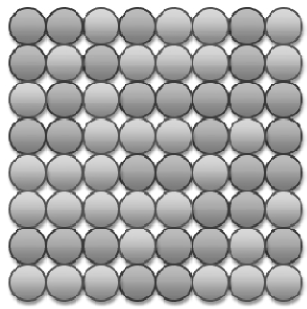
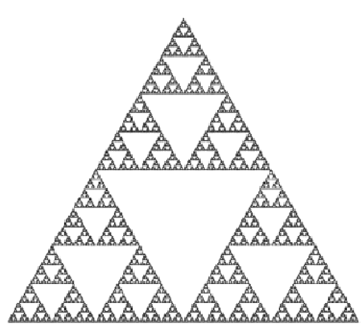


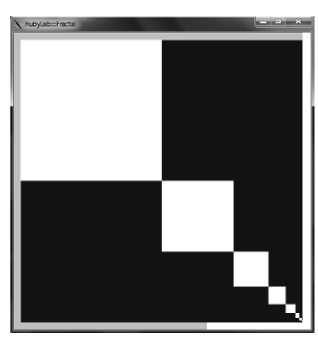
UNIT 9C

Randomness in Computation:
More Fractals and Cellular Automata

Fractals

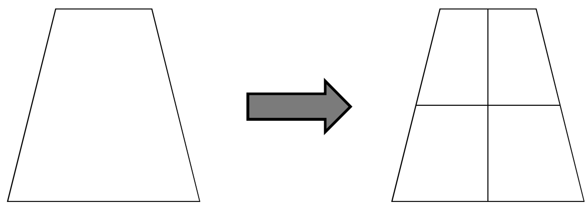
- Recall: A fractal is an image that is self-similar.
- Fractals are typically generated using recursion.





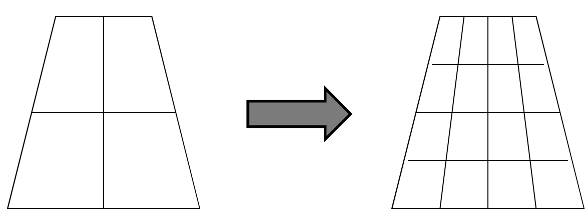
Simple Fractal

Connect midpoints of the quadrilateral



Simple Fractal

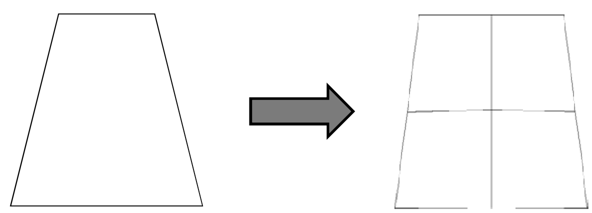
- Connect midpoints of each quadrilateral recursively



It makes a disco floor!

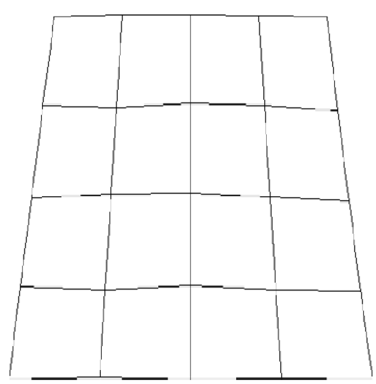
Fractal with Randomness

- Randomly move midpoints slightly and then connect.



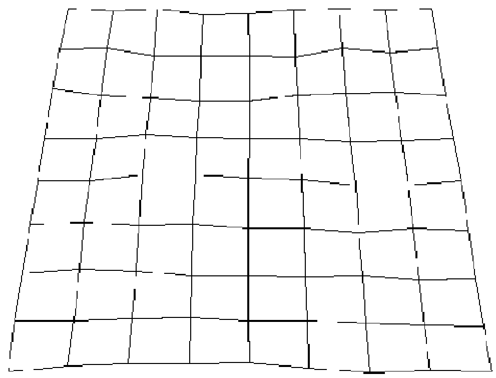
Fractal with Randomness

- Randomly move midpoints slightly and then connect.



