# Algorithms for solving sequential (zero-sum) complete-information games

Tuomas Sandholm

# CHESS,

# MINIMAX SEARCH,

# AND IMPROVEMENTS TO MINIMAX SEARCH


1996

1996

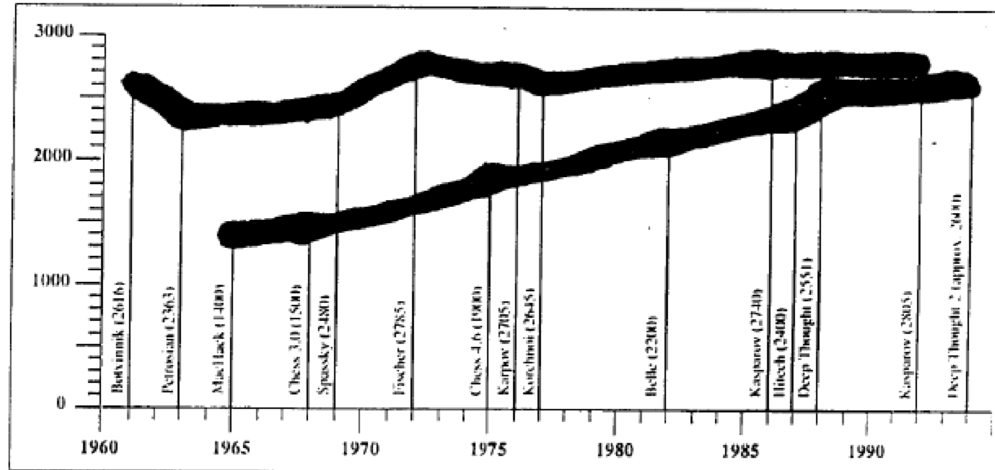



**Figure 5.12** Ratings of human and machine chess champions.

*Deep Blue team. Front, left to right: Joel Benjamin, Chung-Jen Tan. Back, left to right: Jerry Brody, Murray Campbell, Feng-Hsiung Hsu, and Joe Hoane.*

1997

3½ - 2½

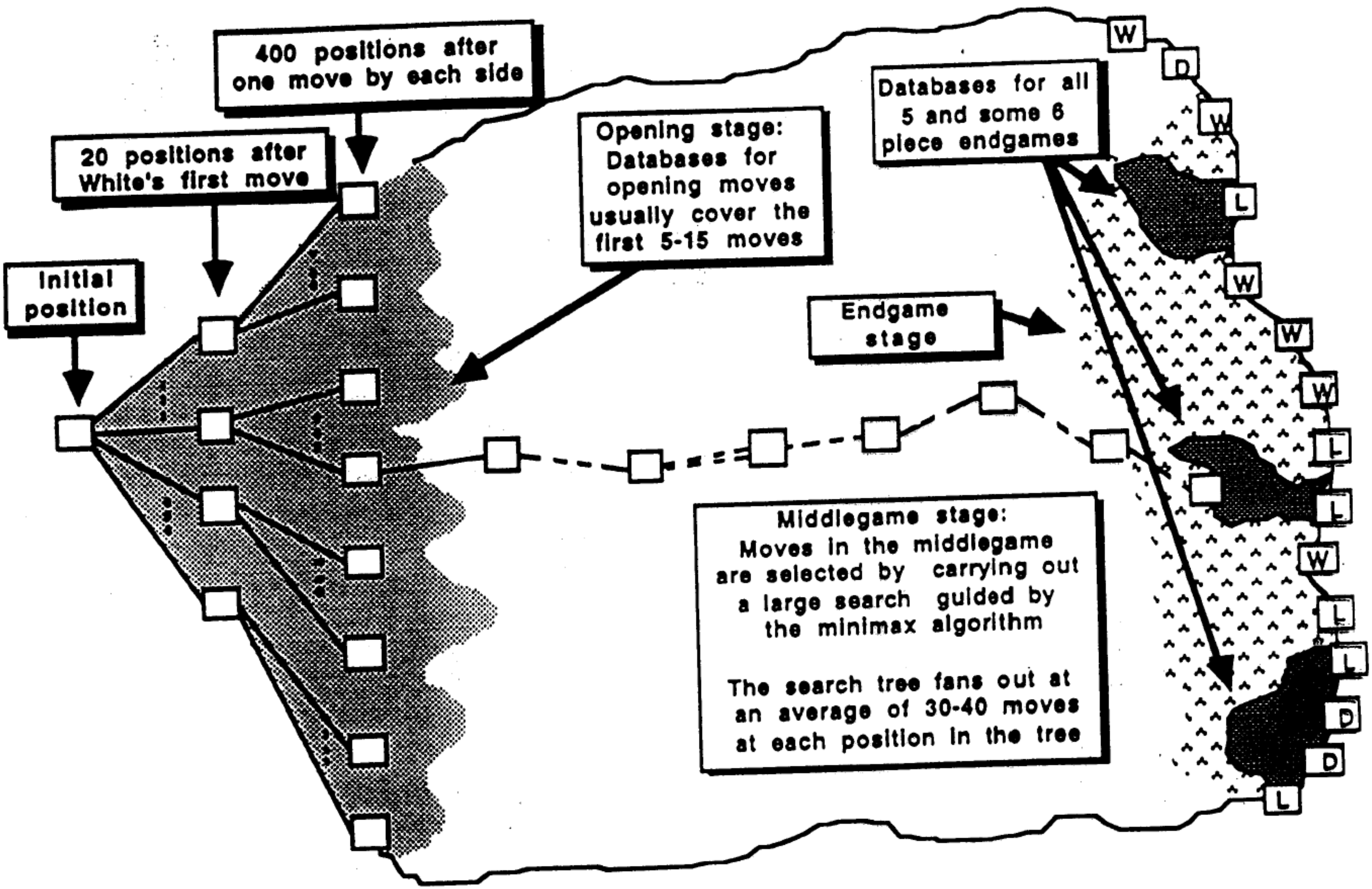
Loss-win-draw-draw-draw-win

# Rich history of cumulative ideas

| | | |
|---|---|---|
| Claude Shannon, Alan Turing | Minimax search with scoring function | 1950 |
| Kotok/McCarthy Program & ITEP Program | Alpha-beta search, brute force search | 1966 |
| Mac Hack | Transposition tables | 1967 |
| Chess 3.0–Chess 4.9 | Iteratively-deepening depth-first search | 1975 |
| Belle | Special-purpose circuitry | 1978 |
| Cray Blitz | Parallel search | 1983 |
| Hitech | Parallel evaluation | 1985 |
| Deep Blue | Parallel search and special-purpose circuitry | 1987 |

Quiescence search 1960's?
End game databases via dynamic programming 1977
Conspiracy numbers 1988
Singular extension 1980's
Opening books
Evaluation function learning & engineering 1950's
⋮

# Chess game tree



# Opening books (available electronically too)

*Example opening where the book goes 16 moves (32 plies) deep*

## RUY LOPEZ

### *Marshall (Counter) Attack*

**1 e4 e5 2 Nf3 Nc6 3 Bb5 a6 4 Ba4 Nf6 5 0–0 Be7 6 Re1 b5 7 Bb3 0–0 8 c3 d5 9 exd5**

| | 97 | 98 | 99 | 100 | 101 | 102 |
|---|---|---|---|---|---|---|
| | Nxd5 ........ | ........ | ........ | ........ | ........ | ........e4 |
| 10 | Nxe5<br>Nxe5 | | | | | dxc6(p)<br>exf3 |
| 11 | Rxe5<br>c6! ........ | <br>........ | <br>........ | <br>........ | <br>........Nf6(l) | d4!(q)<br>fxg2(r) |
| 12 | d4 ........<br>Bd6 | ........ | ........Bxd5<br>cxd5 | ........g3(h)<br>Bd6(i) | d4<br>Bd6 | Qf3<br>Be6 |
| 13 | Re1 ........<br>Qh4 | ........Re2<br>Bg4(c) | d4<br>Bd6 | Re1<br>Qd7!(j) | Re1<br>Ng4 | Bf4<br>Nd5 |
| 14 | g3<br>Qh3 | f3<br>Bh5 | Re3<br>Qh4(f) | d3<br>Qh3 | h3<br>Qh4(m) | Bg3<br>a5 |
| 15 | Be3(a)<br>Bg4 | Bxd5(d)<br>cxd5 | h3<br>Qf4 | Re4<br>Qf5 | Qf3<br>Nxf2 | Nd2 ± |
| 16 | Qd3<br>Rae8(b) | Nd2<br>Qc7(e) | Re5<br>Qf6(g) | Nd2<br>Qg6(k) | Re2(n)<br>Ng4(o) | |

