

This lecture is being recorded

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**18-452/18-750**

**Wireless Networks and Applications**

**Lecture 5: Physical Layer  
Modulation and Diversity**

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**Spring Semester 2021**

**<http://www.cs.cmu.edu/~prs/wireless21/>**

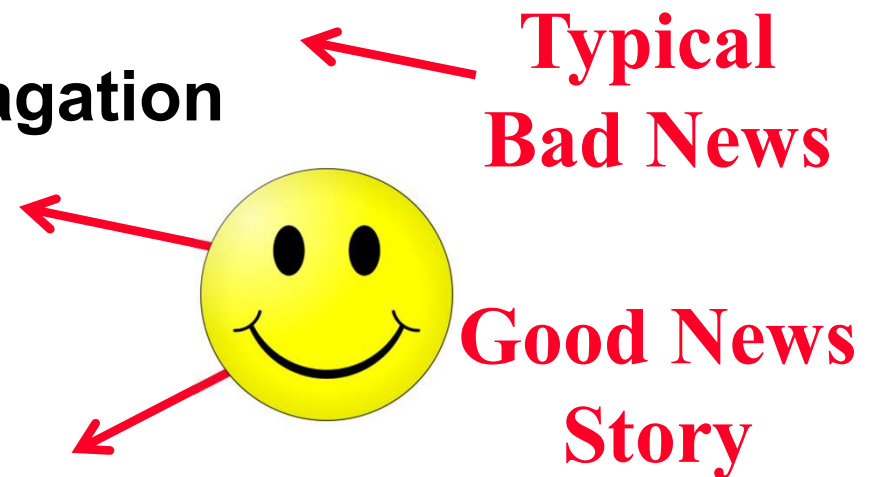
# Announcements

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- **Waiting list status**
  - » Only two people left!
  - » There are several open slots so everybody will be able to get in
- **We will post the Project 1 handout today**

# Outline

- RF introduction
- Modulation and multiplexing
- Channel capacity
- Antennas and signal propagation
- Modulation
- Coding and diversity
- OFDM



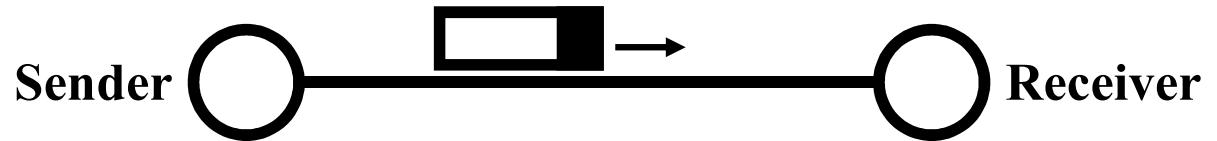
# (Limited) Goals

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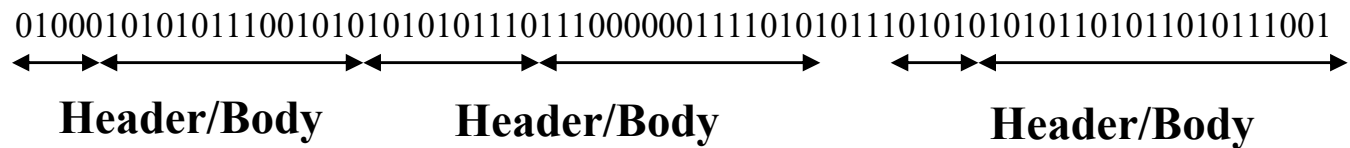
- **Non-goal: turn you into electrical engineers**
  - » Of course, some of you already are electrical engineers
- **Basic understanding of how modulation can be done**
- **Understand the tradeoffs involved in increasing the bit rate**

