
18-452/18-750
Wireless Networks and Applications
Lecture 3: Physical Layer
Capacity and Signal Propagation

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Spring 2024

<http://www.cs.cmu.edu/~prs/wirelessS24/>

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Outline

- **Challenges in Wireless Networking**
- **RF introduction**
- **Modulation and multiplexing**
- **Channel capacity**
- **Antennas and signal propagation**
- **Modulation**
- **Diversity and coding**
- **OFDM and MIMO**

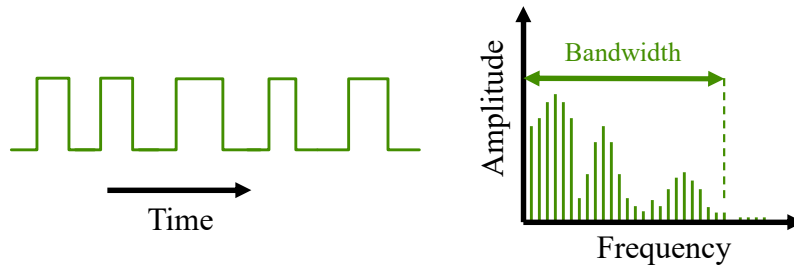
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From Previous Lecture: The Frequency Domain

- A (periodic) signal can be viewed as a sum of sine waves of different strengths.
 - Corresponds to energy at a certain frequency
- Every signal has an equivalent representation in the frequency domain.
 - What frequencies are present and what is their strength (energy)
- Our focus here is on the baseband signal



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Relationship between Data Rate and Spectral Bandwidth

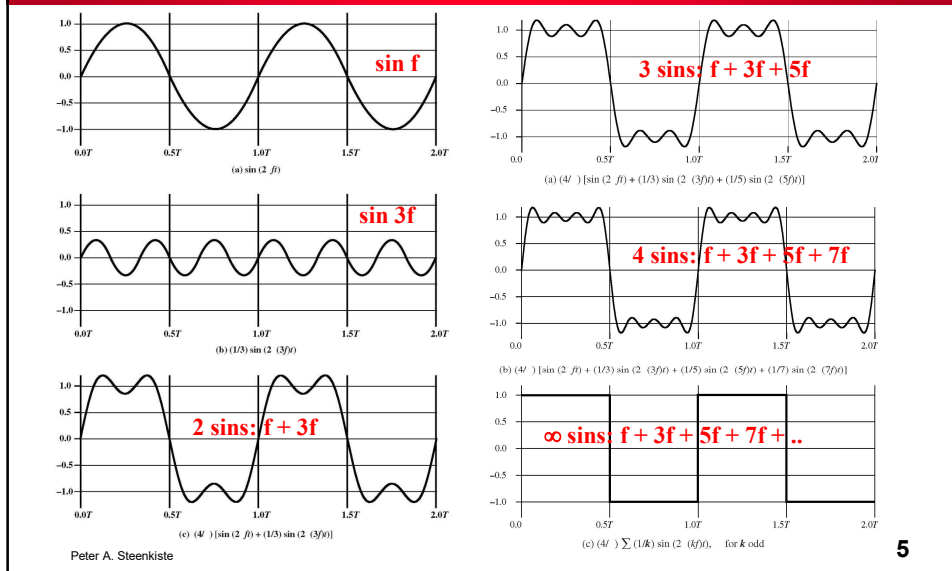
- The larger the (spectral) bandwidth is, the higher the information-carrying capacity of the signal
- Intuition: if a signal can change faster, it can be modulated in a more detailed way and it can thus carry more more information
 - » E.g. more bits or higher fidelity music
- Extreme example: a signal that only changes once a second will not be able to carry a lot of bits or convey a very interesting movie
- Can we make this more precise?