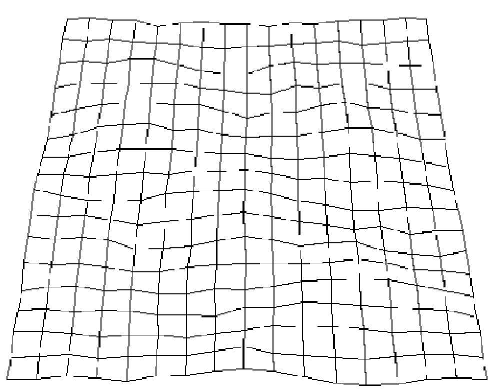
Fractal with Randomness

- Randomly move midpoints slightly and then connect.



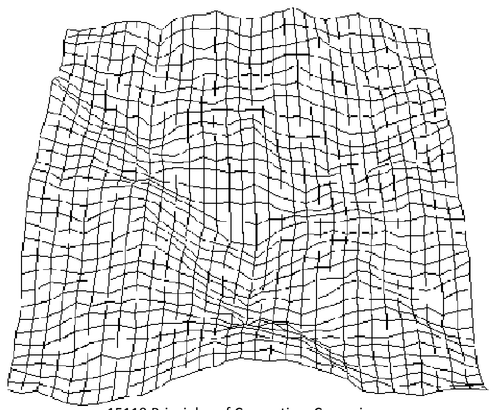
Fractal with Randomness

- Randomly move midpoints slightly and then connect.



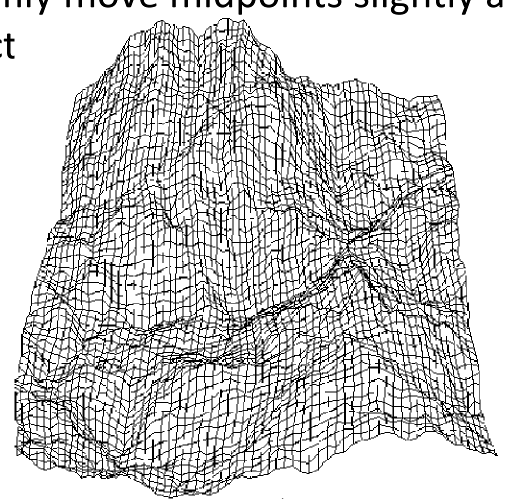
Fractal with Randomness

- Randomly move midpoints slightly and then connect.



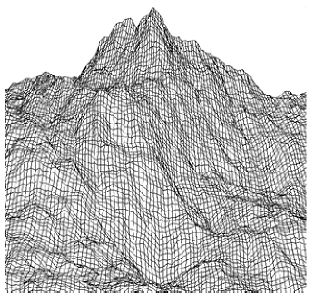
Fractal with Randomness

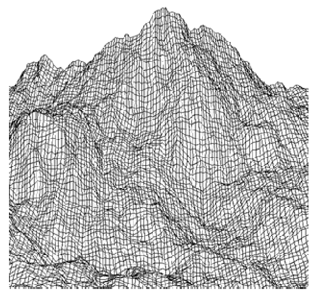
- Randomly move midpoints slightly and then connect



Fractal with Randomness

- This technique can be used to create some realistic mountain ranges.





Fractals in Nature

- Approximate fractals are easily found in nature. These objects display self-similar structure over an extended, but finite, scale range. Examples include clouds, snow flakes, crystals, mountain ranges, lightning, river networks, cauliflower or broccoli, systems of blood vessels and pulmonary vessels, coastlines, tree branches, galaxies, etc. (borrowed from Wikipedia)

"Elevated"

- Was produced from somebody's 4 kilobyte computer program.

Determinism

- A computer is deterministic. It follows rules, step by step.
- Does that mean a program does the same thing every time, given the same input?
- Can a computer behave randomly?

Cellular Automata

- A cellular automaton is a collection of cells on a grid that evolves through a number of discrete time steps according to a set of rules based on the states of neighboring cells.
- The rules are then applied iteratively for as many time steps as desired.
  - John von Neumann was one of the first people to consider such a model.