# From Signals to Packets

Packet  
Transmission



Packets



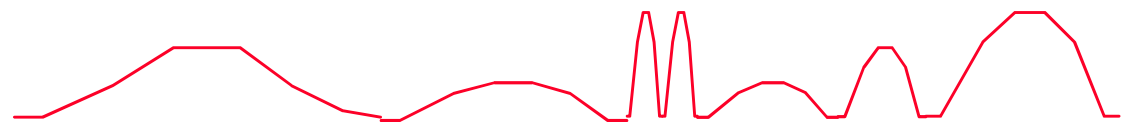
Bit Stream

**0 0 1 0 1 1 1 0 0 0 1**

“Digital” Signal

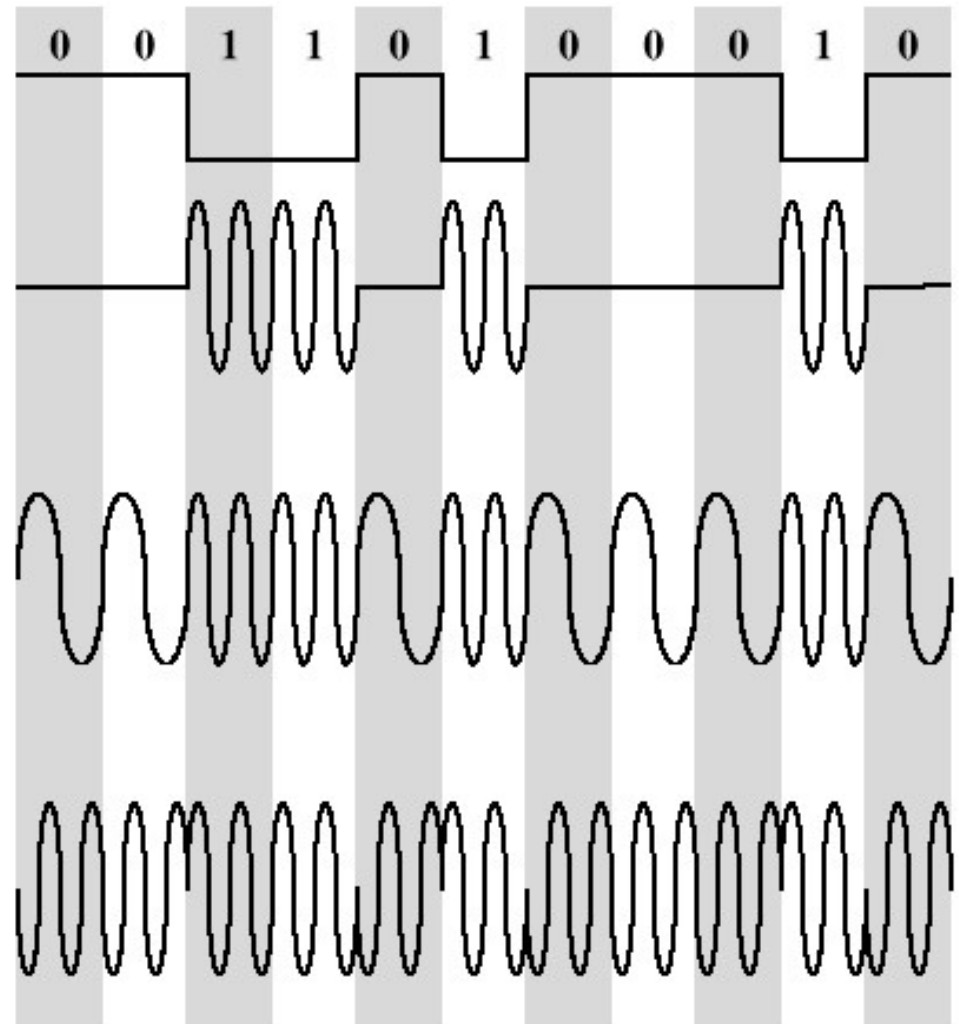


Analog Signal



# Basic Modulation Techniques

- **Encode digital data in an analog signal**
- **Amplitude-shift keying (ASK)**
  - » Amplitude difference of carrier frequency
- **Frequency-shift keying (FSK)**
  - » Frequency difference near carrier frequency
- **Phase-shift keying (PSK)**
  - » Phase of carrier signal shifted



# Amplitude-Shift Keying

- **One binary digit represented by presence of carrier, at constant amplitude**
- **Other binary digit represented by absence of carrier**

$$s(t) = \begin{cases} A \cos(2\pi f_c t) & \text{binary 1} \\ 0 & \text{binary 0} \end{cases}$$

– where the carrier signal is  $A \cos(2\pi f_c t)$

- **Inefficient because of sudden gain changes**
  - » Only used when bandwidth is not a concern, e.g. on voice lines (< 1200 bps) or on digital fiber
- **A can be a multi-bit symbol**

# How Can We Go Faster?

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- **Increase the rate at which we modulate the signal, or ...**
  - » I.e., a higher frequency base signal
  - » Signal time becomes short
- **Modulate the signal with “symbols” that send multiple bits**
  - » I.e., each symbol represents more information
  - » Longer signal time but more sensitive to distortion
- **Which solution is the best depends on the many factors**
  - » We will not worry about that in this course



# Binary Frequency-Shift Keying (BFSK)

- **Two binary digits represented by two different frequencies near the carrier frequency**

$$s(t) = \begin{cases} A \cos(2\pi f_1 t) & \text{binary 1} \\ A \cos(2\pi f_2 t) & \text{binary 0} \end{cases}$$

– where  $f_1$  and  $f_2$  are offset from carrier frequency  $f_c$  by equal but opposite amounts

- **Less susceptible to error than ASK**
- **Sometimes used for radio or on coax**
- **Demodulator looks for power around  $f_1$  and  $f_2$**

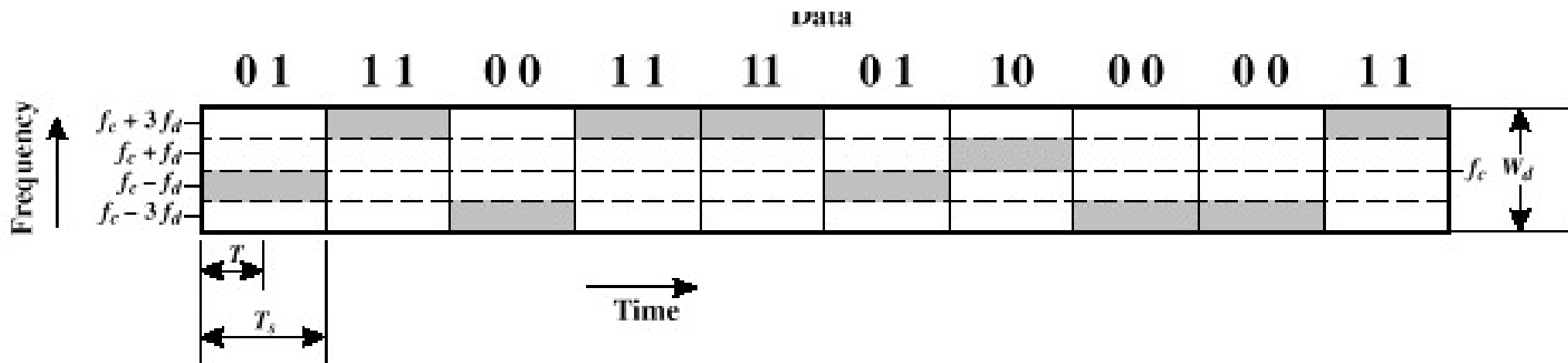
# Multiple Frequency-Shift Keying (MFSK)