(a) 15 Re4? g5 16 Qf3 (16 Bxg5?? Qf5) 16 . . . Bf5 17 Bc2 (17 Bf4!?) 17 . . . Bxe4 18 Bxe4 Qe6 19 Bxg5 (19 Bf5? Qe1† 20 Kg2 Qxc1 21 Na3 Qd2 wins) 19 . . . f5 20 Bd3 h6 ∓ (Gutman).

(b) Short–Pinter, Rotterdam, 1988 continued 17 Nd2 Re6 18 a4 bxa4 19 Rxa4 f5 20 Qf1 Qh5 21 f4 Rb8 22 Bxd5 cxd5 23 Rxa6 Rbe8 24 Qb5 Qf7 25 h3! with complications favoring White.

(c) 13 . . . Qh4 14 g3 Qh5 (14 . . . Qh3 15 Nd2 Bf5 16 Ne4!?) 15 Nd2 Bg4 16 f3 Bxf3 17 Nxf3 Qxf3 18 Rf2 Qe4 19 Qf3 ⩲, Sax–P. Nikolić, Plovdiv 1983.

(d) If 15 Nd2 Nf4 is annoying.

(e) 17 Nf1 Rfe8 18 Be3 Qc4 ∞, van der Sterren–Pein, Brussels 1984. Black has good play for the pawn.

(f) 14 . . . f5 15 Nd2 f4 16 Re1 Qg5 17 Nf3 Qh5 18 Ne5 f3 19 gxf3 Bh3 20 f4 ± (Tal).

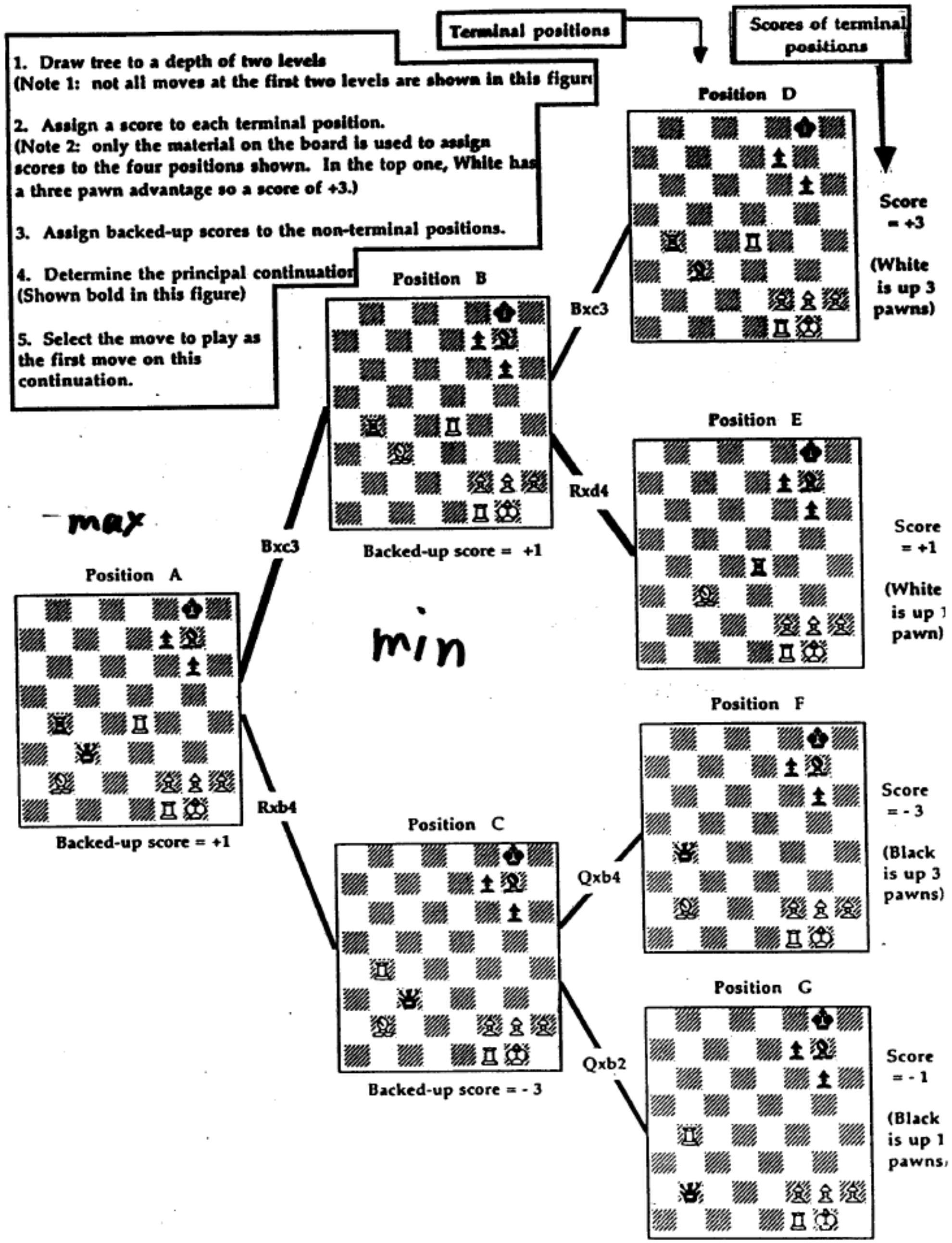
(g) 17 Re1 Qg6 18 Qf3 Be6 19 Bf4 Bxf4 20 Qxf4 Bxh3 21 Qg3 Qxg3 =, Tal–Spassky, match 1965.

(h) 12 d3 Bd6 13 Re1 (13 . . . Qh4 14 g3 Qh3 transposes back into the column) 13 . . . Bf5! 14 Nd2 Nf4 15 Ne4 Nxd3 16 Bg5 Qd7 17 Re3 Bxe4 18 Rxe4 Rae8 =, Kir. Georgiev–Nunn, Dubai 1986.

(i) Geller's 12 . . . Bf6 13 Re1 c5 14 d4 Bb7, playing for central control, is a reasonable alternative.

(j) 13 . . . Nf6 14 d4 Bg4 15 Qd3 c5 16 Bc2 is better for White, according to Fischer.

