## 15110 PRINCIPLES OF COMPUTING – EXAM 2A – FALL 2011

Name _____ Section _____

*Directions: Answer each question neatly in the space provided. Please read each question carefully. You have 50 minutes for this exam. No electronic devices allowed. Good luck!*

1 _____

2 _____

3 _____

4 _____

5 _____

TOTAL _____

1. (20 pts) This problem focuses on the representation of data in a computer. Here is a table with powers of two that you may find helpful during the exam:

| $2^{10}$ | $2^9$ | $2^8$ | $2^7$ | $2^6$ | $2^5$ | $2^4$ | $2^3$ | $2^2$ | $2^1$ | $2^0$ |
|---|---|---|---|---|---|---|---|---|---|---|
| 1024 | 512 | 256 | 128 | 64 | 32 | 16 | 8 | 4 | 2 | 1 |

(a) (2 pts) Show how the decimal integer 68 would be stored as an unsigned integer in binary using 8 bits.                    _____

(b) (3 pts) Show how the decimal integer -68 would be stored as a signed 2's complement integer in binary using 8 bits.              _____

(c) (5 pts) We received the following transmission sent using the principle of parity. Unfortunately, one of the bits was corrupted during transmission. Circle the bit that was transmitted in error and briefly explain why it is wrong.

```
1 0 1 1 0 1 1 0
1 1 1 0 0 1 1 0
1 0 1 1 1 1 1 1
0 0 0 0 1 0 0 0
1 1 1 0 0 0 0 0
1 0 0 1 1 0 1 0
0 1 1 1 1 1 1 1
0 0 1 1 1 1 0 1
```

(d) (2 pts) To improve digital sound quality, which of the following should be done? Check the best answer.

_____ increase the number of samples per second and decrease the number of bits per sample

_____ increase the number of samples per second and increase the number of bits per sample

_____ decrease the number of samples per second and decrease the number of bits per sample

_____ decrease the number of samples per second and increase the number of bits per sample

(e) (3 pts)  Consider a pixel represented by an array of three values [ R, G, B ]. Show how the RGB pixel [127, 65, 200] in decimal format would be represented in hexadecimal format. (To help you, we've given you a table of each hexadecimal symbol and its equivalent value in binary.)

| Bin | 0000 | 0001 | 0010 | 0011 | 0100 | 0101 | 0110 | 0111 | 1000 | 1001 | 1010 | 1011 | 1100 | 1101 | 1110 | 1111 |
|-----|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|
| Hex | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | A | B | C | D | E | F |

[127, 65, 200] = [ _____, _____, _____]

(hexadecimal values)

(f) (5 pts) An image is simply an array of arrays of arrays of RGB values. For example, here is a 3 X 2 image consisting of a red and green pixel in the top row, a blue and white pixel in the middle row, and two black pixels in the bottom row:

```
[ [ [255,0,0] , [0,255,0] ],[ [0,0,255] , [255,255,255] ],[ [0,0,0] , [0,0,0] ] ]
```

Complete the following Ruby function that takes an arbitrary image represented in this format and removes all of the blue from every pixel of the image.

```
def remove_blue(image)

  num_rows = _____

  num_columns = _____

  for row in 0..num_rows-1 do

    for column in 0..num_columns-1 do




    end

  end

  return nil

end
```

2. (20 pts) This problem focuses on the structuring of data in a computer.

(a) (2 pts) A linked list of integers is stored as follows in a computer:

| Address | Data | Next |
|---------|------|------|
| 100 | 0 | 108 |
| 104 | 116 | 112 |
| 108 | 104 | 0 |
| 112 | 108 | 116 |
| 116 | 112 | 100 |
| 120 | | |

The linked list starts at address 104. What is the third data value of the list?        _____

(b) (3 pts) Suppose we wish to append the data value 200 to the end of the linked list starting at address 104. The memory at address 120 is available. Modify the table above to show how this can be done in the computer, modifying only those entries in the computer's memory that would need to be changed to append the new value to the linked list.

