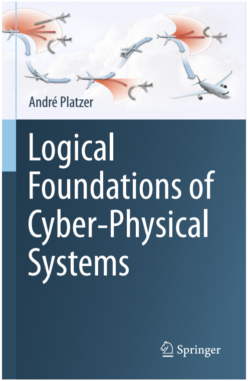
08: Events & Responses

Logical Foundations of Cyber-Physical Systems



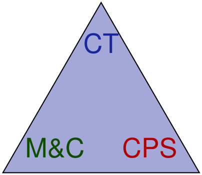
Stefan Mitsch

Carnegie Mellon University
Computer Science Department

Outline

Outline

using loop invariants
design event-triggered control

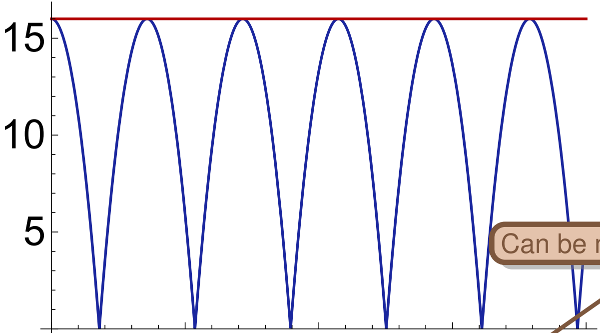


modeling CPS
event-triggered control
continuous sensing
feedback mechanisms
control vs. physics

semantics of event-triggered control
operational effects
model-predictive control

Outline

Quantum the Safely Bored Bouncing Ball



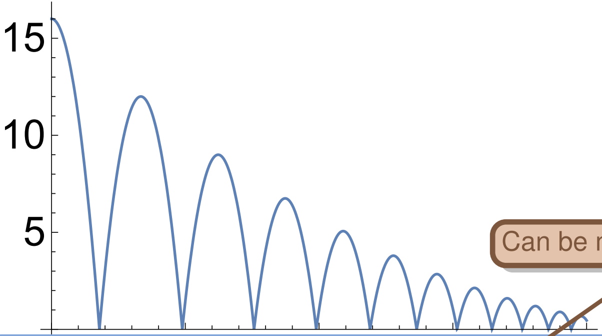
Can be made more realistic...

Proposition (Quantum can bounce around safely)

$$0 \le x \land x = H \land v = 0 \land g > 0 \land 1 = c \rightarrow$$
$$[(\{x' = v, v' = -g \,\&\, x \ge 0\}; (?x{=}0; v := -cv \cup ?x{\ne}0))^*](0 \le x \land x \le H)$$

Proof **@invariant**$(2gx = 2gH - v^2 \land x \ge 0)$

Quantum the Safely Bored Bouncing Ball



Can be made more exciting…

Proposition (Quantum can bounce around safely)

$$0 \le x \land x = H \land v = 0 \land g > 0 \land 1 \ge c \ge 0 \to$$
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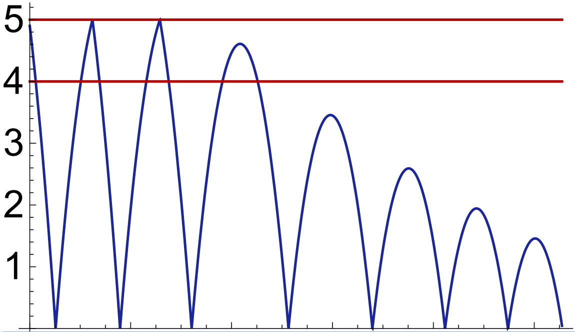
Quantum the Daring Ping-Pong Ball



Conjecture (Quantum can play ping-pong safely)

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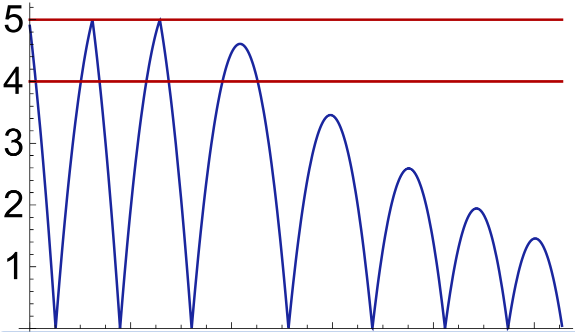

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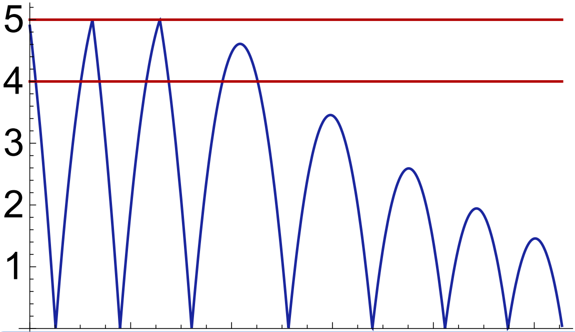
Correct? Ask René Descartes

Outwit the Cartesian Demon

Skeptical about the truth of all beliefs until justification has been found.