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Adding Detail to the Signal



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Some Intuition

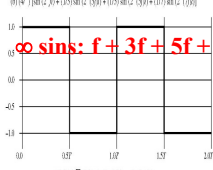
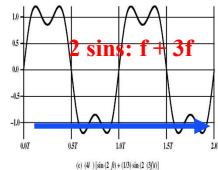
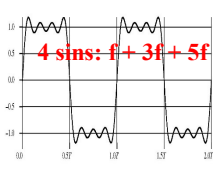
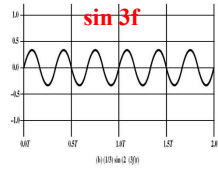
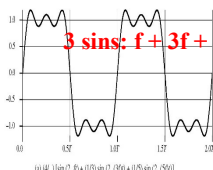
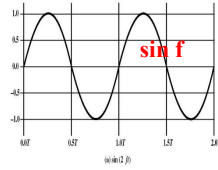
- **A signal that is smooth in the time domain has narrow frequency range**
 - » Sine wave → pulse at exactly one frequency
- **Adding detail widens the frequency range**
 - » Need to add additional frequencies to represent details
 - » Very sharp edges are especially bad (many frequencies)
- **The opposite is also true**
 - » A ulse (very sharp edge!) in time domain has a very wide spectrum
 - » Same is true for random noise (“noise floor”)
- **Implication: modulation has a big impact on how much (scarce) spectrum is used**

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What happens when I Double the Bandwidth?



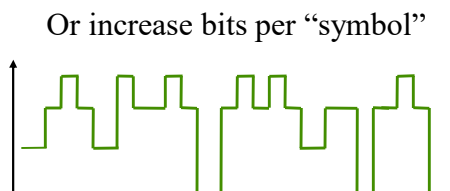
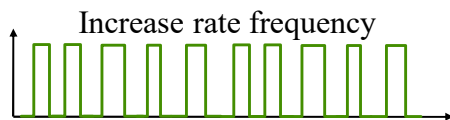
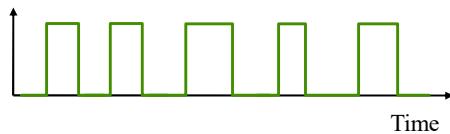
- Shown here by scaling by two along time axis
- What happens to spectrum use?
- We have to double frequencies: $f \rightarrow 2f$
- This means that we double the frequency range

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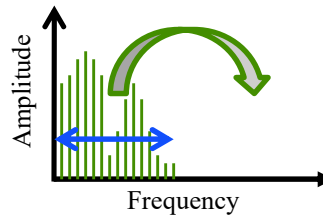
Increasing the Bit Rate



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2 bit symbols – 4 values

- Increases the rate at which the signal changes.
 - » Proportionally increases all signals present, and thus the spectral bandwidth
- Increase the number of bits per change in the signal
 - » Adds detail to the signal, which also increases the spectral BW

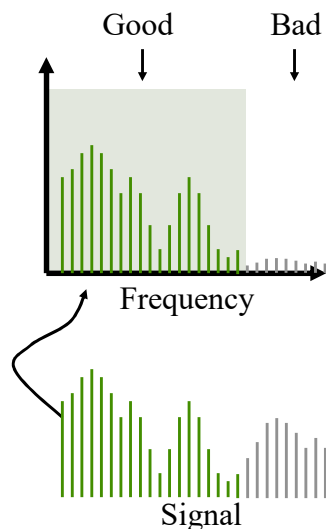


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Transmission Channel Considerations

- **Example: grey frequencies get attenuated significantly**
- **For wired networks, channel limits are an inherent property of the wires**
 - Different types of fiber and copper have different properties
 - Capacity also depends on the radio and modulation used
 - Improves over time, even for same wire
- **For wireless networks, limits are often imposed by policy**
 - Can only use certain part of the spectrum
 - Radio uses filters to comply



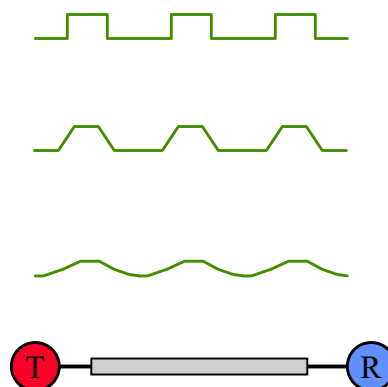
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So Why Don't we Always Send a Very High Bandwidth Signal?