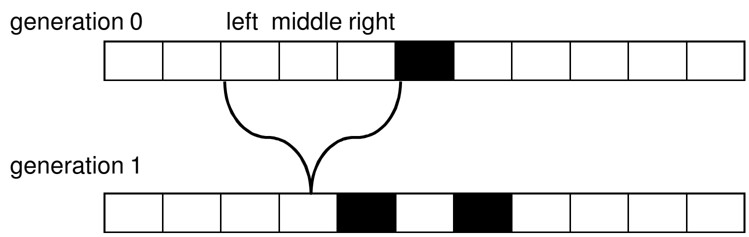
(from Wolfram MathWorld)

Simple Example

- For each cell, look at the 3 cells on the row immediately above it (immediately above, above-and-to-the-left, and above-and-to-the-right).
- If the middle is white and either the left or the right is black (but not both), then this cell will become black. Otherwise, it will be white.

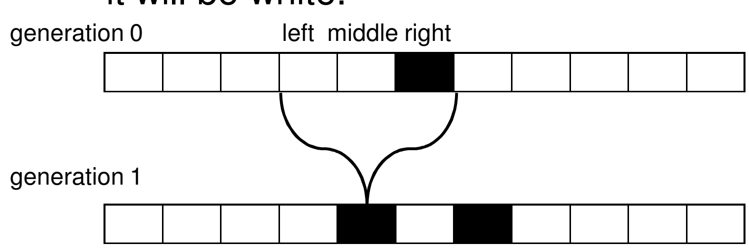
How it works

If the middle is white and either the left or the right is black (but not both), then this cell will become black. Otherwise, it will be white.



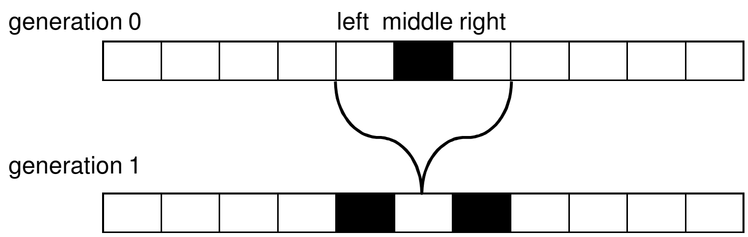
How it works

If the middle is white and either the left or the right is black (but not both), then this cell will become black. Otherwise, it will be white.



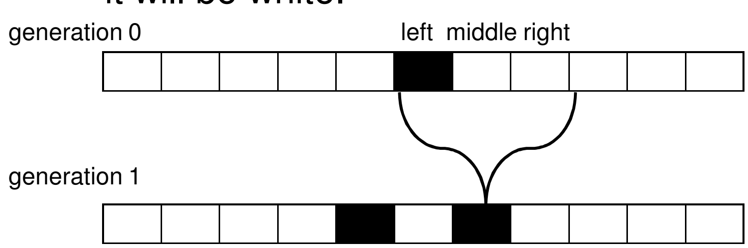
How it works

If the middle is white and either the left or the right is black (but not both), then this cell will become black. Otherwise, it will be white.



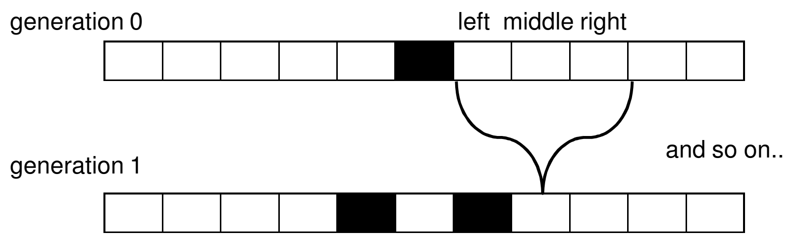
How it works

If the middle is white and either the left or the right is black (but not both), then this cell will become black. Otherwise, it will be white.



How it works

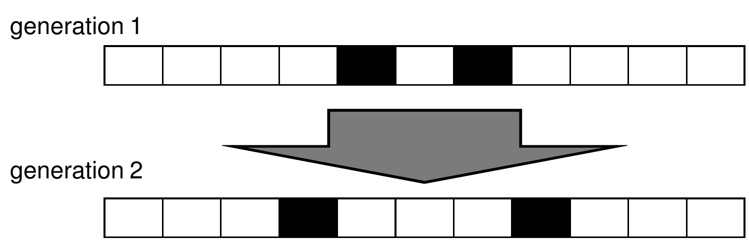
If the middle is white and either the left or the right is black (but not both), then this cell will become black. Otherwise, it will be white.