- More than two frequencies are used
- Each symbol represents  $L$  bits

$$s_i(t) = A \cos 2\pi f_i t \quad 1 \leq i \leq M$$

- $f_i = f_c + (2i - 1 - M)f_d$
  - $L$  = number of bits per signal element
  - $M$  = number of different signal elements =  $2^L$
  - $f_c$  = the carrier frequency
  - $f_d$  = the difference frequency
- More bandwidth efficient but more susceptible to error
    - » Symbol length is  $T_s = LT$  seconds, where  $T$  is bit period

# Multiple Frequency-Shift Keying (MFSK)



# Phase-Shift Keying (PSK)

- **Two-level PSK (BPSK)**

- » Uses two phases to represent binary digits

$$s(t) = \begin{cases} A \cos(2\pi f_c t) & \text{binary 1} \\ A \cos(2\pi f_c t + \pi) & \text{binary 0} \end{cases}$$

$$= \begin{cases} A \cos(2\pi f_c t) & \text{binary 1} \\ -A \cos(2\pi f_c t) & \text{binary 0} \end{cases}$$

- **Differential PSK (DPSK)**

- » Phase shift with reference to previous bit

- Binary 0 – signal of same phase as previous signal burst
- Binary 1 – signal of opposite phase to previous signal burst

# Phase-Shift Keying

## Four Level PSK

- Each element represents 2 (or more) bits

$$s(t) = \begin{cases} A \cos\left(2\pi f_c t + \frac{\pi}{4}\right) & 11 \\ A \cos\left(2\pi f_c t + \frac{3\pi}{4}\right) & 01 \\ A \cos\left(2\pi f_c t - \frac{3\pi}{4}\right) & 00 \\ A \cos\left(2\pi f_c t - \frac{\pi}{4}\right) & 10 \end{cases}$$

# Quadrature Amplitude Modulation - QAM

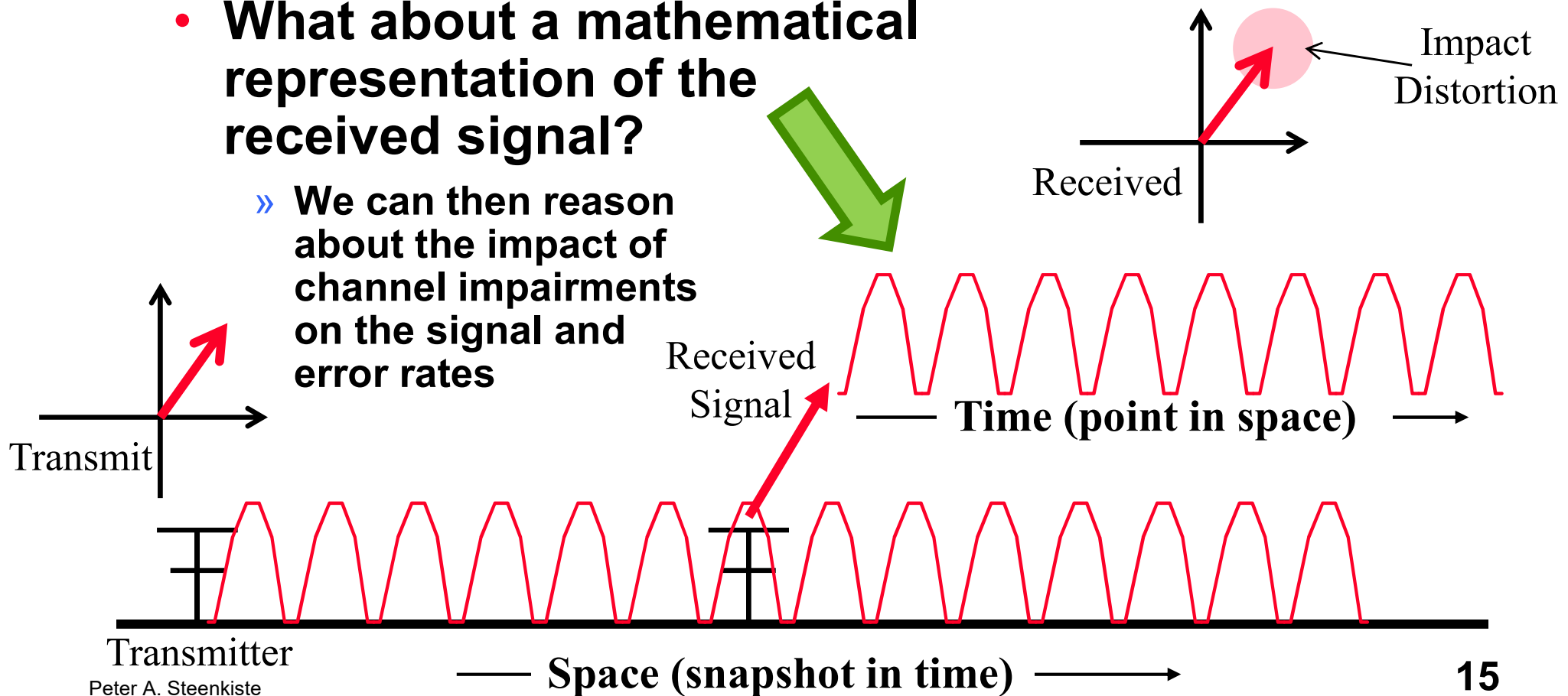
- **Modulation is based on both the phase and amplitude**
- **Has many benefits**
  - » Since two signal properties are used, it can be used for a wide range of symbol sizes
  - » It has a simple mathematical representation (next slide)
  - » Bonus: it has a very intuitive presentation (figures)
- **QAM is the dominating modulation technique for modern, high performance wireless technologies**
  - » 4G, 5G, all recent WiFi standards

# Time and Point View of Signal

- **Remember: communication is based on the transmission of a modulated carrier signal**
  - » Focus on amplitude-phase modulation – very common!