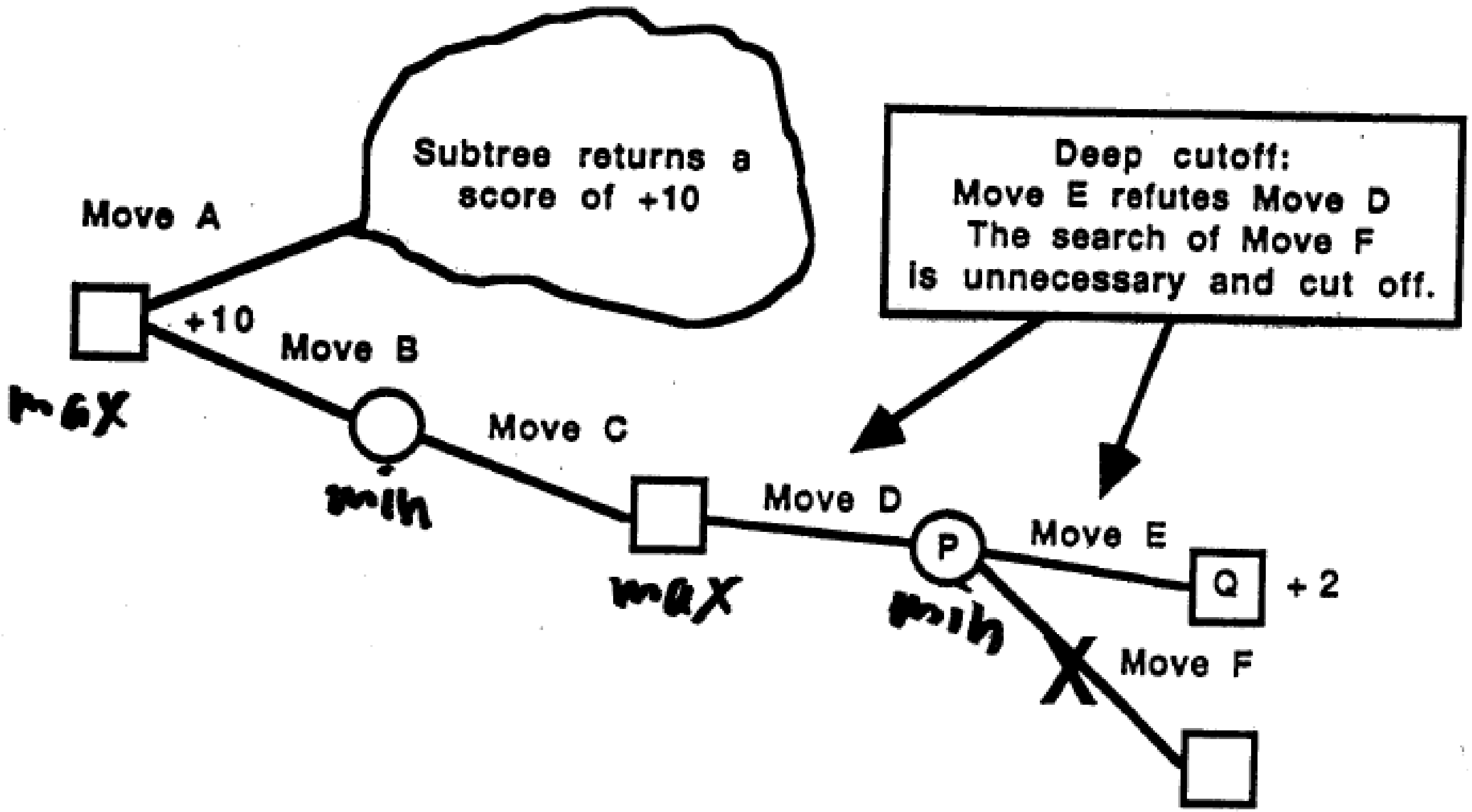
# Minimax algorithm (not all branches are shown)



# Search depth pathology
(although in practice in games deeper search yields stronger play)

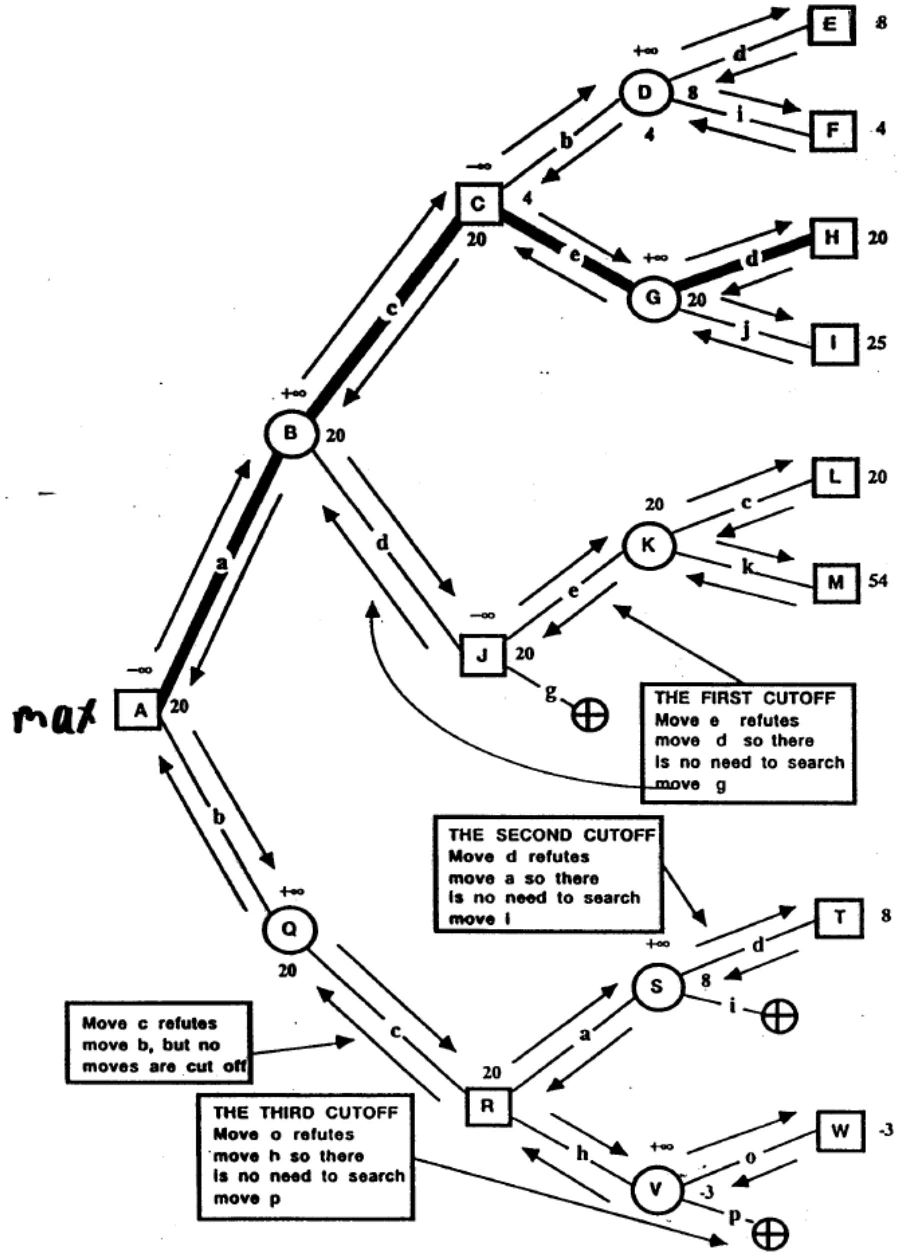
- Beal (1980) and Nau (1982, 83) analyzed whether values backed up by minimax search are more trustworthy than the heuristic values themselves. The analyses of the model showed that backed-up values are somewhat less trustworthy
- Anomaly goes away if sibling nodes' values are highly correlated [Beal 1982, Bratko & Gams 1982, Nau 1982]
- Pearl (1984) partly disagreed with this conclusion, and claimed that while strong dependencies between sibling nodes can eliminate the pathology, practical games like chess don't possess dependencies of sufficient strength
  - He pointed out that few chess positions are so strong that they cannot be spoiled abruptly if one really tries hard to do so
  - He concluded that success of minimax is "based on the fact that common games do not possess a uniform structure but are riddled with early terminal positions, colloquially named blunders, pitfalls or traps. Close ancestors of such traps carry more reliable evaluations than the rest of the nodes, and when more of these ancestors are exposed by the search, the decisions become more valid."
- Still not fully understood. For newer results, see:
  - Sadikov, Bratko, Kononenko. (2003) Search versus Knowledge: An Empirical Study of Minimax on KRK, In: van den Herik, Iida and Heinz (eds.) Advances in Computer Games: Many Games, Many Challenges, Kluwer Academic Publishers, pp. 33-44
  - Understanding Sampling Style Adversarial Search Methods [PDF]. Raghuram Ramanujan, Ashish Sabharwal, Bart Selman. UAI-2010, pp 474-483.
  - On Adversarial Search Spaces and Sampling-Based Planning [PDF]. Raghuram Ramanujan, Ashish Sabharwal, Bart Selman. ICAPS-2010, pp 242-245.
- Also present in imperfect-information games when one party has limited lookahead [Kroer & Sandholm IJCAI-15; Kroer, Farina & Sandholm AAAI-18]