How it works

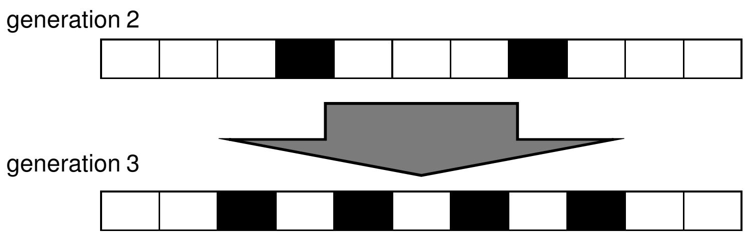
Once the next generation is created, use that to create a new generation.

If the middle is white and either the left or the right is black (but not both), then this cell will become black. Otherwise, it will be white.



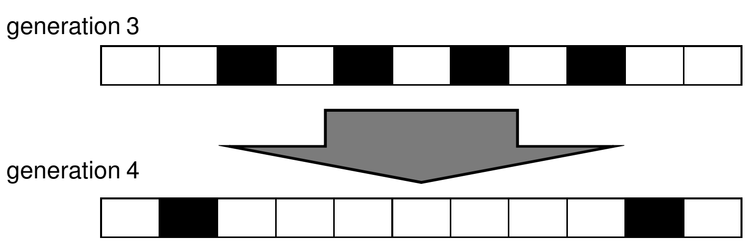
How it works

If the middle is white and either the left or the right is black (but not both), then this cell will become black. Otherwise, it will be white.



How it works

If the middle is white and either the left or the right is black (but not both), then this cell will become black. Otherwise, it will be white.

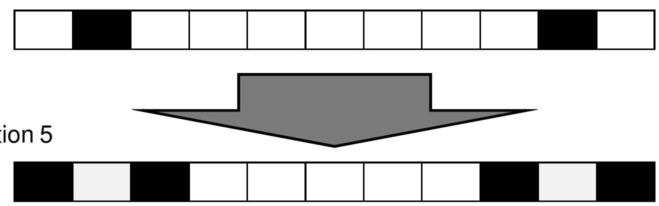


How it works

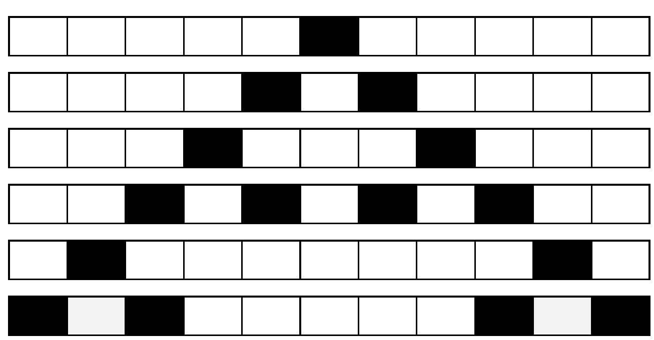
If the middle is white and either the left or the right is black (but not both), then this cell will become black. Otherwise, it will be white.

generation 4

generation 5



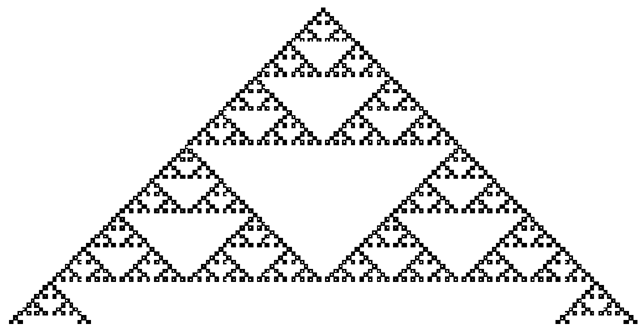
What we have so far



Keep going... what do we get?
(assume each row is infinite in length)

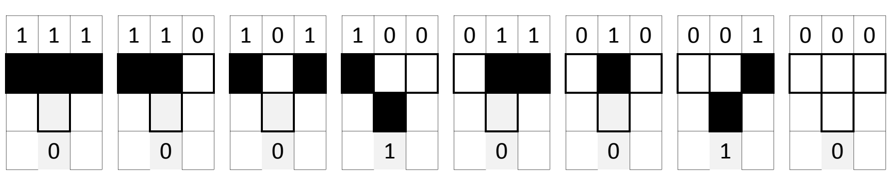
Results

Look familiar?