Quantum the Daring Ping-Pong Ball



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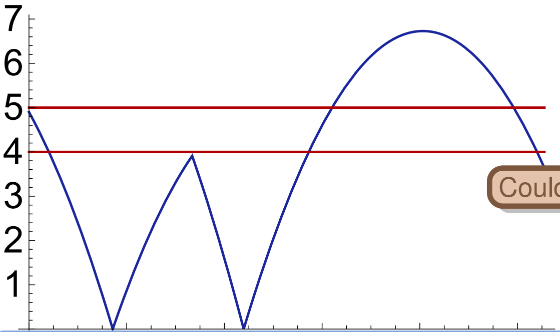

Could run instead of control

Conjecture (Quantum can play ping-pong safely)

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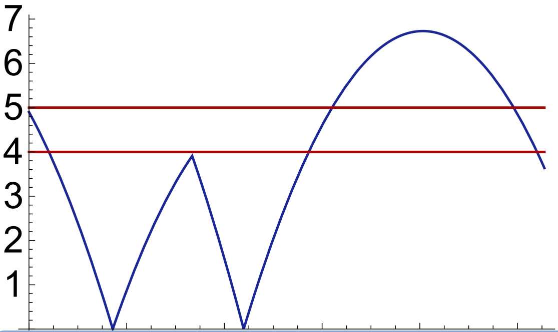

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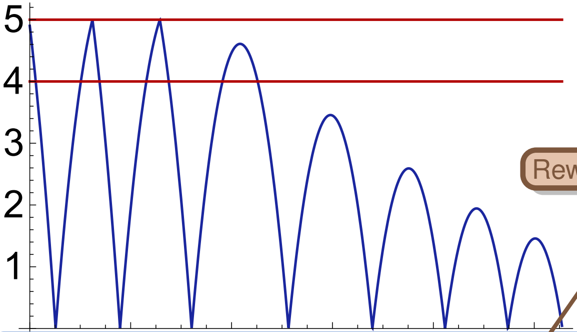
Quantum the Deterministically Daring Ping-Pong Ball



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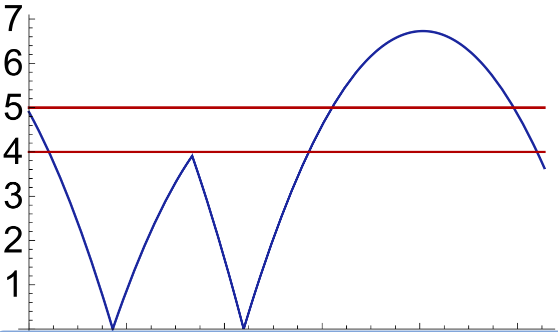

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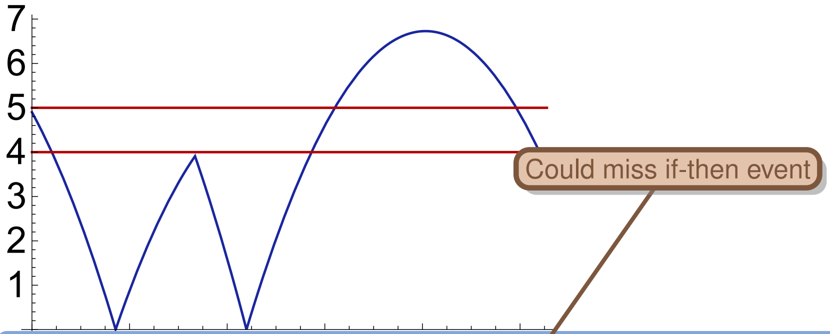

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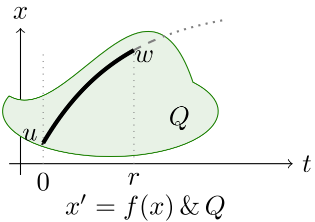
Outline

Evolution domains detect events

$$x' = f(x) \,\&\, Q$$

Evolution domain Q of a differential equation is responsible for detecting events. Q can stop physics whenever an event happens on which the control wants to take action.



