sinc decays as 1/time, so good approximations are expensive but at least possible.