(c) (4 pts) Let data = [1, 5, 2, 4, 3]. What is returned from the following function?

```ruby
def mystery(data)
      stack = []
      for i in 0..data.length-1 do
            stack.push(data[i])
      end
      result = []
      while (stack.length != 0) do
            result << stack.pop
      end
      return result
end
```

result = _____

(d) (4 pts) Recall that the function `slice!(index)` removes and returns the element at the given index from an array. For example, if `data = [1,5,2,4,3]` and we execute `data.slice!(3)`, we will get the value `4` and the `data` array will now be `[1,5,2,3]`.

Suppose we implement a queue using an array such that the first element in the array is the front of the queue and the last element of the array is the back (rear) of the queue. Show in Ruby how you can implement the following queue operations on an array called `q`:

enqueue element x          _____

dequeue          _____

(e) (3 pts) An amusement park has 5 rides, all connected with walking paths as shown below. The numbers on each path indicate the cost to build that path in thousands of dollars.
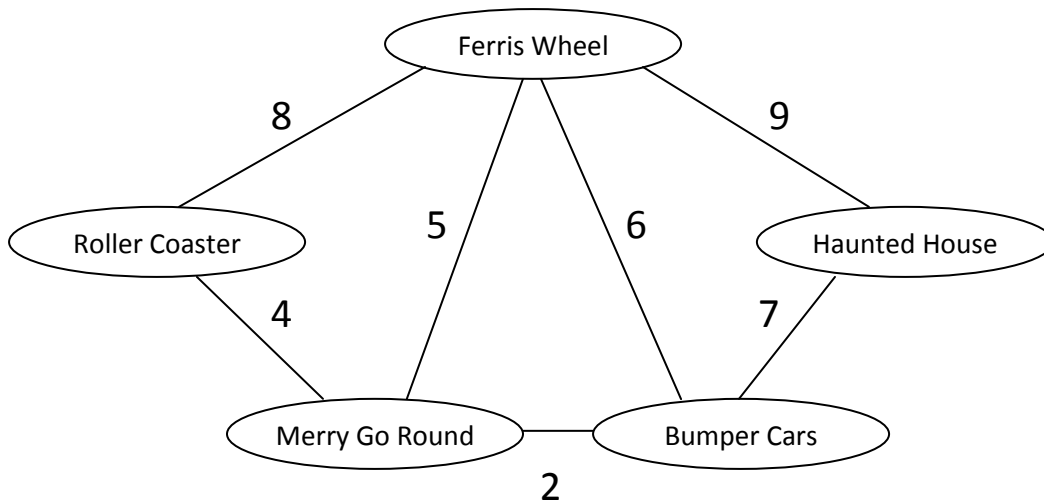


Show how this graph can be represented as a 2-dimensional array. If there is no direct connection between two different rides, set its cost to infinity (∞).

| | Ride |
|---|---|
| 0 | Roller Coaster |
| 1 | Merry Go Round |
| 2 | Bumper Cars |
| 3 | Haunted House |
| 4 | Ferris Wheel |

| | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| 0 | 0 | | | | |
| 1 | | 0 | | | |
| 2 | | | 0 | | |
| 3 | | | | 0 | |
| 4 | | | | | 0 |

(f) (2 pts) TRUE OR FALSE: The graph in part (e) is also a minimum spanning tree. _____

(g) (2 pts) A hash table of size 10 is used to store integer keys. The hash table is implemented as an array of "buckets" with a hash function of h(key) = key modulo 10. A collision occurs when an element is stored in a bucket that already has 1 or more elements. If the following integer keys are stored in the hash table, how many collisions occur?

```
18, 29, 34, 27, 98, 40, 78, 64, 21, 96, 48
```

| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | |

Number of collisions = _____

3. (20 pts)The following question deals with recursion.