- **Wires only transmit signals in certain frequency ranges**
 - » Stronger attenuation and distortion outside of range
- **Wireless radios are only allowed to use certain parts of the spectrum**
 - » The radios are optimized for that frequency band
- **Attenuation and distortion makes it hard for receiver to extract the information**
 - » A major challenge in wireless



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Propagation Degrades RF Signals

- **Attenuation in free space: signal gets weaker as it travels over longer distances**
 - » Radio signal spreads out – free space loss
 - » Refraction and absorption in the atmosphere
- **Obstacles can weaken signal through absorption or reflection.**
 - » Reflection redirects part of the signal's energy
- **Multi-path effects: multiple copies of the signal interfere with each other at the receiver**
 - » Similar to an unplanned directional antenna
- **Mobility: moving the radios or other objects changes how signal copies add up**
 - » Node moves $\frac{1}{2}$ wavelength -> big change in signal strength

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Channel Capacity

- **Data rate - rate at which data can be communicated (bps)**
 - » Channel Capacity – the maximum rate at which data can be transmitted over a channel under given conditions
- **Bandwidth - the bandwidth of the signal as constrained by the transmitter and the nature of the transmission medium (Hertz)**
 - » Confusing term – network people use the term bandwidth for data rate (in Mbps)
- **Noise – amount of noise in the spectrum band used by the communications channel**
- **Error rate - rate at which errors occur**
 - » Error = transmit 1 and receive 0; transmit 0 and receive 1

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Decibels

- **Decibels: ratio between signal powers**
decibels (db) = $10\log_{10}(P_1 / P_2)$
- **Term is used in many contexts:**
 - » The loss of a wireless channel, gain of an amplifier, ...
 - » Useful when doing a lot of multiplication/division
- **Note that dB is a relative value.**
- **Absolute value requires a reference point.**
 - » Decibel-milliwatt – power relative to 1 milliwatt (dbm)
- **Some example values (WiFi):**
 - » Noise floor -90 dbm
 - » Received signal strength: -70 to -65 dbm
 - » Transmit power (2.4 GHz): 20 dbm

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Signal-to-Noise Ratio

- **Ratio of the power in a signal to the power contained in the noise that is present at a particular point in the transmission**
 - » Typically measured at a receiver
- **Signal-to-noise ratio (SNR, or S/N)**
$$(SNR)_{dB} = 10\log_{10} \frac{\text{signal power}}{\text{noise power}}$$
- **A high SNR means a high-quality signal**
- **Low SNR means that it may be hard to “extract” the signal from the noise**
- **SNR sets upper bound on achievable data rate**

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The Nyquist Limit

- **A noiseless channel of bandwidth B can at most transmit a binary signal at a capacity 2B**
 - » E.g. a 3000 Hz channel can transmit data at a rate of at most 6000 bits/second
 - » Assumes binary amplitude encoding
- **More aggressive encoding can increase the actual channel bandwidth**
 - » More bits per symbol
- **For M levels: $C = 2B \log_2 M$**
 - » M discrete signal levels
- **Factors such as noise can reduce the capacity**

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Shannon Capacity Formula

- **Equation:**
$$C = B \log_2(1 + \text{SNR})$$
- **Represents error free capacity**
 - » It is possible to design a suitable signal code that will achieve error free transmission (you design the code)
- **This result is based on many assumptions**
 - » Formula assumes white noise (thermal noise)
 - » Impulse noise is not accounted for
 - » Various types of distortion are also not accounted for
- **We can also use Shannon's theorem to calculate the noise that can be tolerated to achieve a certain data rate over a channel**

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Shannon Discussion

- **Bandwidth B and noise N are not independent**
 - » N is the noise in the signal band, so it increases with the bandwidth
- **Shannon does not provide the coding that will meet the limit, but the formula is still useful**
- **The performance gap between Shannon and a practical system can be accounted for by a gap parameter**
 - » Gap depends on error rate, coding, modulation, etc.

$$C = B \log_2(1 + \text{SNR}/\Gamma)$$

- » Over time, the gap has been shrinking due to technology improvements

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Example of Nyquist and Shannon Formulations