# α-β -pruning




Partially drawn game tree showing deep alpha-beta cutoff

# α-β -search on ongoing example



# α-β -search

**function** MAX-VALUE(*state, game*, $\alpha$, $\beta$) **returns** the minimax value of *state*
  **inputs**: *state*, current state in game
        *game*, game description
        $\alpha$, the best score for MAX along the path to *state*
        $\beta$, the best score for MIN along the path to *state*

  **if** CUTOFF-TEST(*state*) **then return** EVAL(*state*)
  **for each** *s* **in** SUCCESSORS(*state*) **do**
    $\alpha \leftarrow$ MAX($\alpha$, MIN-VALUE(*s, game*, $\alpha$, $\beta$))
    **if** $\alpha \geq \beta$ **then return** $\beta$
  **end**
  **return** $\alpha$

---

**function** MIN-VALUE(*state, game*, $\alpha$, $\beta$) **returns** the minimax value of *state*

  **if** CUTOFF-TEST(*state*) **then return** EVAL(*state*)
  **for each** *s* **in** SUCCESSORS(*state*) **do**
    $\beta \leftarrow$ MIN($\beta$, MAX-VALUE(*s, game*, $\alpha$, $\beta$))
    **if** $\beta \leq \alpha$ **then return** $\alpha$
  **end**
  **return** $\beta$

# Complexity of α-β -search

Best case

| Search Depth (DMAX) | Minimum number of terminal positions in an alpha-beta search | |
|---|---|---|
| 2 | $\sim 2 \times 30^1 \approx 6 \times 10^1$ | = 60 |
| 4 | $\sim 2 \times 30^2 \approx 2 \times 10^3$ | = 2,000 |
| 6 | $\sim 2 \times 30^3 \approx 6 \times 10^4$ | = 60,000 |
| 8 | $\sim 2 \times 30^4 \approx 2 \times 10^6$ | = 2,000,000 |
| 10 | $\sim 2 \times 30^5 \approx 6 \times 10^7$ | = 60,000,000 |
| 12 | $\sim 2 \times 30^6 \approx 2 \times 10^9$ | = 2,000,000,000 |
| 14 | $\sim 2 \times 30^7 \approx 6 \times 10^{10}$ ≈ Deep Blue | = 60,000,000,000 |
| 16 | $\sim 2 \times 30^8 \approx 2 \times 10^{12}$ | = 2,000,000,000,000 |

[Knuth & Moore 1975]

Best case: α-β allows search 2x as deep as minimax.

Worst case: α-β does not prune a single node.

Average case based on random order of moves $O(b^d) \rightarrow O((b/\log b)^d)$,

Close to best case by exploring better moves first

- captures → threats → forward moves → backward moves

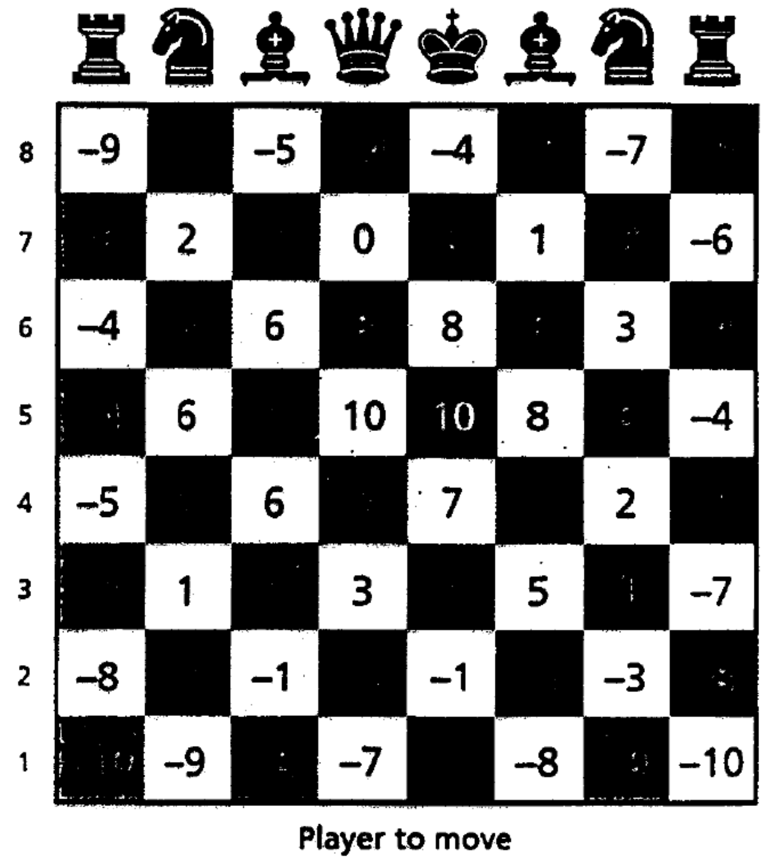
hash table → - iterative deepening search and use backed up values from one iteration to determine the ordering of successors in the next iteration.

Variance in search time (due to α-β and quiescence search)

⇒ iterative deepening (used by all major chess programs).

# Evaluation function

- **Difference (between player and opponent) of**
  - **Material**
  - **Mobility**
  - **King position**
  - **Bishop pair**
  - **Rook pair**
  - **Open rook files**
  - **Control of center (piecewise)**
  - **Others**




Values of knight's position in Deep Blue

# Evaluation function...

- Deep Blue used ~6,000 different features in its evaluation function (in hardware)
- A different weighting of these features is downloaded to the chips after every real world move (based on current situation on the board)
  - Contributed to strong positional play
- Acquiring the weights for Deep Blue
  - Weight learning based on a database of 900 grand master games (~120 features)
    - Alter weight of one feature => 5-6 ply search => if matches better with grand master play, then alter that parameter in the same direction further
    - Least-squares with no search
  - Manually: Grand master Joel Benjamin played take-back chess. At possible errors, the evaluation was broken down, visualized, and weighting possibly changed

Deep Blue ~~is~~ brute force

Smart search and knowledge engineered evaluation

  - Other learning is possible, e.g., Tesauro's Backgammon programs
    - Neurogammon [1989]
      - Taught using supervised learning on 400 games
      - Level: intermediate human player
    - TD-Gammon [1992]: Reinforcement learning; Level: world-class human tournament player

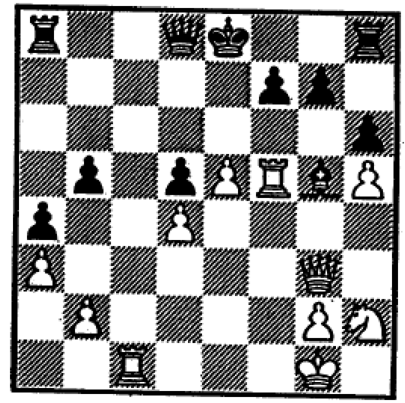
## Databases of expert games
- Deep Blue does not use these during play
- Deep Blue uses them offline to learn evaluation f.