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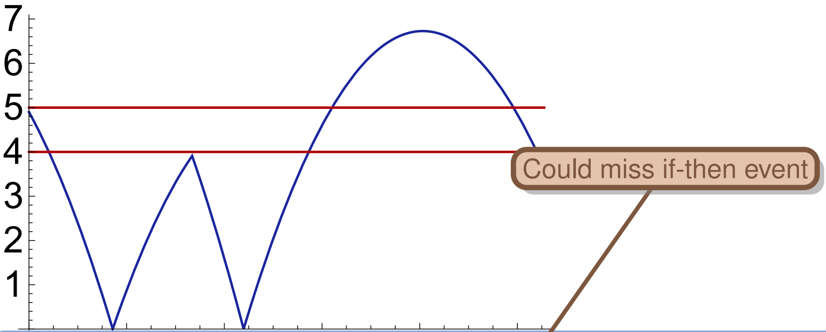

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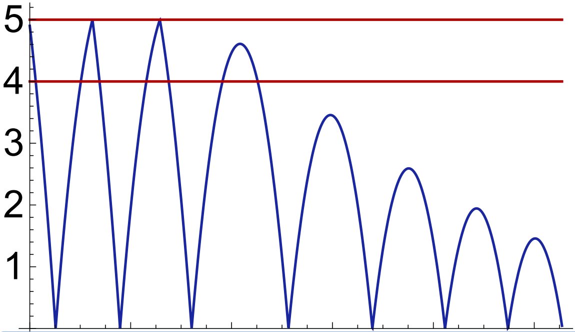

Domain as event trap?

Conjecture (Quantum can play ping-pong safely)

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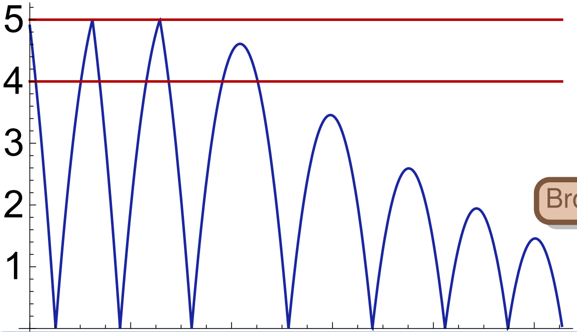

Broken physics: Always event

Conjecture (Quantum can play ping-pong safely)

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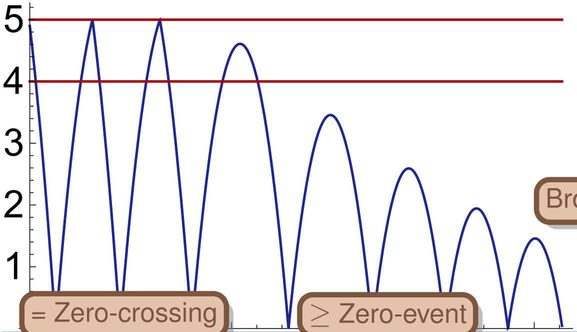

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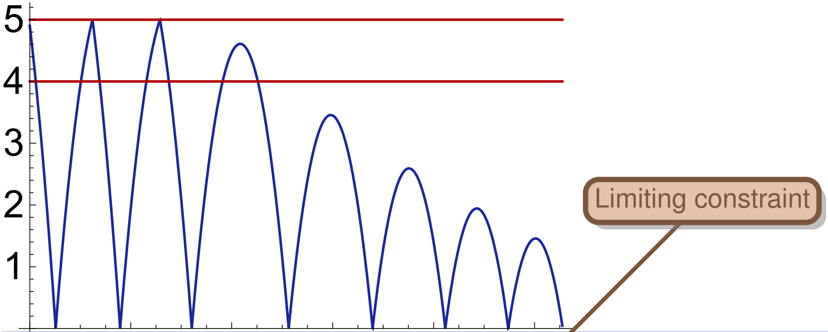

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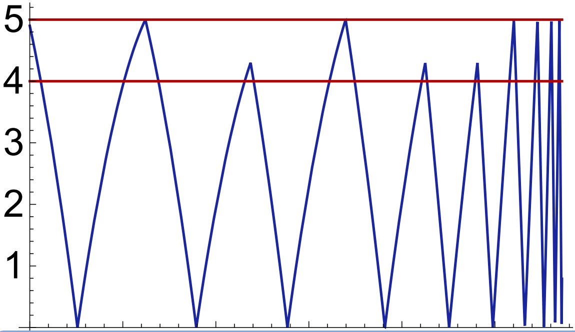

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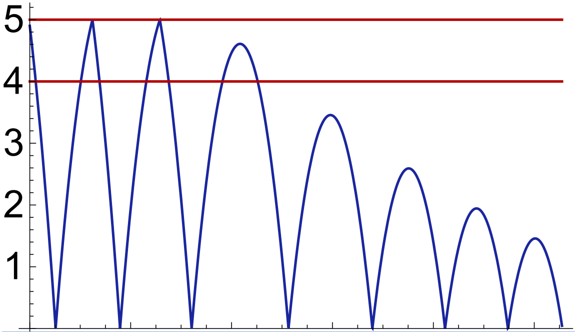