(a) (6 pts) The function $f$ is defined recursively as follows:

$$f(n) = \begin{cases} n & if\ n = 1\ or\ n = 2 \\ n + f(n-1) + f(n-2) & if\ n > 2 \end{cases}$$

Compute f(6). You must show your work for full credit.

(b) (4 pts) Suppose you want to determine if any particular value is in a sorted array of 1023 elements. How many elements will you have to examine in the worst case if you use binary search? Briefly explain your answer. (HINT: 1023 = $2^{10} - 1$.)

(c) (4 pts) Here is an improved version of the `merge` function for the recursive Merge Sort algorithm, written in Ruby. Fill in the missing conditions. (The expression a + b is the array formed by the elements of a followed by the elements of b.)

```ruby
def merge(a, b)

    return a + b if _____

    return b + a if _____

    index_a = 0
    index_b = 0
    c = []
    while index_a < a.length and index_b < b.length do
        if a[index_a] <= b[index_b] then
            c << a[index_a]
            index_a = index_a + 1
        else
            c << b[index_b]
            index_b = index_b + 1
        end
    end
    if index_a < a.length then
        for i in (index_a..a.length-1) do
            c << a[i]
        end
    else
        for i in (index_b..b.length-1) do
            c << b[i]
        end
    end
    return c
end
```

(d) (6 pts) Complete the following Ruby function <u>recursively</u> so that it computes the sum of the first n positive integers. You may assume that n is greater than or equal to 1. Do not use a loop in your answer. HINT: 1 + 2 + … + n-1 + n = (1 + 2 + … + n-1) + n.

```ruby
def sum(n)

    if _____ then

        return _____

    else

        return _____

    end
end
```

4. (20 pts) The following question involves Boolean logic and computer organization.

(a) (12 pts) Let S = (A ∨ B) ∧ ((¬B) ∨ C), where A, B and C are Boolean variables. Fill in the truth table below to compute S.

| A | B | C | A ∨ B | (¬B) ∨ C | S |
|---|---|---|---|---|---|
| 0 | 0 | 0 | | | |
| 0 | 0 | 1 | | | |
| 0 | 1 | 0 | | | |
| 0 | 1 | 1 | | | |
| 1 | 0 | 0 | | | |
| 1 | 0 | 1 | | | |
| 1 | 1 | 0 | | | |
| 1 | 1 | 1 | | | |

(b) (4 pts) The Boolean value S can be computed by an electronic circuit. Draw this circuit at the gate level of abstraction.

(c) (4 pts) Use DeMorgan's Law to write an equivalent logical expression in Ruby for the given expression.
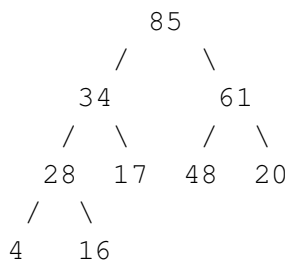
```
not(x >= 4 and y != 8)    ≡  _____
```

5. (20 pts) This question deals with binary trees.

(a) (4 pts) Draw the binary search tree that results by inserting the following integers into the tree in the order shown.

```
35   68   94   20   76   14   19   54
```

(b) (4 pts) Given the following max-heap, redraw the max-heap after the integer 59 is inserted

```
        85
      /     \
    34        61
   /  \      /  \
  28  17   48   20
 /  \
4    16
```

(c) (4 pts) Suppose you have n elements in a max-heap. What is the worst-case order of complexity using big O notation for each of the following operations?

Inserting a new element _____         Removing the maximum element _____

Finding the maximum element _____         Finding the minimum element _____

(d) (8 pts) Consider the Huffman tree shown for words in a language with 7 possible letters:

Which letter occurs most frequently in words
of this language based on the tree?         _____

If we used this Huffman tree, how many bits
would be needed to encode the word RSSERN?         _____

If we used a minimum fixed width coding for the
7 letters, how many bits would be needed to
encode the alien word RSSERN?         _____

TRUE OR FALSE: For any Huffman tree, each node
must have exactly 2 children or must be a leaf.         _____