**332.*** **C 02**

KUPREJČIK 2520 — VLADO KOVAČEVIĆ 2545
Ljubljana/Rogaška Slatina 1989

**1. e4 e6 2. d4 d5 3. e5 c5 4. c3 ♘e7 5. ♘f3 ♘ec6 6. ♗e3!?** N [6. h4 — 46/343; RR 6. ♗d3 N b6 7. ♗g5 ♕d7 8. 0—0 ♗a6 9. dc5 bc5 10. ♗a6 ♘a6 11. c4 h6 12. ♗h4 ♘c7 13. ♘c3 ♗e7 14. ♗e7 ♘e7 15. ♖c1 ♖c8 16. ♕e2 0—0 17. ♖fd1 ♕c6 18. b3± Svešnikov 2435 — Lputjan 2610, Moskva (GMA) 1989] **♘d7** [6... b6] **7. ♗d3 a5** [7... ♗e7] **8. ♘bd2** [8. ♘g5!? cd4 9. cd4 ♗e7 (9... h6?! 10. ♕h5 hg5 11. ♕h8 ♘b4 12. ♕h7 g6 13. ♗g6+—) 10. h4!? (10. ♕h5? ♗g5! 11. ♗g5 ♕b6∓) ♕b6 (10... h6 11. ♕h5) 11. ♘c3±] **cd4 9. cd4 a4 10. a3** [10. ♘g5!] **♗e7 11. h4** [11. 0—0] **h6 12. h5 ♘b6∞ 13. ♘h2 ♘a5 14. ♕g4 ♗f8** [14... ♔f8 15. ♖c1 △ 0—0, f4-f5↑] **15. ♖c1** [⌓ 15. ♕e2 ♗d7 16. f4] **♗d7 16. 0—0 ♘bc4! 17. ♘c4 ♘c4 18. ♕e2** [18. ♗c4 dc4 19. d5 ed5 20. ♕d4 ♗f5! 21. g4 ♗d3∓; 19. f4!?] **b5** [18... ♖c8!?] **19. f4 ♗e7 20. f5!?** [20. ♗c4 dc4 (20... bc4 21. g4↑) 21. f5!? (21. d5 ed5 22. f5 d4! 23. ♗d4 ♗f5∓; 22. ♗d4!?⩱) ef5 22. d5∞] **ef5** [20... ♗g5? 21. ♗c4 bc4 (21... dc4 22. d5↑) 22. ♗g5 ♕g5 23. f6±] **21. ♗f5 ♘e3 22. ♕e3 ♗g5 23. ♕g3 ♗f5 24. ♖f5**

(diagram)



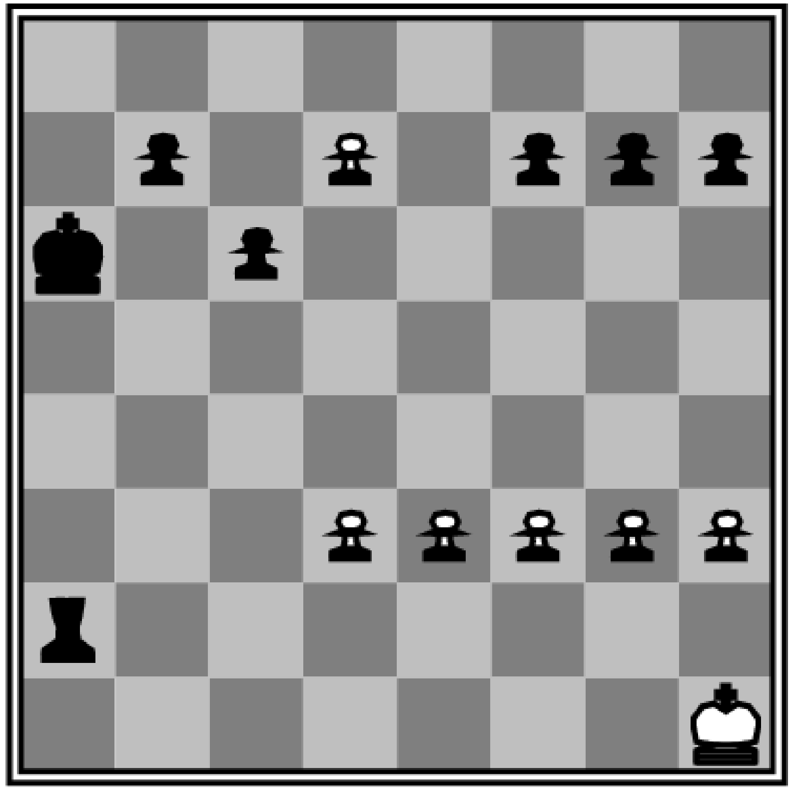
**24... ♖c8?** [24... ♗c1! 25. ♕g7 ♖f8 *a)* 26. ♘g4 ♖a6 (26... ♗g5 27. e6 ♖a7 28. ♘c5 ♕d6 29. ef7 ♔d8 30. ♘c6±) 27. ♘f6 ♖f6 (27... ♔e7 28. ♘g8 ♔d7 29. ♖f7 ♖f7 30. ♕f7 ♔c8 31. e6 ♗e3 32. ♔f1∞) 28. ef6 ♕d6□ 29. ♖e5 ♔d8 30. ♖e7 ♖e8 31. ♖e8 ♔e8 32. ♕g8 ♕f8 33. ♕g3! ♔d7 34. ♕h3 ♔d8 35. ♕g3=; *b)* 26. e6!? ♕d6! 27. ef7!? (27. ♖f7 0-0-0 28. e7 ♖f7 29. ♕f7 ♗b2! 30. e8♕ ♗d4 31. ♔h1 ♖e8 32. ♕e8 ♔c7∓) ♔d7 28. ♘f3 (28. ♘g4!? △ ♖d5) ♔c7 29. ♖f6! ♕e7 30. ♕g6∞] **25. ♖cf1 0—0 26. e6!± ♕c7** [26... f6 27. ♕f3±] **27. ♕e1! ♕e7** [27... ♗f6 28. ♖f6 gf6 29. ♘g4 fe6 30. ♕e6 ♔g7 31. ♖f6±; 27... f6 28. ♖d5±] **28. ♖f7 ♖f7 29. ♖f7 ♖c1** [29... ♕d6 30. ♖d7 ♕b6 31. ♕e5 ♗f6 32. ♕d5+—] **30. ♕c1 ♕e6 31. ♖f4 1 : 0** **[Kuprejčik]**