- **What about a mathematical representation of the received signal?**

- » We can then reason about the impact of channel impairments on the signal and error rates

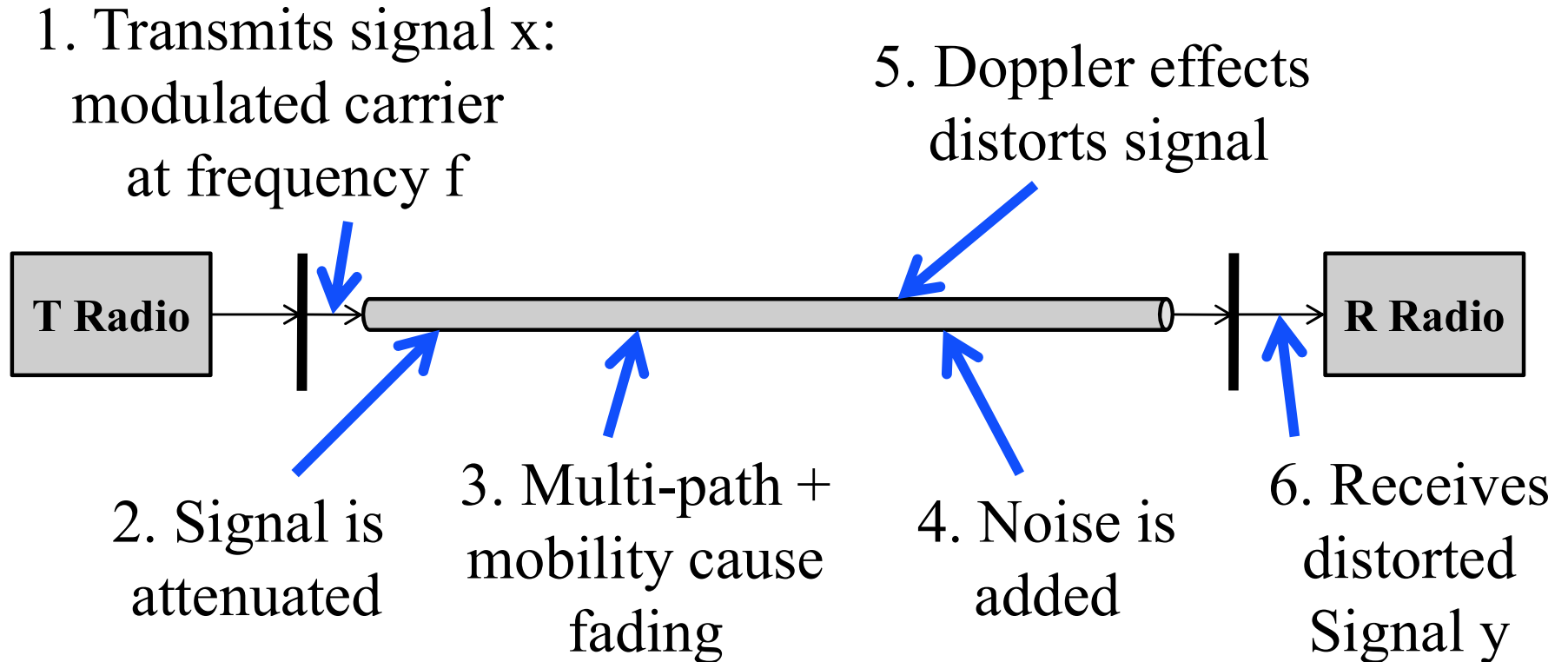


# QAM Signals and Channel State

- **A signal is a complex number that represents the signal's amplitude and phase**
- **The channel state captures how it changes a signal's attenuation and phase**
  - » The two main channel properties relevant to wireless communication
- **$c$  changes over time due to mobility:  $c(t)$ ,**
  - » Change is continuous; captured as a sequence of samples  $c_i$
- **$c$  typically depends on carrier frequency:  $c(f)$** 
  - » Frequency selective fading or attenuation
  - » The dependency on  $f$  is a concern for wide-band signals
- **$c$  is sampled in frequency and time domains**



# Channel Model



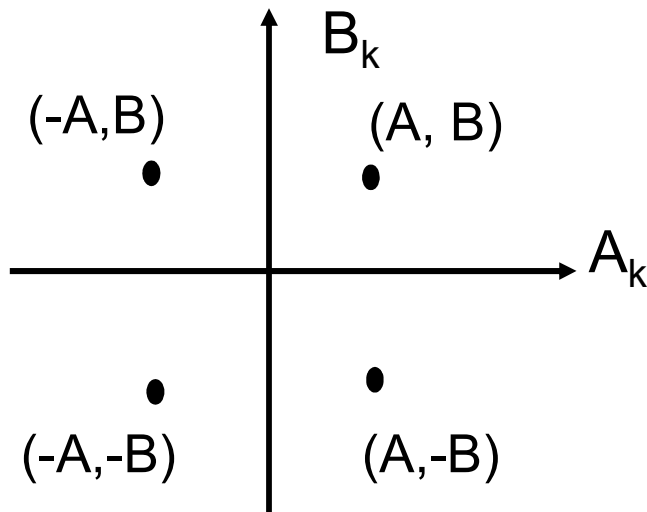
$$\mathbf{x} \quad \mathbf{x} \quad \mathbf{c} \quad + \quad \mathbf{n} \quad = \quad \mathbf{y}$$