- **Spectrum of a channel between 3 MHz and 4 MHz ; $\text{SNR}_{\text{dB}} = 24 \text{ dB}$**

$$B = 4 \text{ MHz} - 3 \text{ MHz} = 1 \text{ MHz}$$

$$\text{SNR}_{\text{dB}} = 24 \text{ dB} = 10 \log_{10}(\text{SNR})$$

$$\text{SNR} = 251$$

- **Using Shannon's formula**

$$C = 10^6 \times \log_2(1 + 251) \approx 10^6 \times 8 = 8 \text{ Mbps}$$

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Example of Nyquist and Shannon Formulations

- How many signaling levels are required using Nyquist?

$$C = 2B \log_2 M$$

$$8 \times 10^6 = 2 \times (10^6) \times \log_2 M$$

$$4 = \log_2 M$$

$$M = 16$$

- Look out for: dB versus linear values and \log_2 versus \log_{10}

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Outline

- Challenges in Wireless Networking
- RF introduction
- Modulation and multiplexing
- Channel capacity
- Antennas and signal propagation
 - » How do antennas work
 - » Propagation properties of RF signals
 - » Modeling the channel
- Modulation
- Diversity and coding
- OFDM and MIMO

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What is an Antenna?

- **Conductor that carries an electrical signal and radiates an RF signal.**
 - » The RF signal “is a copy of” the electrical signal in the conductor
- **Also the inverse process: RF signals are “captured” by the antenna and create an electrical signal in the conductor.**
 - » This signal can be interpreted (i.e. decoded)
- **Efficiency of the antenna when transmitting or receiving depends on its size and orientation**
 - » Transmitting: How much energy does it radiate?
 - » Receiving: How much of the energy radiated by the transmitter does it capture?

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Types of Antennas

- **Abstract view: antenna is a point source that radiates with the same power level in all directions – omni-directional or isotropic.**
 - » In practice, the shape of the conductor creates a specific radiation pattern
 - » Note that isotropic antennas are not very efficient!!
 - Unless you have a very large number of receivers
- **A common shape is a straight conductor.**
 - » Creates a “disk” pattern, e.g. dipole
- **Shaped antennas can be used to direct the energy in a certain direction.**
 - » Well-known case: a parabolic antenna

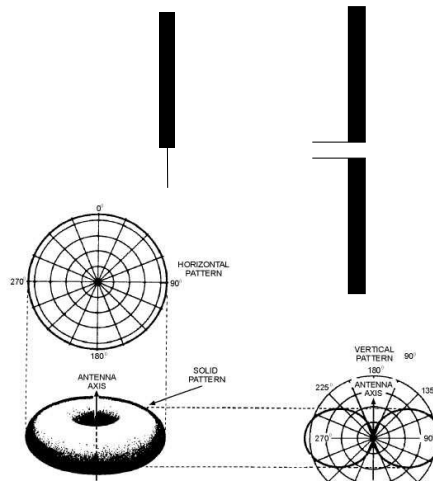
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Antenna Types: Dipoles

- **General rule: length of the antenna should be ~half a wavelength**
 - » Length depends on the carrier frequency!
 - » Wavelength at 900MHz: ~1 foot
- **Simplest antenna: half wave, quarter wave or dipole antenna**
 - » Very simple and very common
 - » Quarter wavelength is replicated using a reflective surface
 - » Donut shape radiation

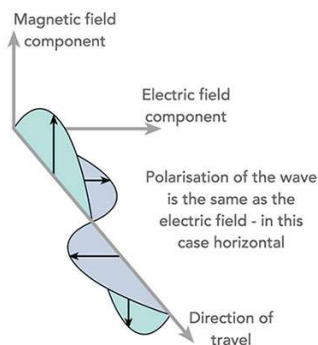


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What about Antenna Orientation? Polarization?