**333.**** **C 02**

KUPREJČIK 2520 — KOSTEN 2505
Torcy 1989

**1. e4 e6 2. d4 d5 3. e5 c5 4. c3 ♘c6 5. ♘f3 ♗d7 6. ♗e2** [RR 6. ♗d3 ♘ge7 7. 0—0 cd4 8. cd4 ♘c8 N 9. ♘c3 ♗e7 10.

# Horizon problem



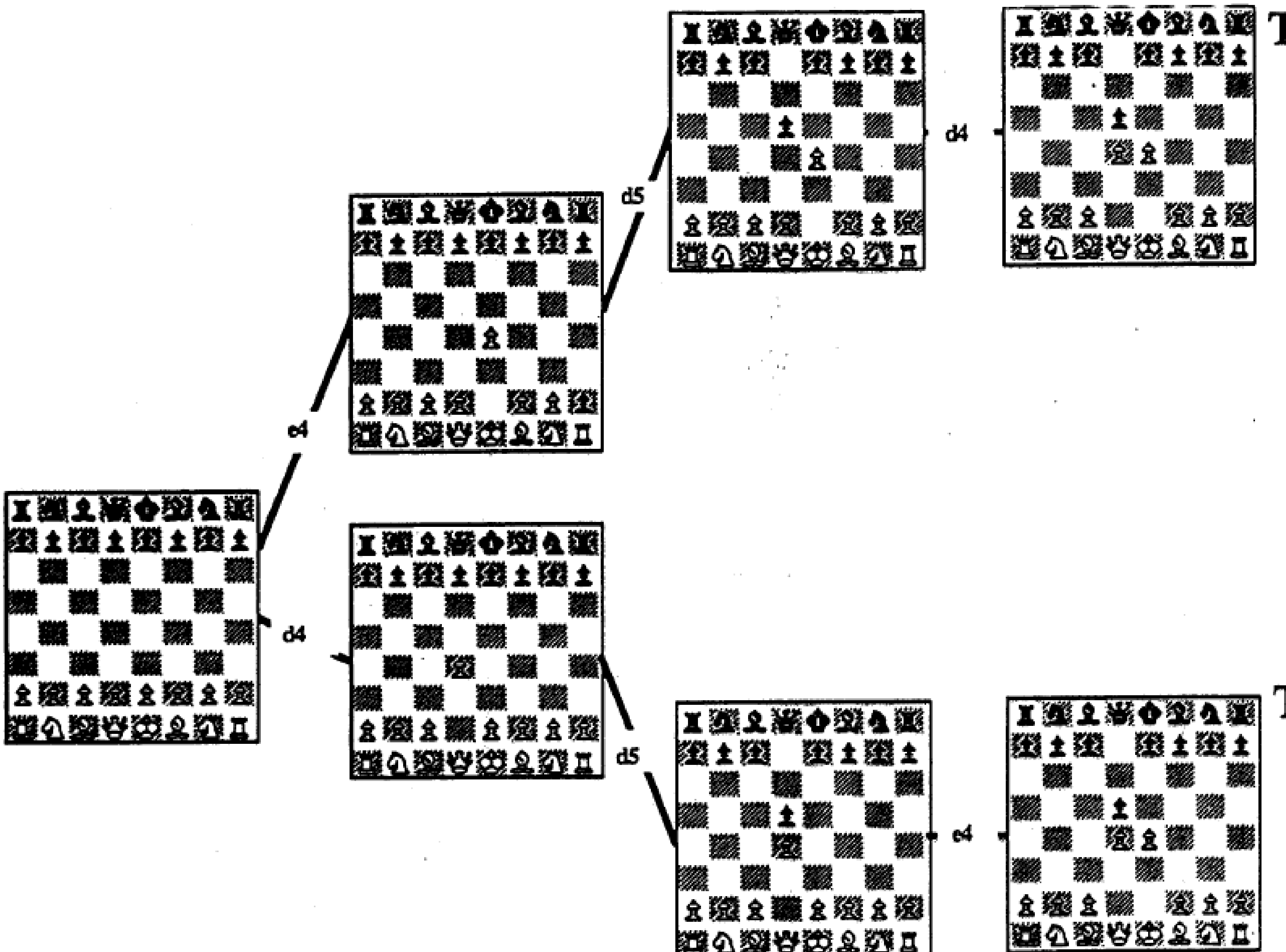
Black to move

A series of checks by the black rook forces the inevitable queening move by white "over the horizon" and makes this position look like a slight advantage for black, when it is really a sure win for white.

# Ways to tame the horizon problem

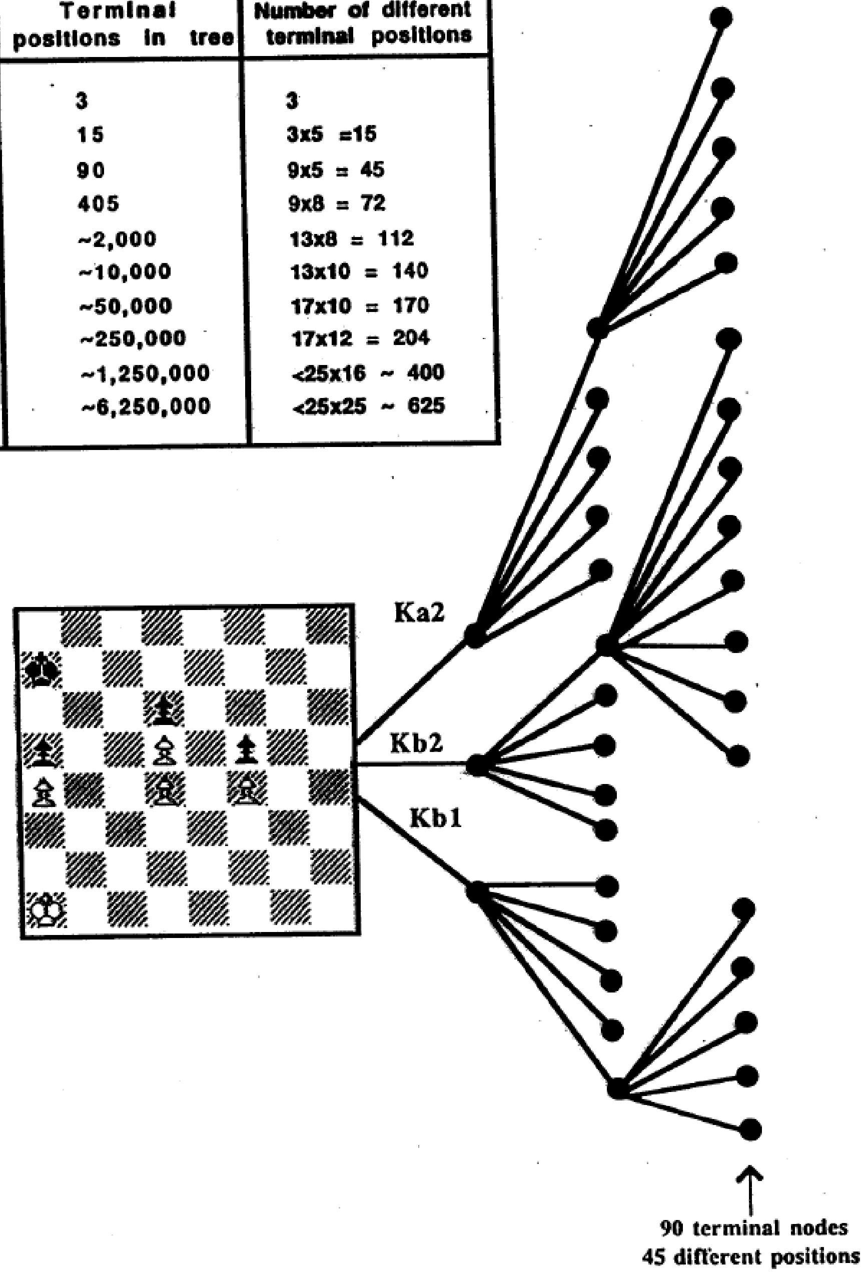
- Quiescence search
  - Evaluation function (domain specific) returns another number in addition to evaluation: stability
    - Threats
    - Other
  - Continue search (beyond normal horizon) if position is unstable
  - Introduces variance in search time
- Singular extension
  - Domain independent
  - A node is searched deeper if its value is much better than its siblings’
  - Even 30-40 ply
  - A variant is used by Deep Blue

# Transpositions



# Transpositions are important

| Depth of Search | Terminal positions in tree | Number of different terminal positions |
|---|---|---|
| 1 | 3 | 3 |
| 2 | 15 | 3x5 =15 |
| 3 | 90 | 9x5 = 45 |
| 4 | 405 | 9x8 = 72 |
| 5 | ~2,000 | 13x8 = 112 |
| 6 | ~10,000 | 13x10 = 140 |
| 7 | ~50,000 | 17x10 = 170 |
| 8 | ~250,000 | 17x12 = 204 |
| 9 | ~1,250,000 | <25x16 ~ 400 |
| 10 | ~6,250,000 | <25x25 ~ 625 |



# Transposition table

- Store millions of positions in a hash table to avoid searching them again
  - Position
  - Hash code
  - Score
  - Exact / upper bound / lower bound
  - Depth of searched tree rooted at the position
  - Best move to make at the position
- Algorithm
  - When a position P is arrived at, the hash table is probed
  - If there is a match, and
    - new_depth(P) $\geq$ stored_depth(P), and
    - score in the table is exact, or the bound on the score is sufficient to cause the move leading to P to be inferior to some other choice
  - then P is assigned the attributes from the table
  - else computer scores (by direct evaluation or search (old best move searched first)) P and stores the new attributes in the table
- Fills up => replacement strategies
  - Keep positions with greater searched tree depth under them
  - Keep positions with more searched nodes under them