May miss 4 but not 5

Conjecture (Quantum can play ping-pong safely)

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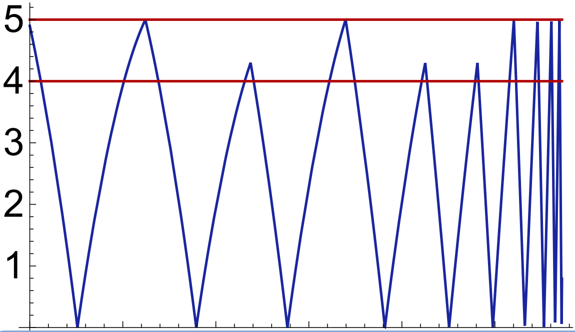

Domain by construction

Conjecture (Quantum can play ping-pong safely)

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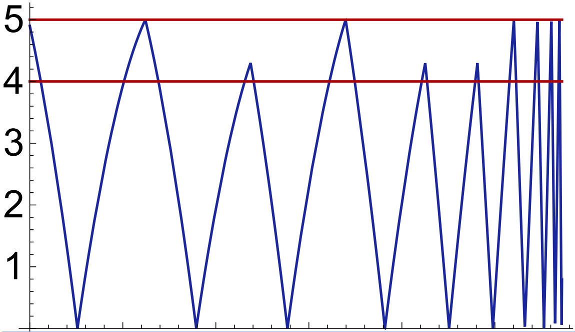

Non-negotiable physics

Conjecture (Quantum can play ping-pong safely)

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On the Nuisance of Nuances of Physics

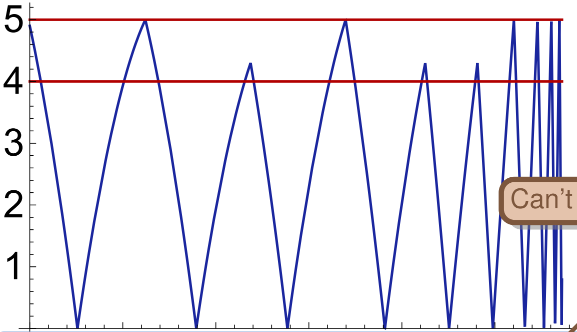
Non-negotiability of Physics

1. Making systems safe by construction is a great idea. For control!
2. But not by changing the laws of physics.
3. Physics is unpleasantly non-negotiable.
4. If models are safe because we forgot to include all behavior of physical reality, then correctness statements only hold in that other universe.

Despite control We don't get to boss physics around

We don't make this world any safer by writing CPS programs for another universe.

Quantum the Deterministically Daring Ping-Pong Ball



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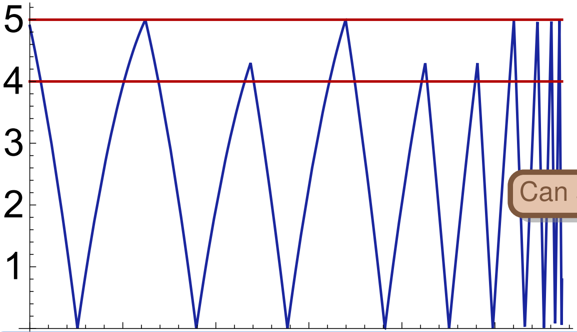

Can split the world for an event

Conjecture (Quantum can play ping-pong safely)

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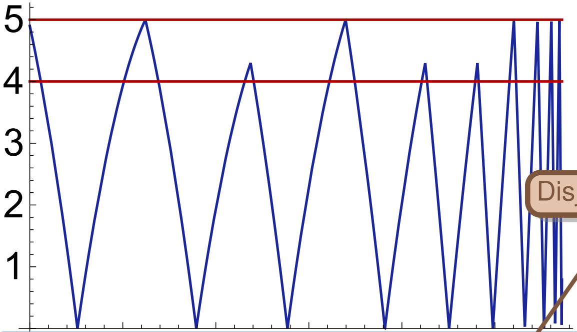

Disjoint domains

Shattered the world

Conjecture (Quantum can play ping-pong safely)

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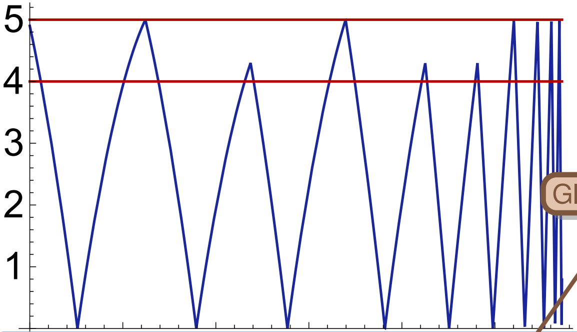