# Tradeoff: Bit Rate versus Error Rate - Informal

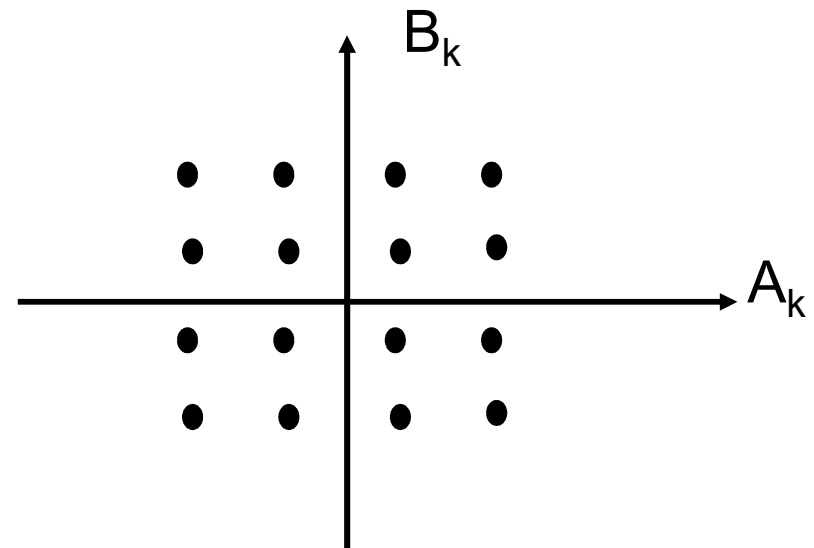
- **Amplitude and phase modulation places transmitted symbols into 2D space**
  - » Represented by a complex number
- **Channel distortion “moves” the symbol**
  - » Large shift can map it onto another symbol
- **Large symbols means denser packing of symbols in the plane** Good channels
  - » Results in high bit rate but distortions are more likely to result in errors
- **Smaller symbols are more conservative** Bad channels
  - » Lower bit rate but more resistant to errors

# Signal Constellations

- Each pair  $(A_k, B_k)$  defines a point in the plane
- **Signal constellation** set of signaling points

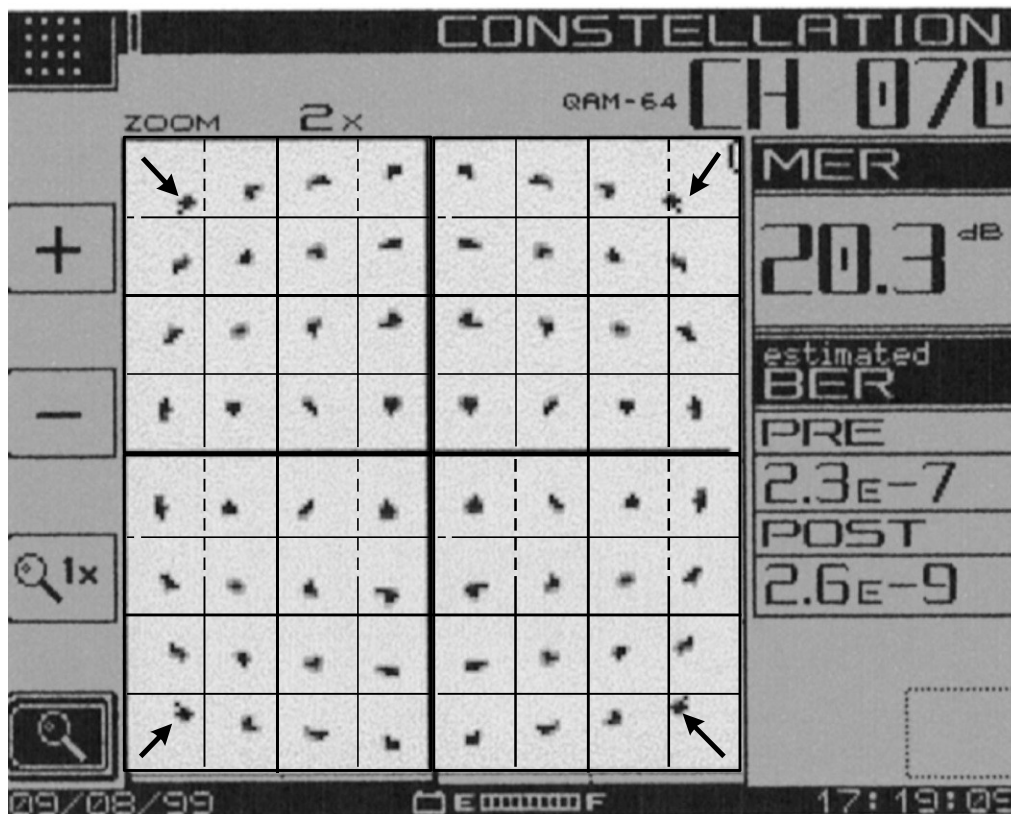


4 possible points per  $T$  sec.  
2 bits QAM (see earlier slide)



16 possible points per  $T$  sec.  
4 bits / pulse

# How Does Distortion Impact a Constellation Diagram?



- Changes in amplitude, phase or frequency move the points in the diagram
- Large shifts can create uncertainty on what symbol was transmitted
- Larger symbols are more susceptible
- Can Adapt symbol size to channel conditions to optimize throughput

# Adapting to Channel Conditions

- **Channel conditions can be very diverse**
  - » Affected by the physical environment of the channel
  - » Changes over time as a result of slow and fast fading
- **Fixed coding/modulation scheme will often be inefficient**
  - » Too conservative for good channels, i.e. lost opportunity
  - » Too aggressive for bad channels, i.e. lots of packet loss
- **Adjust coding/modulation based on channel conditions – “rate” adaptation**
  - » Controlled by the MAC protocol
  - » E.g. 802.11a: BPSK – QPSK – 16-QAM – 64 QAM

← Bad → Good Channel

# Summary

- **Key properties for channels are:**
  - » Channel state that concisely captures many of the factors degrading the channel
  - » The power budget expresses the power at the receiver
  - » Channel reciprocity
- **Modulation changes the signal based on the data to be transmitted**
  - » Can change amplitude, phase or frequency
  - » The transmission rate can be increased by using symbols that represent multiple bits
    - Can use hybrid modulation, e.g., phase and amplitude
  - » The symbol size can be adapted based on the channel conditions – results in a variable bit rate transmission
  - » Details do not matter!