Rule 18

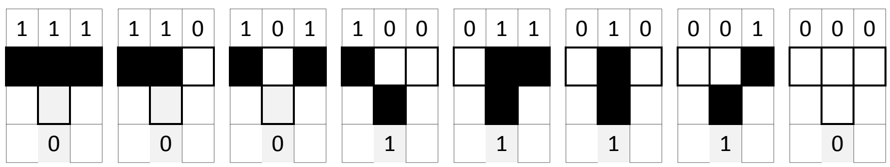
- This is known as “Rule 18” for 1-dimensional cellular automata.
  - Rule: If the middle is white and either the left or the right is black (but not both), then this cell will become black. Otherwise, it will be white.



00010010 in binary = 18

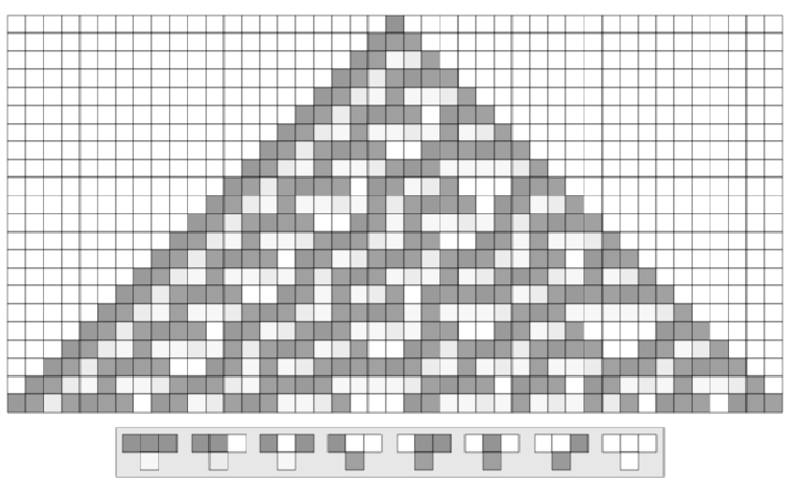
Rule 30

- How would you describe this rule?
- Try this rule using a random initial phase.
- Try this rule with a single black cell in the center.