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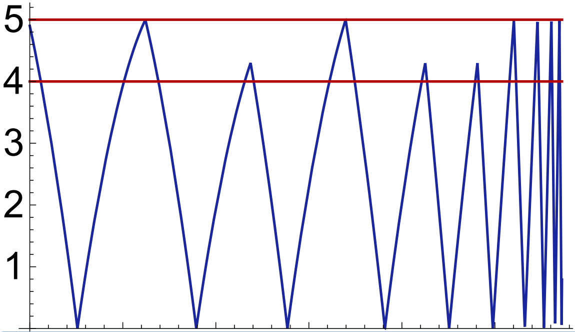
Connected Evolution Domains

Connected evolution domains

1. Evolution domain constraints need care.
2. Determine regions within which the system can evolve.
3. Disconnected/disjoint disallows continuous transitions.

1. Splitting the state space into different regions to detect events is fine.
2. Destroying the world is not.
3. Not even by poking infinitesimal holes into the time-space continuum.

Quantum the Deterministically Daring Ping-Pong Ball



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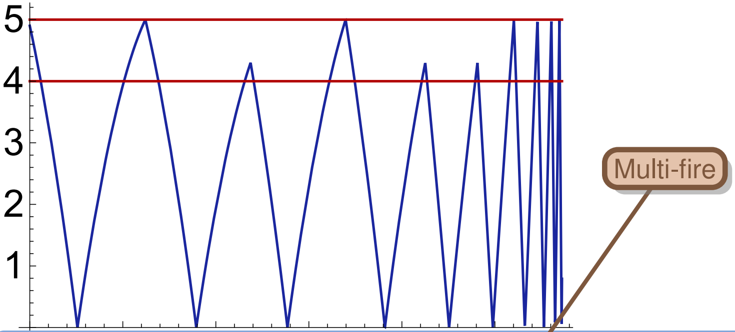

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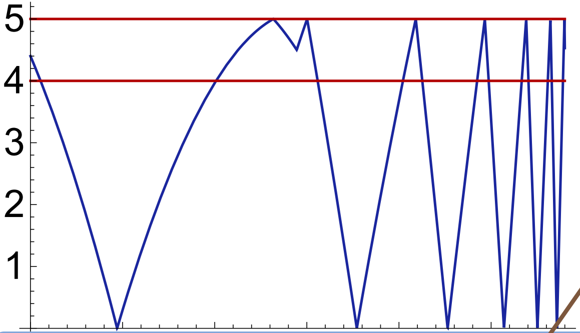

Multi-fire

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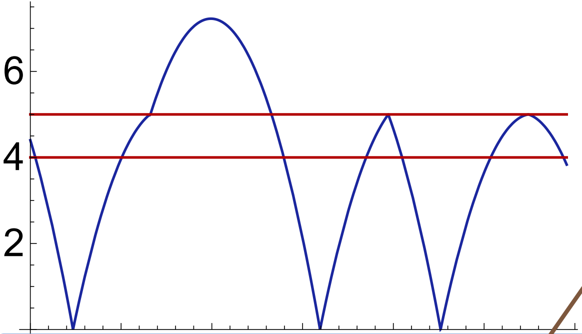

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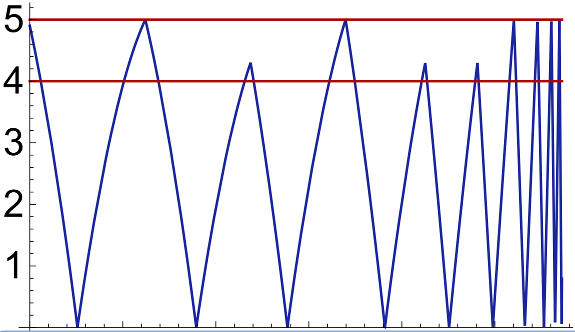
$$\text{if}(x{=}0)\, v := -cv \,\text{else}\, \text{if}(4 \le x \le 5)\, v := -fv\Big)^*\Big](0 \le x \le 5)$$

Correct? Ask René Descartes who definitely says no!