# End game databases

## Torres y Quevedo's Mating Algorithm

Torres' scheme for effecting mate in the KRK endgame assumes an initial position with the automaton's White King on **a8**, Rook on **b8**, and the opponent's King on any unchecked square in the first six ranks. His algorithm for moving can be described in programming notation:

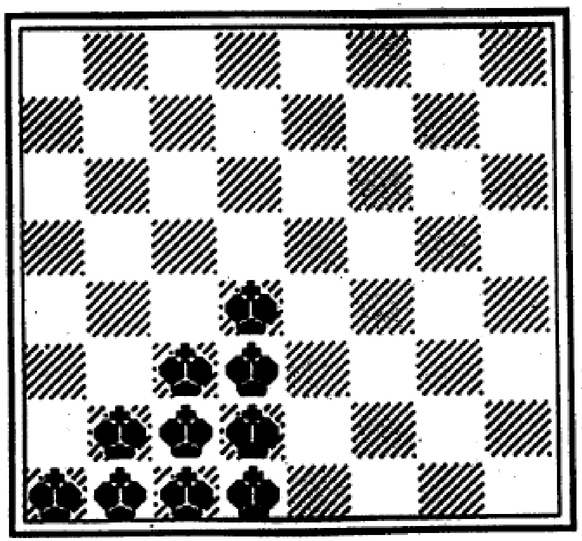
```
if          both BK and R are on left side {files a,b,c}
  then      move R to file h {keep R out of reach of K}
elseif      both BK and R are on right side {files f,g,h}
  then      move rook to file a {keep R away from K}
elseif      rank of R exceeds rank of BK by more than one
  then      move R down one rank {limit scope of BK}
elseif      rank of WK exceeds rank of BK by more than two
  then      move WK down one {WK approaches to support R}
elseif      horizontal distance between kings is odd
  then      {make tempo move with R}
            if       R is on a file then move R to b file
            elseif   R is on b file then move R to a file
            elseif   R is on g file then move R to h file
            else     {R is on h file} move R to g file
            endif
elseif      horizontal distance between kings is not zero
  then      move WK horizontally toward BK {keep opposition}
else        give check by moving rook down
            {and if on first rank, it's mate}
endif
```

If the opponent's King is placed on **a6**, with best delaying tactics mate can be staved off for 61 moves.

# Generating endgame databases automatically

- State space = {WTM, BTM} x {all possible configurations of remaining pieces}
- BTM table, WTM table, legal moves connect states between these
- Start at terminal positions: mate, stalemate, immediate capture without compensation (=reduction). Mark white's wins by won-in-0
- Mark unclassified WTM positions that allow a move to a won-in-0 by won-in-1 (store the associated move)
- Mark unclassified BTM positions as won-in-2 if forced moved to won-in-1 position
- Repeat this until no more labelings occurred
- Do the same for black
- Remaining positions are draws

# Compact representation methods to help endgame database representation & generation



Squares for Black's king that must be considered in KRK database.

| Position | Information on position |
|---|---|
| <a1-a1-a1> | 0 |
| <a1-a1-b1> | 0 |
| ... | ... |
| ... | ... |
| <a1-a1-h8> | 0 |
| <a1-b1-a1> | 0 |
| <a1-b1-b1> | 0 |
| ... | ... |
| ... | ... |
| <a1-c1-a1> | 0 |
| <a1-c1-b1> | 0 |
| ... | ... |
| ... | ... |
| <a1-c1-h8> | 0 |
| ... | ... |
| ... | ... |
| <d4-h8-h8> | 0 |

(a)

| Position | Information on position |
|---|---|
| <a1-a1-a1> | Illegitimate |
| <a1-a1-b1> | Illegitimate |
| ... | ... |
| ... | ... |
| <a1-a1-h8> | Illegitimate |
| <a1-b1-a1> | Illegitimate |
| <a1-b1-b1> | Illegitimate |
| ... | ... |
| ... | ... |
| <a1-c1-a1> | Illegitimate |
| <a1-c1-b1> | In check |
| ... | ... |
| ... | ... |
| <a1-c1-h8> | In check |
| ... | ... |
| ... | ... |
| <d4-h8-h8> | In check |

(b)

Building a KQK database: (a) initial contents of database, and (b) contents after performing the first step.

# Endgame databases…

1977

Game 1 [Ken Thompson]

White: Walter Browne Black: BELLE

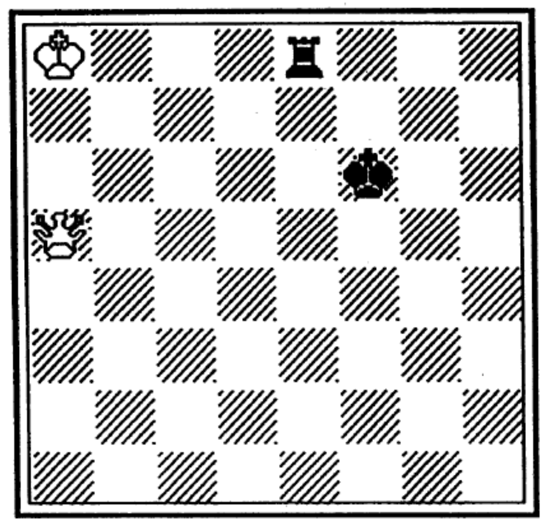


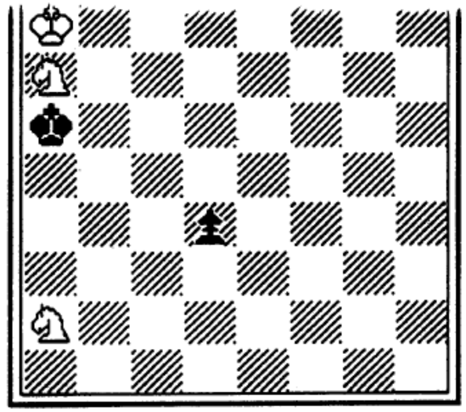
**Figure 6.17. Position from BELLE's database: White to play and win in thirty moves.**

Computer could hold a lost position against IM Hans Berliner.

Separated rook & king.

When playing against a computer with an endgame DB, don't exchange pieces when the exchange would bring the game to a position that is in the DB!