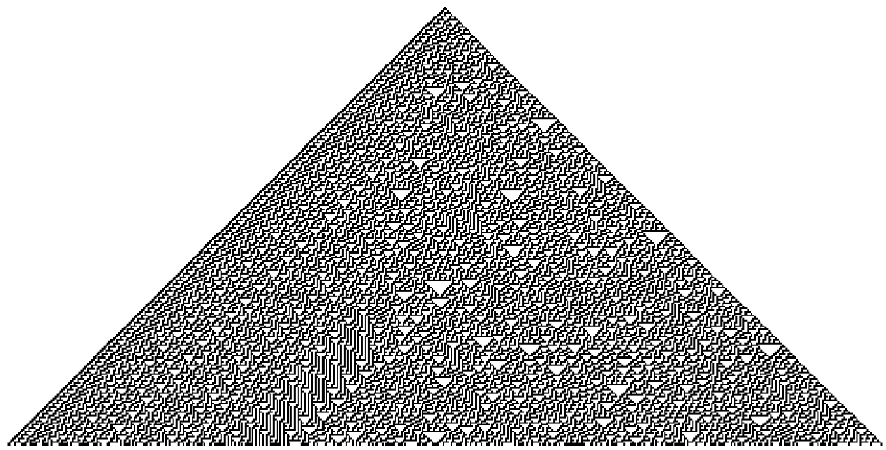


00011110 in binary = 30

Rule 30



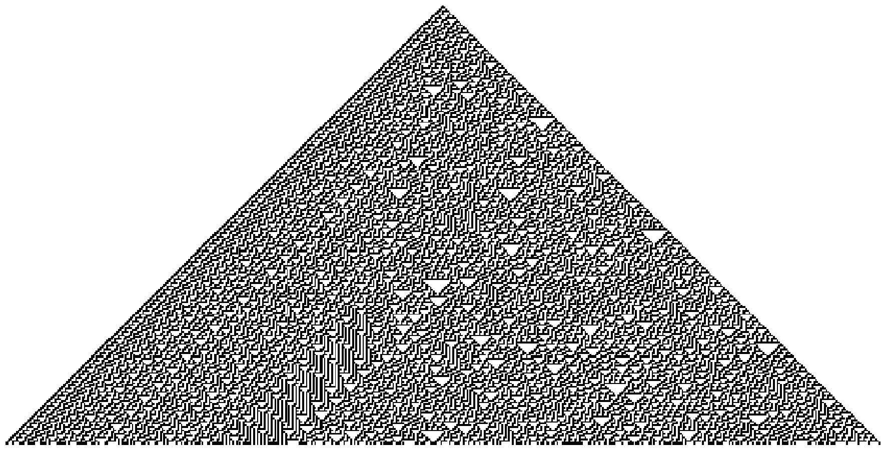
Rule 30



Rule 30

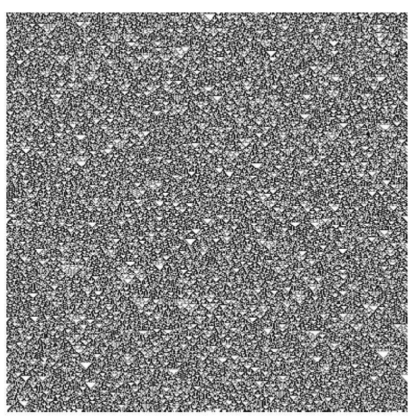
- Both look very random.
- Where does the randomness come from?
- Read off the sequence down the middle column. Can you find a pattern?
  11011100110001011001001110101110011101010110000110
- Rule 30 exhibits pseudo-randomness.
- Generated by an algorithm, but the output appears random to us.

Rule 30



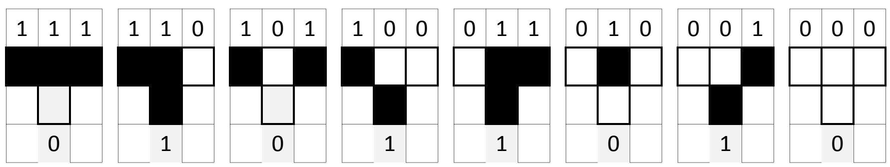
Rule 90

- Results starting with a random initial phase



Rule 90

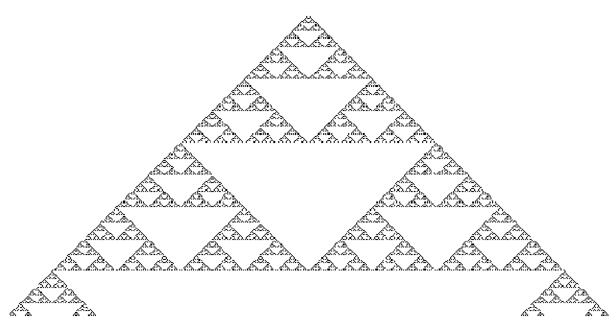
- How would you describe this rule?
- Try this rule using a random initial phase.
- Try this rule with a single black cell in the center.



01011010 in binary = 90

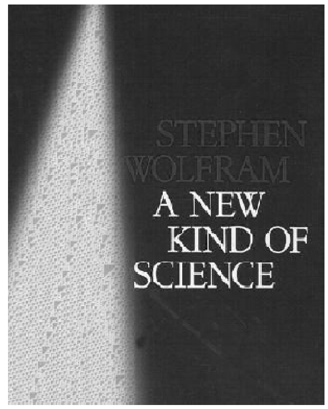
Rule 90

- Results starting with a single cell in the center of the first phase



Cellular Automata

- For more information:



Game of Life

- An infinite two-dimensional cellular automaton devised by the mathematician John Horton Conway.
- The automaton consists of an infinite two-dimensional orthogonal grid of square cells, each of which is in one of two possible states, alive (■) or dead (□).
- Every cell interacts with its eight neighbors, which are the cells that are horizontally, vertically, or diagonally adjacent.