# Outline

- RF introduction
- Modulation and multiplexing
- Channel capacity
- Antennas and signal propagation
- Modulation
- Diversity and coding
  - » Space, time and frequency diversity
- OFDM



# Diversity Techniques

- **The quality of the channel depends on time, space, and frequency**
- **Space diversity: use multiple nearby antennas and combine signals**
  - » Both at the sender and the receiver
- **Time diversity: spread data out over time**
  - » Useful for burst errors, i.e., errors are clustered in time
- **Frequency diversity: spread signal over multiple frequencies**
  - » For example, spread spectrum
- **Distribute data over multiple “channels”**
  - » “Channels” experience different frequency selective fading, so only part of the data is affected

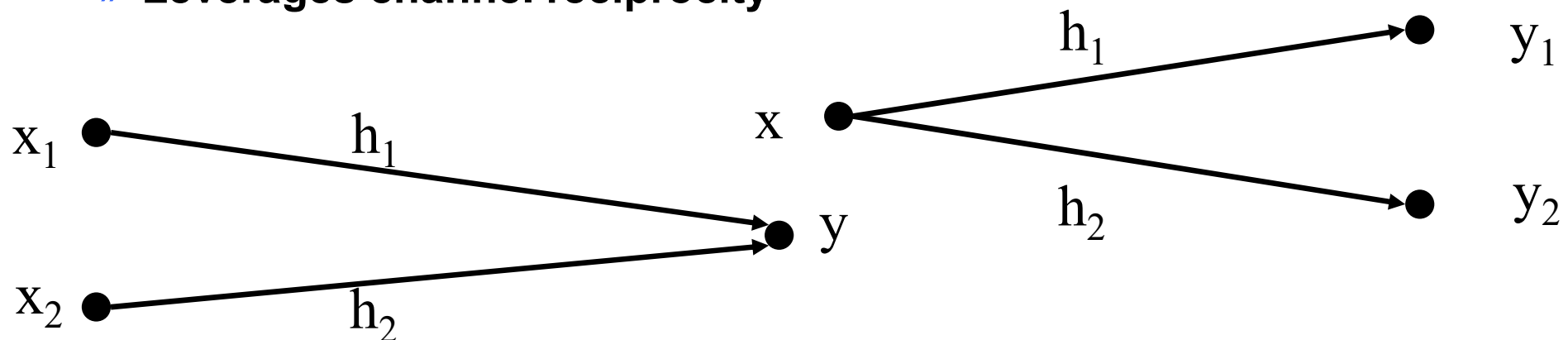


# Space Diversity

- **Use multiple antennas that pick up/transmit the signal in slightly different locations**
- **If antennas are sufficiently separated, instantaneous channel conditions are independent**
  - » **Antennas should be separated by  $\frac{1}{2}$  wavelength or more**
- **If one antenna experiences deep fading, the other antenna has a strong signal**
- **Represents a wide class of techniques**
  - » **Use on transmit and receive side - channels are symmetric**
  - » **Level of sophistication of the algorithms used**
  - » **Can use more than two antennas!**

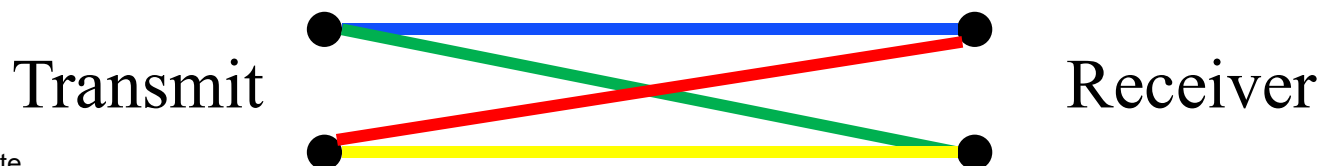
# Selection Diversity

- **Receiver diversity: receiver picks the antenna with the best SNR**
  - » Very easy
- **Transmit diversity: sender picks the antenna that offers the best channel to the receiver**
  - » Transmitter can learn the channel conditions based on signals sent by the receiver
  - » Leverages channel reciprocity



# Simple Algorithm in (older) 802.11

- **Combine transmit + receive selection diversity**
  - » Assume packets are acknowledged – why?
- **How to explore all channels to find the best one ... or at least the best transmit antenna**
- **Receiver:**
  - » Uses the antenna with the strongest signal
  - » Always use the same antenna to send the acknowledgement – gives feedback to the sender
- **Sender:**
  - » Picks an antenna to transmit and learns about the channel quality based on the ACK
  - » Needs to occasionally try the other antenna to explore the channel between all four channel pairs