Only upsense event

Conjecture (Quantum can play ping-pong safely)

$$0 \le x \land x \le 5 \land v \le 0 \land g > 0 \land 1 \ge c \ge 0 \land f \ge 0 \to$$
$$\Big[\Big(\big(\{x' = v, v' = -g \,\&\, x \ge 0 \land x \le 5\} \cup \{x' = v, v' = -g \,\&\, x \ge 5\}\big);$$
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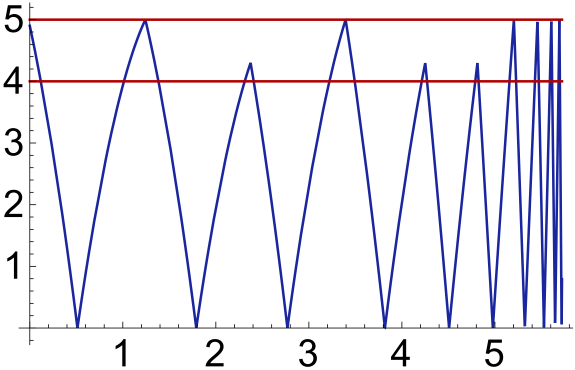
Correct? Ask René Descartes

Multi-firing Events

Multi-firing of events

1. If the same event is detected multiple times:
2. Are multiple responses acceptable?
3. Or is a single response crucial?

Physics vs. Control: Classification



control: robust, all cases

physics: precise

Conjecture (Quantum can play ping-pong safely)

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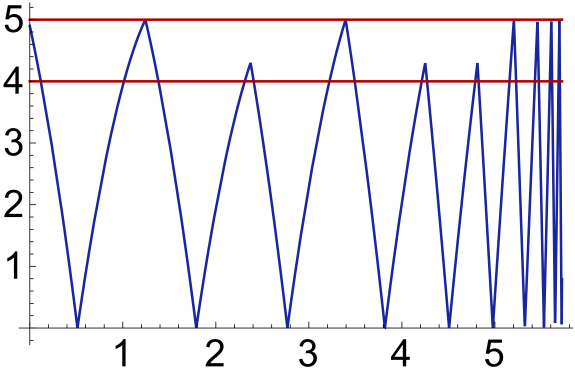

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Proposition (Quantum can play ping-pong safely)

$$0 \le x \land x \le 5 \land v \le 0 \land g > 0 \land 1 \ge c \ge 0 \land f \ge 0 \to$$
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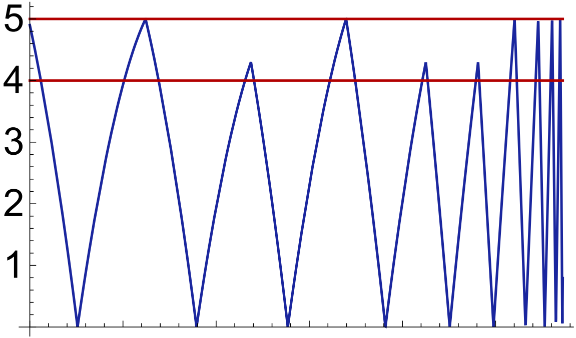

Quantum's Ping-Pong Proof Invariants

Proposition (Quantum can play ping-pong safely)

$$0 \le x \land x \le 5 \land v \le 0 \land g > 0 \land 1 \ge c \ge 0 \land f \ge 0 \rightarrow$$
$$\Big[\Big(\big(\{x' = v, v' = -g \,\&\, x \ge 0 \land x \le 5\} \cup \{x' = v, v' = -g \,\&\, x \ge 5\}\big);$$
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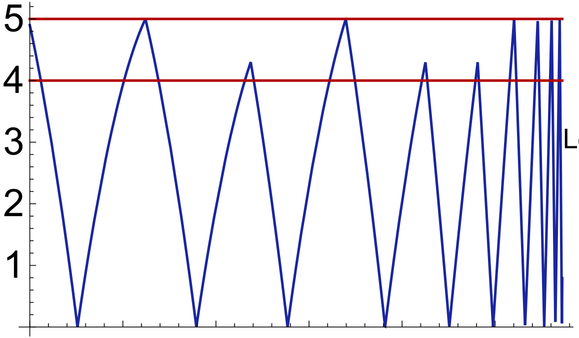

Loop invariant $j(x, v)$:

Quantum's Ping-Pong Proof Invariants

Proposition (Quantum can play ping-pong safely)

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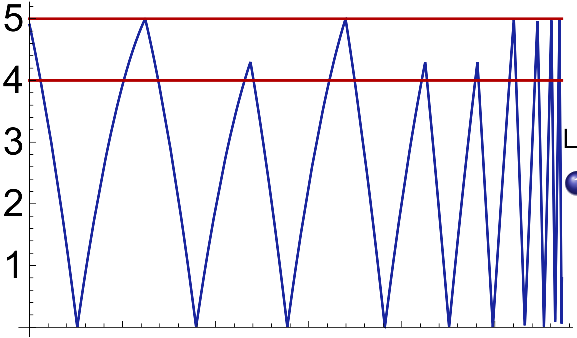
$$\Big[\Big(\big(\{x' = v, v' = -g \,\&\, x \ge 0 \wedge x \le 5\} \cup \{x' = v, v' = -g \,\&\, x \ge 5\}\big);$$
$$\text{if}(x=0)\, v := -cv \,\text{else if}(4 \le x \le 5 \wedge v \ge 0)\, v := -fv\Big)^*\Big](0 \le x \le 5)$$