- A “wireless” signal consists of an electric and magnetic field
 - » “electro magnetic” signal
- They can both be viewed as sine waves traveling in orthogonal planes
- The orientation of the electric field is called “polarization” of the signal
- The polarization of the signal is determined by the sending antenna
- In order for reception to be effective, the signal and receiving antenna must have the same polarization
 - » But reflections, ... can change polarization

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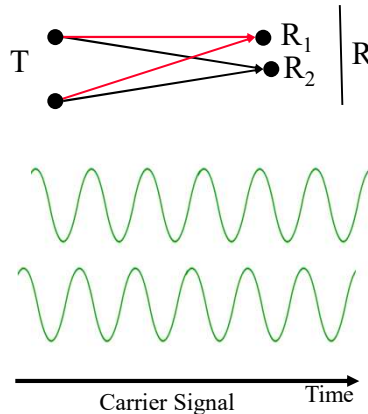
Figure from: <https://www.digikey.com/en/blog/antenna-polarization-what-it-is-and-why-it-matters>

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Preview: Multi-element Antennas

- **Multi-element antennas have multiple, independently controlled conductors.**
 - » Signal is the sum of the individual signals transmitted (or received) by each element
- **Can electronically direct the RF signal by sending different versions of the signal to each element.**
 - » For example, change the phase in a two-element array.
- **There are many variants of multi-element antennas**
 - » Number of elements, relative position of the elements, control over the signals, ...

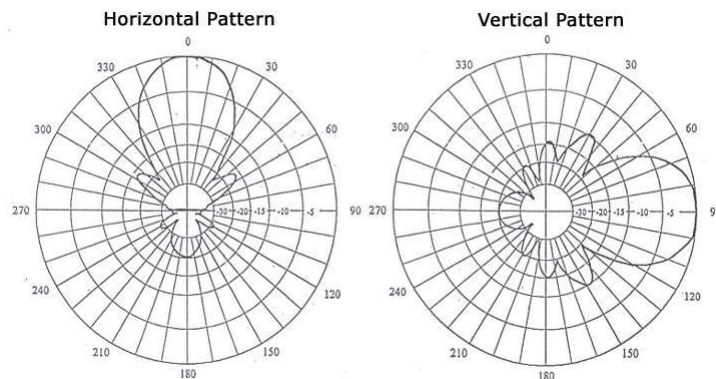


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Directional Antenna Properties



- **dBi: antenna gain in dB relative to an isotropic antenna with the same transmit power**
 - » Example: an 8 dBi Yagi antenna has a gain of a factor of 6.3 ($8 \text{ db} = 10 \log 6.3$)

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Examples 2.4 GHz



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Summary

- **The maximum capacity of a channel depends on the SINR**
 - » How close you get to this maximum depends on the sophistication of the radios
 - » Distortion of the signal also plays a role – next lecture
- **Antennas are responsible for transmitting and receiving the EM signals**
 - » The “ideal” isotropic antenna is a point source that radiates energy in a sphere
 - » Practical antennas are directional in nature, as a result of the antenna shape or the use of multi-element antennas
 - » The antenna gain is expressed in dBi

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- **Antennas and signal propagation**
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- **Modulation**
- **Diversity and coding**
- **OFDM**

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Propagation Modes

- **Line-of-sight (LOS) propagation.**
 - » Most common form of propagation
 - » Happens above ~ 30 MHz
 - » Subject to many forms of degradation (next set of slides)
- **Obstacles can redirect the signal and create multiple copies that all reach the receiver**
 - » Creates multi-path effects
- **Refraction changes direction of the signal due to changes in density**
 - » E.g., changes in air temperature, humidity, ...
 - » If the change in density is gradual, the signal bends!

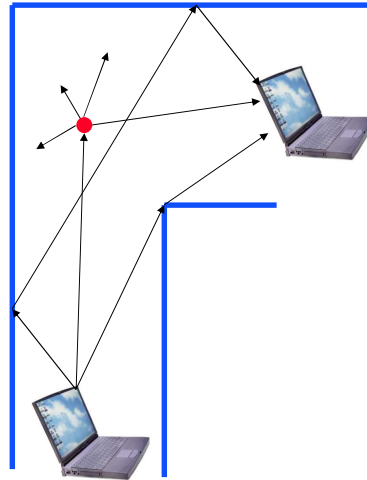
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Impact of Obstacles

- Besides line of sight, signal can reach receiver in three “indirect” ways.
- Reflection: signal is reflected from a large object.
 - » Large: relative to the wavelength
- Scattering: signal is scattered by an object that is small
- Diffraction: signal is scattered by the edge of a large object – “bends”.