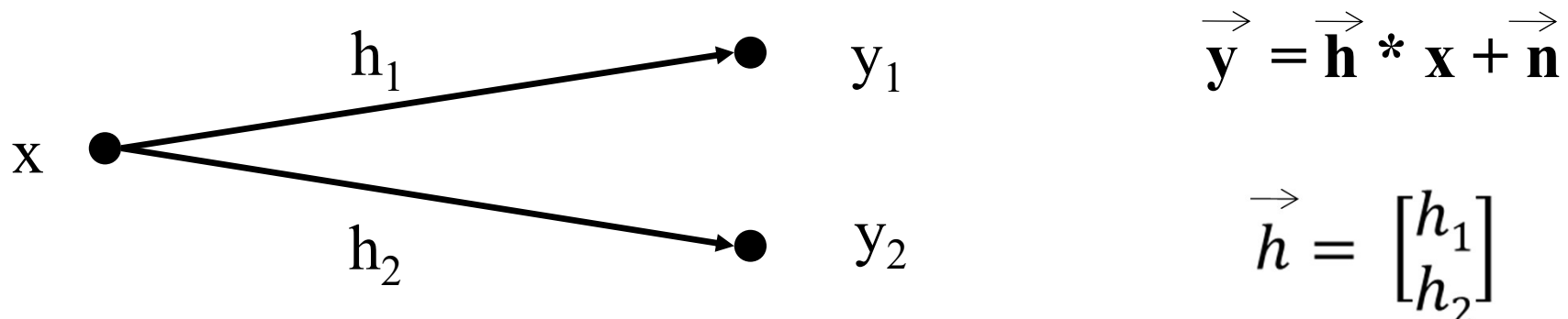


# Receiver Diversity

## Can we Do Better?

- **But why not use both signals?**
  - » 2 Signals contain more information than 1
  - » What can go wrong?
- **Simply adding the two signals has drawbacks:**
  - » Signals may be out of phase, e.g. kind of like multi-path; can reduce the signal strength!
  - » We want to make sure we do not amplify the noise
- **Maximal ratio combining: combine signals with a weight that is based on their SNR**
  - » Weight will favor the strongest signal (highest SNR)
  - » Also: equal gain combining as a quick and dirty alternative

# Receiver Diversity Optimization



- Multiply  $\vec{y}$  with the complex conjugate  $\vec{h}^*$  of the channel vector  $\vec{h}$ 
  - » Aligns the phases of the two signals so they amplify each other
  - » Scales the signals with their magnitude so the effect of noise is not amplified
- Can learn  $\vec{h}$  based on training data

# The Details

- **Complex conjugates: same real part but imaginary parts of opposite signs**

$$\vec{h}^* * \vec{y} = \vec{h}^* * (\vec{h} * \mathbf{x} + \vec{n})$$

Where  $\mathbf{h}^* = [h_1^* \ h_2^*] = [a_1 + b_1i \ a_2 - b_2i]$

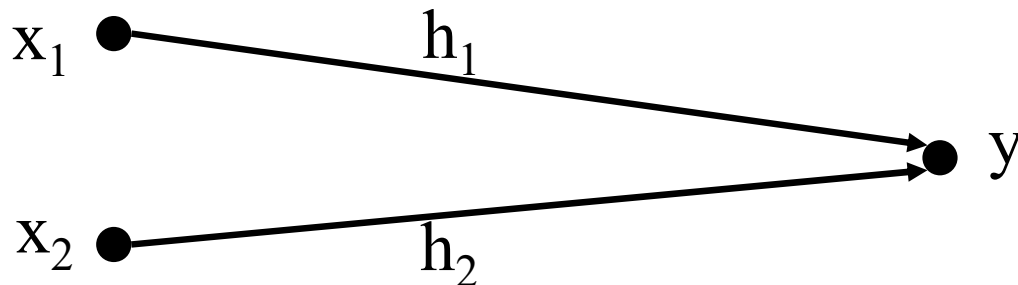
- **Result:**

**signal  $\mathbf{x}$  is scaled by  $a_1^2 + b_1^2 + a_2^2 + b_2^2$**

**noise becomes:  $h_1^* * n_1 + h_2^* * n_2$**

# Transmit Diversity

- Same as receive diversity but the transmitter has multiple antennas
- Maximum ratio combining: sender “precodes” the signal
  - » Pre-align the phases at receiver and distribute power over the transmit antennas (total power fixed)
- How does transmitter learn channel state?
  - » Channel reciprocity: learn from packets received Y