Loop invariant $j(x, v)$:

1. $0 \le x \le 5$

Proposition (Quantum can play ping-pong safely)

$$0 \le x \land x \le 5 \land v \le 0 \land g > 0 \land 1 \ge c \ge 0 \land f \ge 0 \rightarrow$$
$$\Big[\Big(\big(\{x' = v, v' = -g \,\&\, x \ge 0 \land x \le 5\} \cup \{x' = v, v' = -g \,\&\, x \ge 5\}\big);$$
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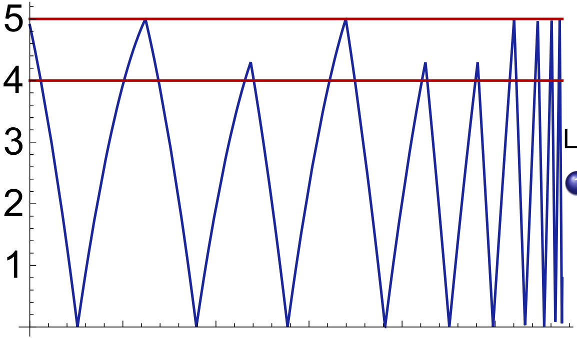
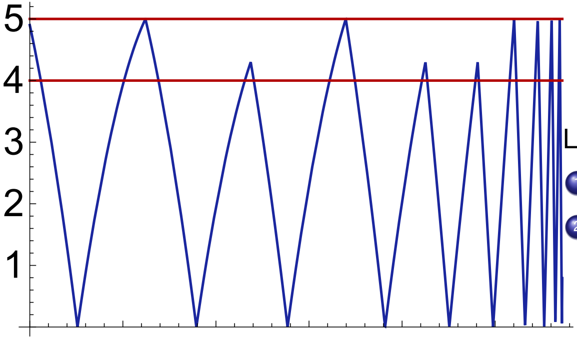


Loop invariant $j(x, v)$:

1. $0 \le x \le 5$ not inductive

Proposition (Quantum can play ping-pong safely)

$$0 \le x \land x \le 5 \land v \le 0 \land g > 0 \land 1 \ge c \ge 0 \land f \ge 0 \rightarrow$$
$$\big[\big((\{x' = v, v' = -g \,\&\, x \ge 0 \land x \le 5\} \cup \{x' = v, v' = -g \,\&\, x \ge 5\});$$
$$\text{if}(x=0)\, v := -cv \,\text{else}\,\text{if}(4 \le x \le 5 \land v \ge 0)\, v := -fv\big)^*\big](0 \le x \le 5)$$



Loop invariant $j(x, v)$:

1. $0 \le x \le 5$ — not inductive
2. $0 \le x \le 5 \land v \le 0$

Quantum's Ping-Pong Proof Invariants

Proposition (Quantum can play ping-pong safely)

$$0 \le x \land x \le 5 \land v \le 0 \land g > 0 \land 1 \ge c \ge 0 \land f \ge 0 \rightarrow$$
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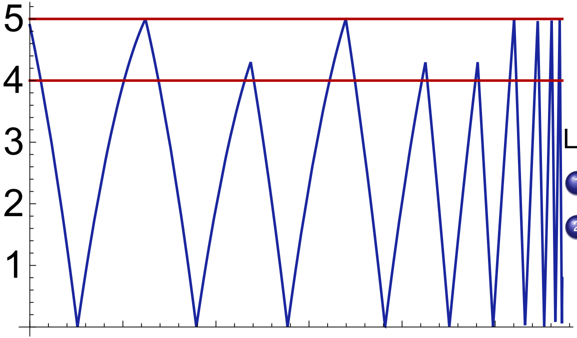

Loop invariant $j(x, v)$:

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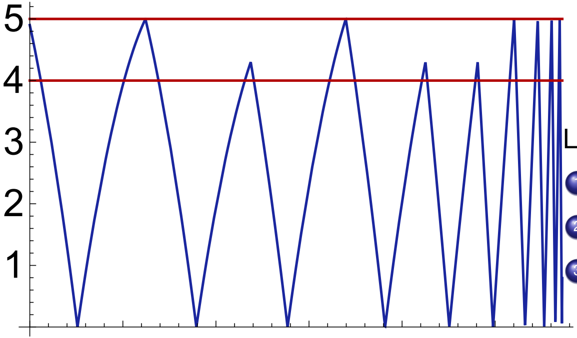


