Sorting 7A

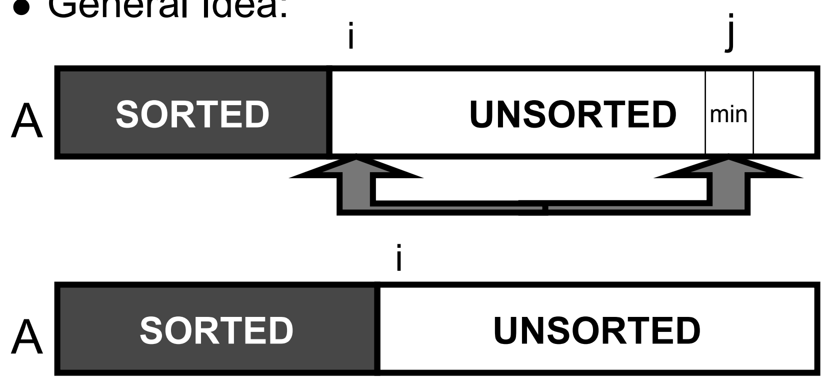
Quadratic Sorts

Selection Sort

- Let A be an array of n elements, and we wish to sort these elements in non-decreasing order.
- Basic Algorithm:
  - Set i = 0.
  - While i < n do the following:
    - Find j, $i \leq j \leq n-1$, such that $A[j] \leq A[k]$, $\forall k$, $i \leq k \leq n-1$.
    - Swap A[j] with A[i]
    - Add 1 to i
- This algorithm works in place, meaning it uses its own storage to perform the sort.

Selection Sort

- General Idea:



Loop invariant: A[0..i-1] are sorted in non-decreasing order.

Selection Sort Example

66	44	99	55	11	88	22	77	33
11	44	99	55	66	88	22	77	33
11	22	99	55	66	88	44	77	33
11	22	33	55	66	88	44	77	99
11	22	33	44	66	88	55	77	99
11	22	33	44	55	88	66	77	99
11	22	33	44	55	66	88	77	99
11	22	33	44	55	66	77	88	99
11	22	33	44	55	66	77	88	99

Run time analysis

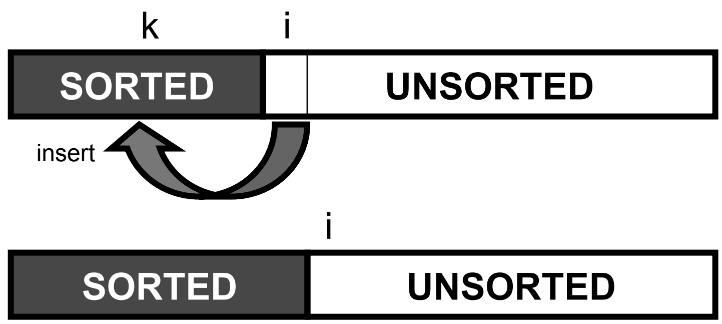
- Worst Case:
  Search for 1st min: n-1 comparisons
  Search for 2nd min: n-2 comparisons
  ...
  Search for 2nd-to-last min: 1 comparison
  Total comparisons:
  (n-1) + (n-2) + ... + 2 + 1 = O(_________)
- Average Case: = O(_________)
- Best Case: = O(_________)

Insertion Sort

- Let A be an array of n elements, and we wish to sort these elements in non-decreasing order.
- Basic Algorithm:
  - Set i = 1
  - While i < n do the following:
    - Set item = A[i].
    - Shift A[j] to A[j+1], ∀j, j < i, where A[j] > item.
    - Let k be the smallest j above, or k=i if no shifts.
    - Set A[k] = item.
    - Add 1 to i.
- This algorithm also works in place.

Insertion Sort

- General Idea:



Loop invariant: A[0..i-1] are sorted in non-decreasing order.

Insertion Sort Example

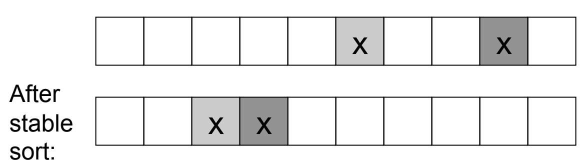
66	44	99	55	11	88	22	77	33
44	66	99	55	11	88	22	77	33
44	66	99	55	11	88	22	77	33
44	55	66	99	11	88	22	77	33
11	44	55	66	99	88	22	77	33
11	44	55	66	88	99	22	77	33
11	22	44	55	66	88	99	77	33
11	22	44	55	66	77	88	99	33
11	22	33	44	55	66	77	88	99

Run time analysis

- Worst Case (when does this occur?):
  Insert 2nd element: 1 comparison
  Insert 3rd element: 2 comparisons
  ...
  Insert last element: n-1 comparisons
  Total comparisons:
  1 + 2 + ... + (n-1) = O(__________)
- Average Case: = O(__________)
- Best Case: = O(__________)

Stable Sorts

- A sort is <u>stable</u> if two elements with the same value maintain their same relative order before and after the sort is performed.

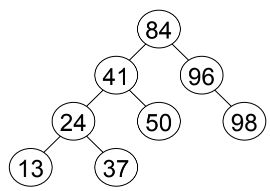


- Is selection sort stable?
- Is insertion sort stable?

Quadratic Sorts

- Quadratic sorts have a worst-case order of complexity of $O(n^2)$
- Selection sort always performs poorly, even on a sequence of sorted elements!
- Insertion sort performs much better if the elements are sorted or nearly sorted.
- Another famous quadratic sort: "Bubble sort"

Tree Sort



- Build a binary search tree out of the elements.
- Traverse the tree using an inorder traversal to get the elements in increasing order.
- Worst case order of complexity:
  - $O(n^2)$ to build the binary search tree (Why?)
  - $O(n)$ to traverse the binary tree. (Why?)
  - Total: $O(n^2) + O(n) =$ _______