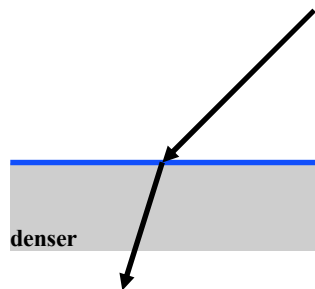
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Refraction

- Speed of EM signals depends on the density of the material
 - » Vacuum: 3×10^8 m/sec
 - » Denser: slower
- Density is captured by refractive index
- Explains “bending” of signals in some environments
 - » E.g. sky wave propagation: Signal “bounces” off the ionosphere back to earth – can go very long distances
 - » But also local, small scale differences in the air density, temperature, etc.



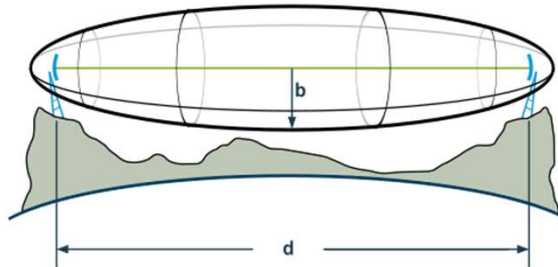
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Fresnel Zones

- Sequence of ellipsoids centered around the LOS path between a transmitter and receiver
- The zones identify areas in which obstacles will have different impact on the signal propagation
 - » Capture the constructive and destructive interference due to multipath caused by obstacles



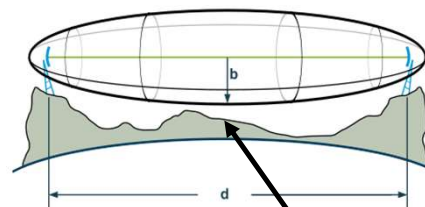
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Fresnel Zones

- Zones create different phase differences between paths
 - » First zone: 0-90
 - » Second zone: 90-270
 - » Third zone: 270-450
 - » Etc.
- Odd zones create constructive interference, even zones destructive
- Also want clear path in most of the first Fresnel zone, e.g. 60%
- The radius F_n of the nth Fresnel zone depends on the distances d_1 and d_2 to the transmitter and receiver and the wavelength



Ground
Buildings
Etc.

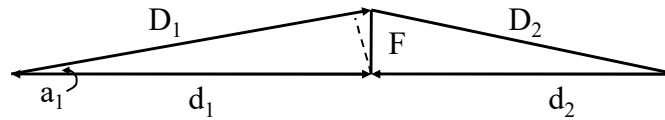
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Sketch of Calculation: Difference in Path Length

FYI only



- Goal is to calculate F
- Difference in path length (a_1 is small)
 - » $D_1 - d_1 \approx F \cdot \sin a_1$
- But for small a_1 we also have
 - » $\sin a_1 = \tan a_1 = F / d_1$
- So $D_1 - d_1 = F^2 / d_1$

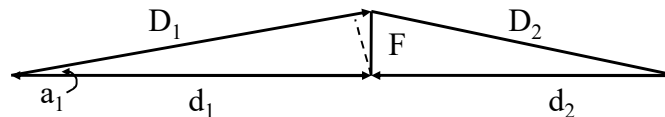
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Sketch of Calculation Fresnel Radios

FYI only



- Given $D_1 - d_1 = F^2 / d_1$
- So, the difference in path length is:
- $(D_1 + D_2) - (d_1 + d_2) = \lambda \cdot n$
 - Or $(D_1 - d_1) + (D_2 - d_2) = F^2 / d_1 + F^2 / d_2$
 - or
$$F_n = \sqrt{\frac{n\lambda d_1 d_2}{d_1 + d_2}}$$

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