Loop invariant $j(x, v)$:

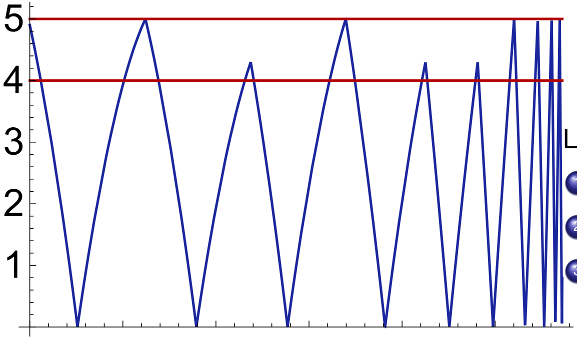
1. $0 \le x \le 5$ — not inductive
2. $0 \le x \le 5 \land v \le 0$ — not inductive
3. $0 \le x \le 5 \land (x{=}5 \rightarrow v \le 0)$

Quantum's Ping-Pong Proof Invariants

Proposition (Quantum can play ping-pong safely)

$$0 \leq x \land x \leq 5 \land v \leq 0 \land g > 0 \land 1 \geq c \geq 0 \land f \geq 0 \rightarrow$$
$$\big[\big(\big(\{x' = v, v' = -g \,\&\, x \geq 0 \land x \leq 5\} \cup \{x' = v, v' = -g \,\&\, x \geq 5\}\big);$$
$$\text{if}(x=0)\, v := -cv \;\text{else if}(4 \leq x \leq 5 \land v \geq 0)\, v := -fv\big)^*\big](0 \leq x \leq 5)$$

Proof **@invariant**$(0 \leq x \leq 5 \land (x = 5 \rightarrow v \leq 0))$



Loop invariant $j(x, v)$:

1. $0 \leq x \leq 5$ — not inductive
2. $0 \leq x \leq 5 \land v \leq 0$ — not inductive
3. $0 \leq x \leq 5 \land (x = 5 \rightarrow v \leq 0)$ — yes!

Quantum's Ping-Pong Proof Invariants

Proposition (Quantum can play ping-pong safely)

$$0 \le x \land x \le 5 \land v \le 0 \land g > 0 \land 1 \ge c \ge 0 \land f \ge 0 \rightarrow$$
$$[((\{x' = v, v' = -g \,\&\, x \ge 0 \land x \le 5\} \cup \{x' = v, v' = -g \,\&\, x \ge 5\});$$
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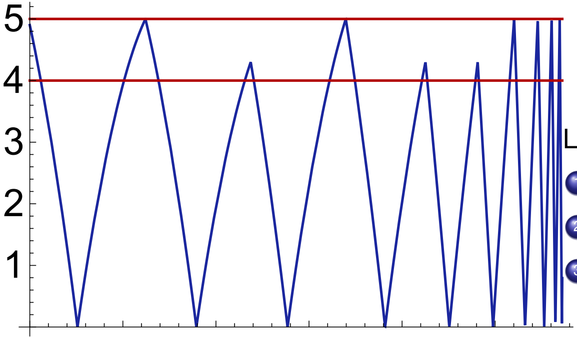

Just can't implement . . .

Loop invariant $j(x, v)$:

1. $0 \le x \le 5$ not inductive
2. $0 \le x \le 5 \land v \le 0$ not inductive
3. $0 \le x \le 5 \land (x=5 \rightarrow v \le 0)$ yes!

Outline

Summary: Event-triggered Control

1. One important principle for designing feedback mechanisms
2. Conceptually simple: detect all relevant events and respond correctly
3. Assumes all events are surely detected
4. Implementation: Requires continuous sensing
   Tell me if you ever find a faithful implementation platform ...
5. Robust events, not just: if$(x = 9.8696)$ ...
6. Events have subtle models, but make design and verification easier!
   Non-negotiability of Physics | Connected domains | Multi-firing
7. Useful abstraction when system evolves slowly but senses quickly
8. Verify event-triggered model as first step
9. Then refine toward realistic implementation based on safe event-triggered design
10. Physics $\neq$ Control

Non-negotiability of Physics

1. Making systems safe by construction is a great idea. For control!
2. But not by changing the laws of physics.
3. Physics is unpleasantly non-negotiable.
4. If models are safe because we forgot to include all behavior of physical reality, then correctness statements only hold in that other universe.

Despite control We don’t get to boss physics around

We don’t make this world any safer by writing CPS programs for another universe.

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