# Endgame databases…



KNNKP(d4) endgame with White to play and win

1 Nb4+ Kb6 2 Nd3 Kc7 3 Nb5+ Kc6 4 Na3 Kb6 5 Kb8 (5 Nc4+ or 5 Nc2) Kc6 6 Nc4 (6 Nc2) Kb5 7 Nce5 Kb6 8 Kc8 Ka6 (8 . . . Ka5 or 8 . . . Kb5) 9 Kc7 (9 Kd7) Kb5 10 Kd6 Ka4 11 Kc5 Kb3 12 Kb5 Kc3 13 Ka4 Kc2 14 Kb4 Kd1 15 Kb3 Kd2 16 Kb2 Kd1 17 Nc4 Ke2 18 Kc2 Kf3 19 Kd2 (19 Kd1) Kg3 (19 . . . Ke4) 20 Ke2 (20 Nce5) Kg2 21 Nce5 Kg3 22 Kf1 Kh4 23 Kg2 (23 Kf2) Kg5 24 Kf3 Kf5 25 Nc4 Kf6 26 Kf4 Ke6 27 Ke4 Kf6 28 Kd5 Ke7 29 Ke5 Kf7 30 Kd6 Kf6 31 Nd2 Kf5 32 Ke7 Kg6 33 Ke6 Kg7 (33 . . . Kg5) 34 Ne4 Kg6 35 Ke5 Kg7 36 Kd6 Kh7 (36 . . . Kh6) 37 Nd2 (37 Nef2) Kg7 38 Ke6 Kf8 39 Ne4 (39 Nc4) Ke8 40 Nf6+ (40 Nd6+) Kf8 (40 . . . Kd8) 41 Nh5 Ke8 42 Ng7+ Kd8 43 Kd6 Kc8 44 Ne6 Kb8 (44 . . . Kb7) 45 Kc5 Ka7 46 Kc6 Ka6 47 Nec5+ (47 Ng5) Ka5 48 Nb3+ (48 Ne4) Ka4 49 Nd2 Ka5 50 Kc5 Ka6 51 Nc4 Kb7 52 Kd6 Kc8 53 Na5 Kd8 54 Nb7+ Ke8 55 Ke6 Kf8 56 Nd6 Kg7 57 Kf5 Kh6 58 Kf6 Kh5 59 Nf7 (59 Ne4) Kg4 60 Ng5 Kh4 61 Kf5 Kg3 62 Ke4 Kg4 63 Nf7 Kh5 (63 . . . Kg3) 64 Kf5 Kh4 65 Nfe5 Kh5 66 Ng4 Kh4 67 Nf6 Kh3 68 Ke5 Kg3 69 Ke4 Kh3 70 Kf3 Kh4 71 Kf4 Kh3 72 Ne8 (72 Ne4 or 72 Nh5) Kh4 73 Ng7 Kh3 74 Nf5 Kg2 (74 . . . Kh2) 75 Kg4 Kh2 (75 . . . Kf1 or 75 . . . Kg1 or 75 . . . Kh1) 76 Nd6 (76 Ng3) Kg2 (76 . . . Kg1 or 76 . . . Kh1) 77 Nc4 (77 Ne4) Kh2 (77 . . . Kg1) 78 Nd2 Kg2 79 Kh4 Kh2 (79 . . . Kg1) 80 Nf4 (80 Ne1) Kg1 81 Kg3 Kh1 82 Nf3 (82 Ne2 or 82 Nh3) d3 followed by 83 Nh3 d2 84 Nf2#.

# How end game databases changed chess

- **All 5 piece endgames solved (can have > $10^8$ states) & many 6 piece**
  - **KRBKNN (~$10^{11}$ states): longest path-to-reduction 223**
- **Rule changes**
  - **Max number of moves from capture/pawn move to completion**
- **Chess knowledge**
  - **Splitting rook from king in KRKQ**
  - **KRKN game was thought to be a draw, but**
    - **White wins in 51% of WTM**
    - **White wins in 87% of BTM**

# Deep Blue's search

- ~200 million moves / second = 3.6 * $10^{10}$ moves in 3 minutes
- 3 min corresponds to
  - ~7 plies of uniform depth minimax search
  - 10-14 plies of uniform depth alpha-beta search
- 1 sec corresponds to 380 years of human thinking time
- Software searches first
  - Selective and singular extensions
- Specialized hardware searches last 5 ply

# Deep Blue's hardware

- 32-node RS6000 SP multicomputer
- Each node had
  - 1 IBM Power2 Super Chip (P2SC)
  - 16 chess chips
    - Move generation (often takes 40-50% of time)
    - Evaluation
    - Some endgame heuristics & small endgame databases
- 32 Gbyte opening & endgame database

# Role of computing power

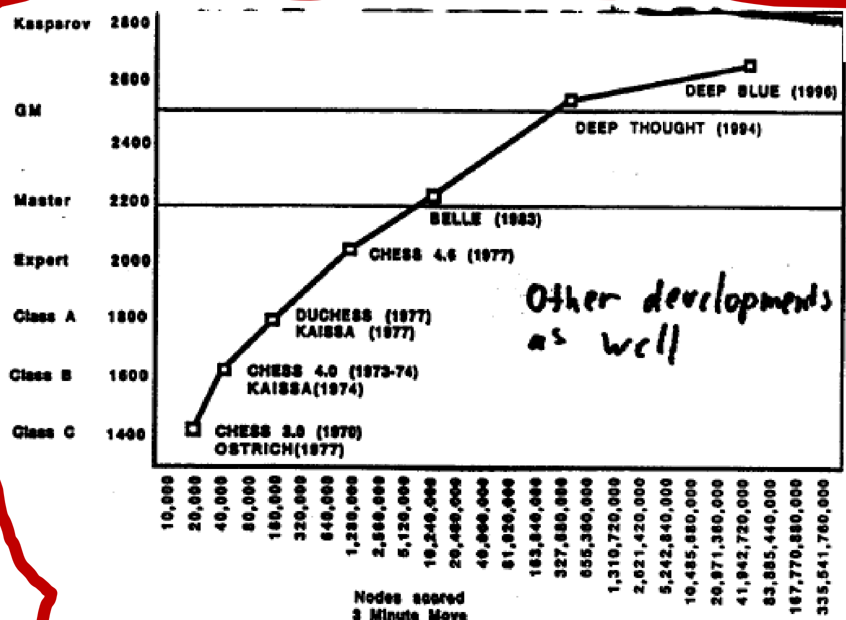



Figure 6.23. Relationship between the level of play by chess programs and the size of the tree searched during a three minute move.

(a)

| | BELLE (3) | BELLE (4) | BELLE (5) | BELLE (6) | BELLE (7) | BELLE (8) | |
|---|---|---|---|---|---|---|---|
| BELLE (3) | | 4 | | | | | 1091 |
| BELLE (4) | 16 | | 5.5 | | | | 1332 |
| BELLE (5) | | 14.5 | | 4.5 | | | 1500 |
| BELLE (6) | | | 15.5 | | 2.5 | | 1714 |
| BELLE (7) | | | | 17.5 | | 3.5 | 2052 |
| BELLE (8) | | | | | 16.5 | | 2320 |

(b)

| | BELLE (4) | BELLE (5) | BELLE (6) | BELLE (7) | BELLE (8) | BELLE (9) | |
|---|---|---|---|---|---|---|---|
| BELLE (4) | | 5 | .5 | 0 | 0 | 0 | 1235 |
| BELLE (5) | 15 | | 3.5 | 3 | .5 | 0 | 1570 |
| BELLE (6) | 19.5 | 16.5 | | 4 | 1.5 | 1.5 | 1826 |
| BELLE (7) | 20 | 17 | 16 | | 5 | 4 | 2031 |
| BELLE (8) | 20 | 19.5 | 18.5 | 15 | | 5.5 | 2208 |
| BELLE (9) | 20 | 20 | 18.5 | 16 | 14.5 | | 2328 |

Figure 6.24. Results of Thompson's two experiments: (a) first experiment, (b) second experiment. Entries in the tables indicate the number of games won by the program heading the row against the program heading the column.

| i | F(i) % of time BELLE(i) picked moves different from BELLE(i − 1) | R(i) Rating of BELLE(i) if R(4) = 1320 and R(5) = 1570 | R(i) Rating of BELLE(i) if R(4) = 1300 and R(5) = 1570 |
|---|---|---|---|
| 4 | 33.1 | 1320 | 1300 |
| 5 | 33.1 | 1570 | 1570 |
| 6 | 27.7 | 1779 | 1796 |
| 7 | 29.5 | 2002 | 2037 |
| 8 | 26.0 | 2198 | 2249 |
| 9 | 22.6 | 2369 | 2433 |
| 10 | 17.7 | 2503 | 2577 |
| 11 | 18.1 | 2639 | 2725 |

Figure 6.25. Percentage of time BELLE(i) picked different moves from BELLE(i − 1) and the corresponding predicted ratings based on expression (1) for two cases: (1) R(4) = 1320 and R(5) = 1570, and (2) R(4) = 1300 and R(5) = 1570.

Diminishing returns to computation power.

# Interestingly… “Freestyle Chess” = centaurs

- Hybrid human-AI chess players were stronger **for a while** than humans or AI alone