$$y = \vec{h} * \vec{x} + n$$

↓

$$\vec{h}^* * x$$

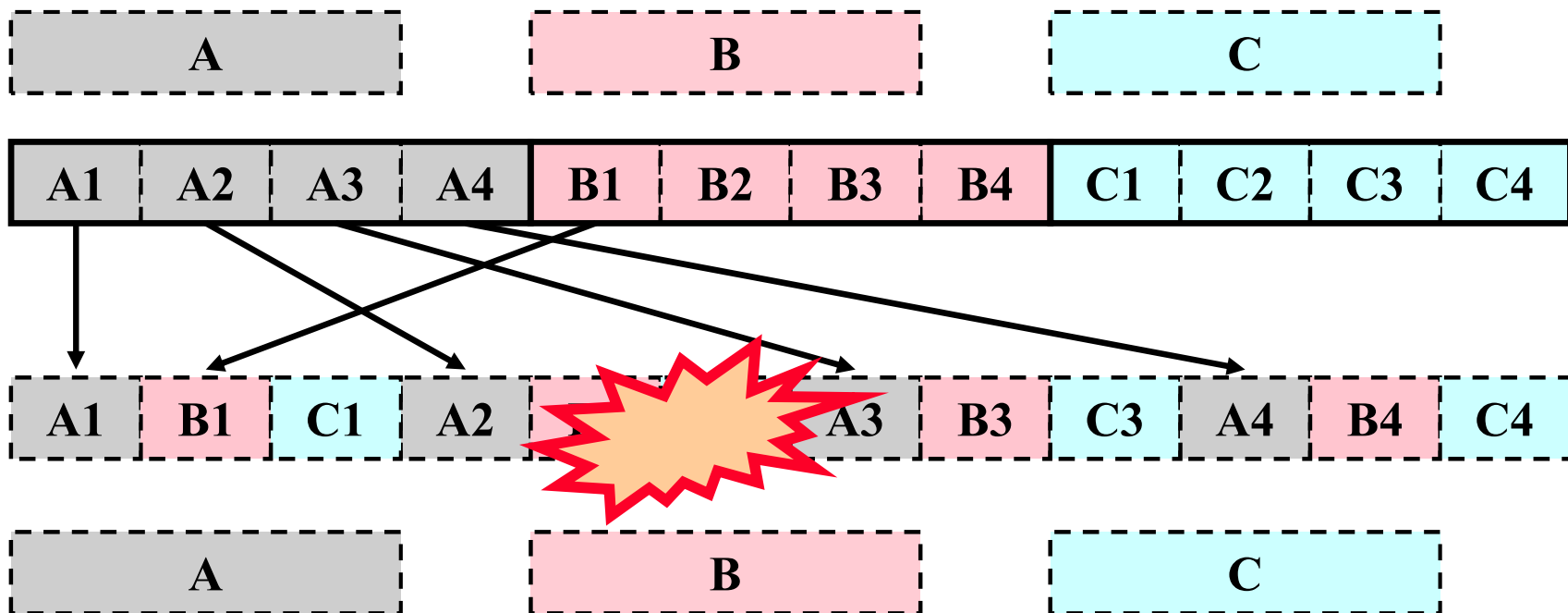
# Adding Redundancy

- **Protects digital data by introducing redundancy in the transmitted data.**
  - » Error detection codes: can identify certain types of errors
  - » Error correction codes: can fix certain types of errors
- **Block codes provide Forward Error Correction (FEC) for blocks of data.**
  - » (n, k) code: n bits are transmitted for k information bits
  - » Simplest example: parity codes
  - » Many different codes exist: Hamming, cyclic, Reed-Solomon, ...
- **Convolutional codes provide protection for a continuous stream of bits.**
  - » Coding gain is  $n/k$
  - » Turbo codes: convolutional code with channel estimation



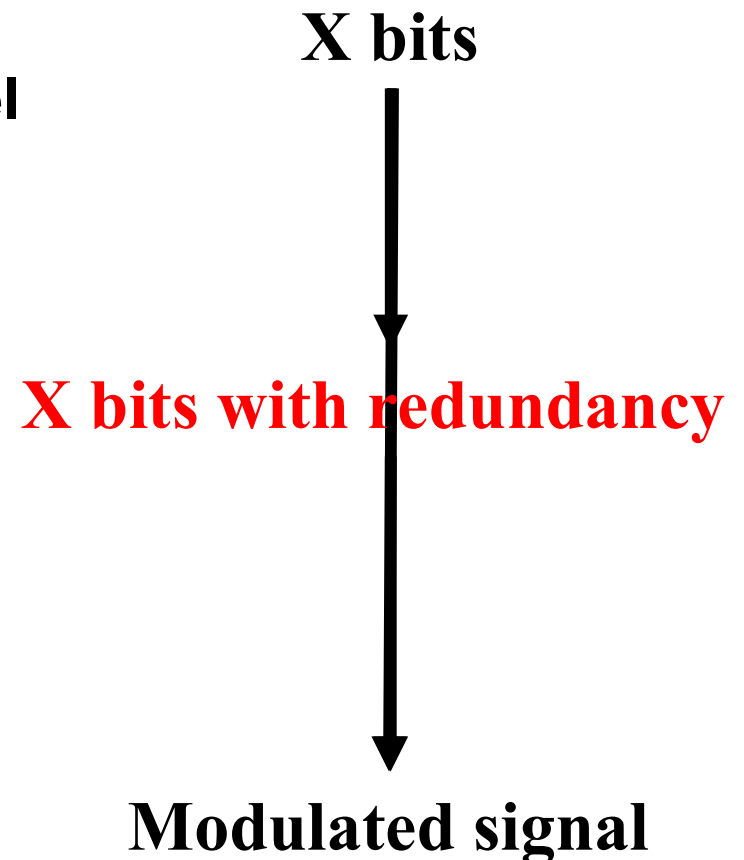
# Combine Redundancy with Time Diversity

- Fading can cause burst errors: a relatively long sequence of bits is corrupted
- Spread blocks of bytes out over time so redundancy can help recover from the burst
  - » Example: only need 3 out of 4 to recover the data



# Bits, Symbols, and Chips

- Redundancy and time diversity can be added easily at the application layer
- Can we do it lower in the stack?
  - » Need to adapt quickly to the channel
- So far: use bits to directly modulate the signal
- Idea: add a coding layer – provides a level of indirection
- Can add redundancy and adjust level of redundancy quickly based on channel conditions



# Discussion

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- **Error coding increases robustness at the expense of having to send more bits**
  - » Technically this means that you need more spectrum
- **But: since you can tolerate some errors, you may be able to increase the bit rate through more aggressive modulation**
- **Coding and modulation combined offer a lot of flexibility to optimize transmission**
- **Next steps:**
  - » Apply a similar idea to frequency diversity - spread spectrum
  - » Combine coding with frequency and time diversity - OFDM

# Summary

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- **Space diversity really helps in overcoming fading**
  - » Very widely deployed
  - » Will build on this when we discuss MIMO
- **Coding is also an effective way to improve throughput**
  - » Widely used in all modern standards
  - » Coding, combined with modulation, can be adapt quickly to channel conditions