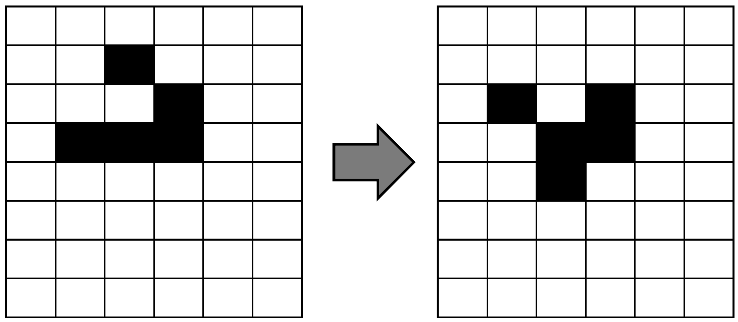
Game of Life: Rules

- At each step in time, the following transitions occur:
  - Any live cell with fewer than two live neighbors dies, as if caused by under-population.
  - Any live cell with two or three live neighbors lives on to the next generation.
  - Any live cell with more than three live neighbors dies, as if by overcrowding.
  - Any dead cell with exactly three live neighbors becomes a live cell, as if by reproduction.

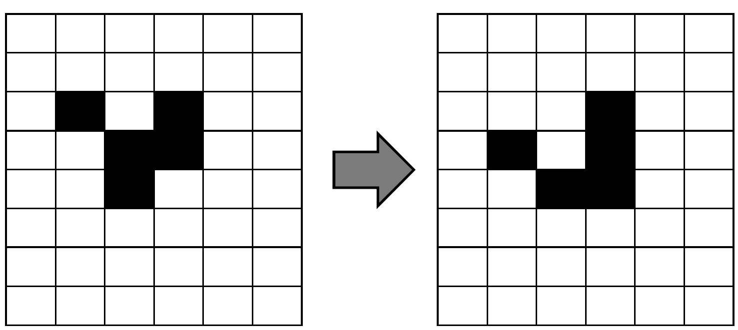
Generations

- The initial pattern constitutes the *seed* of the system.
- The first generation is created by applying the above rules simultaneously to every cell in the seed—births and deaths occur simultaneously, and the discrete moment at which this happens is sometimes called a *tick*.
- The rules continue to be applied repeatedly to create further generations.

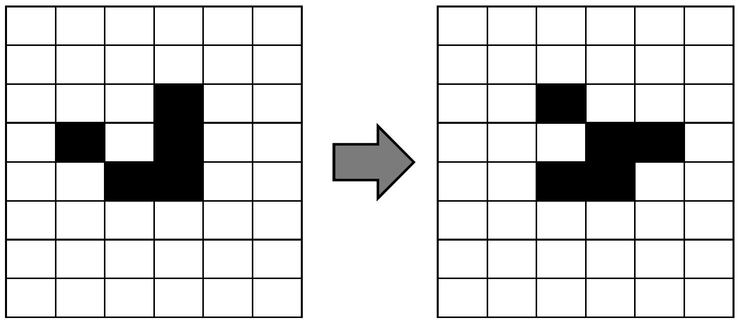
Example: Generation 1



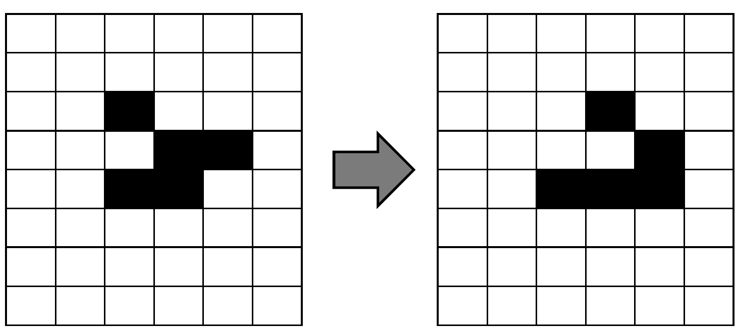
Example: Generation 2



Example: Generation 3



Example: Generation 4



Look familiar?

Game of Life and Randomness

- It was observed early on in the study of the Game of Life that random starting states all seem to stabilize eventually.
- Conway offered a prize for any example of patterns that grow forever. Conway's prize was collected soon after its announcement, when two different ways were discovered for designing a pattern that grows forever.
  (from www.math.com)